

Summary of Methods for Solving $ax^2 + bx + c = 0$

Method	Lesson	Comments
Finding Square Roots	9.2	Efficient way to solve $ax^2 + c = 0$.
Graphing	9.3	Can be used for <i>any</i> quadratic equation, but gives only approximate solutions.
Using Quadratic Formula	9.4	Can be used for <i>any</i> quadratic equation. Always gives exact solutions.
Factoring	10.5	Efficient way to solve equation <i>if</i> quadratic can be factored easily.
Completing the Square	10.6	Can be used for <i>any</i> quadratic equation, but is best suited for quadratics with $a = 1$ and b an even number.

$$(46.) \quad 4x^2 - 13x + 3 = 0$$

$$(4x-1)(x-3) = 0$$

$$\begin{array}{l} \swarrow \quad \searrow \\ 4x-1=0 \quad x-3=0 \\ \Rightarrow 4x=1 \quad \Rightarrow x=3 \\ \Rightarrow x=\frac{1}{4} \text{ or } \Rightarrow x=3 \end{array}$$

Solve by Square Root method

$$-\frac{2}{3}x^2 + 9 = -22$$

$$\text{Sub} \left(-\frac{2}{3}x^2 \quad = -31 \right) -\frac{3}{2}$$

$$\sqrt{x^2} = \sqrt{\frac{93}{2}}$$

$$x = \pm \sqrt{\frac{93}{2}} \approx \pm 6.82$$

Suppose you drop a marshmallow
from the top of the John Hancock
building, then how long will it
take to smush on the ground?
($h = -16t^2 + 790$)

$$\begin{aligned}
 0 &= -16t^2 + 790 \\
 -790 & \quad -790 \\
 \hline
 -790 &= -16t^2 \\
 \frac{-790}{-16} & \quad \frac{-16t^2}{-16} \\
 \hline
 \sqrt{\frac{790}{16}} &= \sqrt{t^2} \\
 \pm \sqrt{\frac{790}{16}} &= t \approx 7.03 \text{ Seconds}
 \end{aligned}$$

20. $1 + x - x^2 = 0$

Solve by Completing the square

$$-1(-x^2 + x + 1 = 0) -1$$

$$x^2 - x - 1 = 0$$

$$\frac{x^2 - x}{+1 \quad +1} = 1$$

$$x^2 - x + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 = 1 + \frac{1}{4}$$

$$\sqrt{\left(x - \frac{1}{2}\right)^2} = \sqrt{\frac{5}{4}}$$

$$x - \frac{1}{2} = \pm \sqrt{\frac{5}{4}}$$

$$x - \frac{1}{2} = \pm \sqrt{\frac{5}{4}}$$

$$\frac{+1}{2} \quad \frac{+1}{2}$$

$$x = \frac{1}{2} \pm \sqrt{\frac{5}{4}}$$

$$x \approx 1.62 \quad x \approx -0.62$$

Solve by Square Root method

$$-\frac{2}{3}x^2 + 4 = -9$$

$$-\frac{2}{3}x^2 + 13 = 0$$

$$\Rightarrow \left(-\frac{2}{3}x^2\right)^{-\frac{3}{2}} = -13^{-\frac{3}{2}}$$

$$\sqrt{x^2}$$

$$= -13\left(-\frac{3}{2}\right)$$

$$x = \pm \sqrt{\frac{39}{2}} \approx \pm 4.42$$

If you drop your new iPhone 5
from the top of the Eiffel Tower,
1454 feet, then how long will it
take to smash on the ground?
($h = -16t^2 + s$)

$$\begin{aligned}
 -16t^2 + 1454 &= 0 \\
 \Rightarrow \quad \frac{1454}{16} &= \frac{16t^2}{16} \\
 \frac{1454}{16} &= t^2
 \end{aligned}$$

$$\sqrt{t^2} = \sqrt{\frac{1454}{16}}$$

$$t = \pm \sqrt{\frac{1454}{16}}$$

$$\approx \pm 9.53$$

$$\approx 9.53 \text{ seconds}$$

How are these two equations
related? $y = 3x^2 + 4x - 5$
and $0 = 3x^2 + 4x - 5$

Solve by factoring

$$3x^2 + 8x + 9 = 4$$

$$\Rightarrow 3x^2 + 8x + 5 = 0$$

$$(3x + 5)(x + 1) = 0$$

or

$$\begin{array}{l} \downarrow \\ 3x + 5 = 0 \\ \Rightarrow 3x = -5 \\ \Rightarrow x = -\frac{5}{3} \end{array} \quad \begin{array}{l} \downarrow \\ x + 1 = 0 \\ \Rightarrow x = -1 \end{array}$$

40. $4x^2 - 13x + 3 = 0$

$$\begin{array}{r} -3 \quad -3 \\ \hline 4x^2 - 13x = -3 \\ \hline 4 \quad 4 \quad 4 \\ \hline x^2 - \frac{13}{4}x = -\frac{3}{4} \end{array}$$

$$\begin{array}{r} \left(-\frac{13}{8}\right)^2 \quad \left(-\frac{13}{8}\right)^2 \\ \hline \left(x - \frac{13}{8}\right)^2 = -\frac{3}{4} + \left(-\frac{13}{8}\right)^2 \\ \left(x - \frac{13}{8}\right)^2 = \frac{121}{64} \end{array}$$

$$\sqrt{\left(x - \frac{13}{8}\right)^2} = \sqrt{\frac{121}{64}}$$

$$\begin{array}{r} x - \frac{13}{8} = \pm \frac{11}{8} \\ +\frac{13}{8} \quad +\frac{13}{8} \\ \hline x = \frac{13}{8} \pm \frac{11}{8} \end{array}$$

$$\begin{array}{l} \swarrow \quad \searrow \\ \frac{24}{8} \text{ (3)} \quad \frac{2}{8} \text{ (}\frac{1}{4}\text{)} \end{array}$$

32. $4x^2 - 25 = 0$

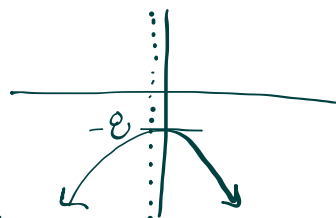
$$\begin{array}{r} +25 +25 \\ \hline 4x^2 = 25 \\ \hline \frac{4}{4} \quad \frac{25}{4} \\ \hline \sqrt{x^2} = \sqrt{\frac{25}{4}} \end{array}$$

$$\sqrt{25} = \sqrt{5} \sqrt{5} \\ = 2\sqrt{5}$$

$$x = \pm \sqrt{\frac{25}{4}} = \pm \frac{\sqrt{25}}{\sqrt{4}} = \pm \frac{5}{2}$$

$$y = -2x^2 - 3x - 8$$

Does it open
up or down? Explain.



Open down, because $a < 0$

vertex? $x = \frac{-b}{2a} = \frac{3}{2(-2)} = -\frac{3}{4}$

$$y = -2\left(-\frac{3}{4}\right)^2 - 3\left(-\frac{3}{4}\right) - 8 = -6.875$$

$$(-0.75, -6.875)$$

en.wikipedia.org/wiki/Empire_State_Building

The **Empire State Building** is a 102-story skyscraper located in Midtown ... meters), and with its antenna spire included, it stands a total of 1,454 ft (443.2 m) high.

How long would it take for your
new iPhone 5 to smash on the ground?
($h = -16t^2 + s$)

$$0 = -16t^2 + 1454$$

$$\Rightarrow -1454 = -16t^2$$

$$\Rightarrow \frac{-1454}{-16} = t^2$$

$$\Rightarrow t \approx 9.53 \text{ seconds}$$

longer "complete the square and derive the quadratic formula" way

$$\begin{aligned}
 & \downarrow \\
 & ax^2 + bx + c = 0 \\
 & x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \\
 & \downarrow \\
 & x^2 + \frac{b}{a}x = -\frac{c}{a} \\
 & \downarrow \\
 & x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 \\
 & \downarrow \\
 & \left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2} \\
 & \downarrow \\
 & \left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2} \\
 & \left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2} \\
 & \downarrow \\
 & \sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{-4ac + b^2}{4a^2}} \\
 & \downarrow \\
 & x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \\
 & x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}} \\
 & x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\
 & x = \pm \frac{\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a} \\
 & x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
 \end{aligned}$$



First I divided both sides of the equation by the coefficient of x^2 . Then I subtracted the constant term from both sides.

I completed the square by adding $\left(\frac{b}{2a}\right)^2$ to both sides of the equation.

I simplified on both sides.

I simplified further on the right-hand side. I found a common denominator for the two fractions on the right, then added them together.

I took the square root on either side..

I simplified and solved for x .

Bonus
I will black out a few lines to this & ask you to fill them in. :D