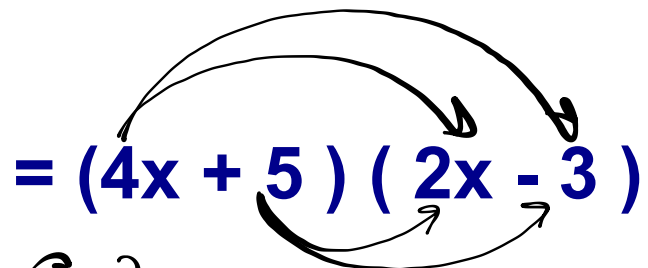


Factor

$$8x^2 - 2x - 15$$

$$= (4x + 5) (2x - 3)$$


The diagram shows two curved arrows above the equation. One arrow starts from the coefficient 8 of $8x^2$ and points to the coefficient 4 of $(4x + 5)$. The other arrow starts from the constant term -15 and points to the constant term -3 of $(2x - 3)$. Below the equation, there are two more curved arrows. One starts from the coefficient 4 of $(4x + 5)$ and points to the coefficient 2 of $(2x - 3)$. The other starts from the coefficient 2 of $(2x - 3)$ and points to the coefficient 4 of $(4x + 5)$, illustrating the cross-multiplication check.

$$8x^2 - 12x + 10x - 15$$

$$8x^2 - 2x - 15$$

Investigation

4

What Is a Quadratic Function?

When you jump from a diving board, gravity pulls you toward Earth. When you throw or kick a ball into the air, gravity brings it back down. For several hundred years, scientists have used mathematical models to describe and predict the effect of gravity on the position, velocity, and acceleration of falling objects.

Did You Know?

Did You Know?

Aristotle, the ancient Greek philosopher and scientist, believed that heavier objects fall faster than lighter objects. In the late 1500s, the great Italian scientist Galileo challenged this idea.

It is said that, while observing a hailstorm, Galileo noticed that large and small hailstones hit the ground at the same time. If Aristotle were correct, this would happen only if the larger stones dropped from a higher point or if the smaller stones started falling first. Galileo didn't think either of these explanations was probable.

A famous story claims that Galileo proved that heavy and light objects fall at the same rate by climbing to the highest point he could find—the top of the Tower of Pisa—and dropping two objects simultaneously. Although they had different weights, the objects hit the ground at the same time.



4.1**Tracking a Ball**

No matter how hard you throw or kick a ball into the air, gravity returns it to Earth. In this problem, you will explore how the height of a thrown ball changes over time.

**Problem 4.1****Interpreting a Table and an Equation**

Height of Thrown Ball

Problem 4.1 and an Equation

Suppose you throw a ball straight up in the air. This table shows how the height of the ball might change as it goes up and then returns to the ground.

- A.**
1. Describe how the height of the ball changes over this 4-second time period.
 2. Without actually making the graph, describe what the graph of these data would look like. Include as many important features as you can.
 3. Do you think these data represent a quadratic function? Explain.
- B.** The height h of the ball in feet after t seconds can be described by the equation $h = -16t^2 + 64t$.
1. Graph this equation on your calculator.
 2. Does the graph match the description you gave in Question A? Explain.
 3. When does the ball reach a height of about 58 feet? Explain.
 4. Use the equation to find the height of the ball after 1.6 seconds.
 5. When will the ball reach the ground? Explain.

Height of Thrown Ball

Time (seconds)	Height (feet)
0.00	0
0.25	15
0.50	28
0.75	39
1.00	48
1.25	55
1.50	60
1.75	63
2.00	64
2.25	63
2.50	60
2.75	55
3.00	48
3.25	39
3.50	28
3.75	15
4.00	0

Factor $8x^2 - 2x - 15$

$$= (4x + 5)(2x - 3)$$

$ab = 0$
 $\swarrow \searrow$
 $a=0 \quad b=0$

So now using the factored form above, can you now use the zero-product property to solve the quadratic equation below?

check

$$8\left(-\frac{5}{4}\right)^2 - 2\left(-\frac{5}{4}\right) - 15 = 0$$

$$0 = 0$$

$$8(1.5)^2 - 2(1.5) - 15 = 0$$

$$0 = 0$$

$$8x^2 - 2x - 15 = 0$$

$$(4x + 5)(2x - 3) = 0$$

\swarrow $4x + 5 = 0$ or \searrow $2x - 3 = 0$

$$\begin{array}{r} 4x + 5 = 0 \\ -5 \quad -5 \\ \hline 4x = -5 \\ \frac{4x}{4} = \frac{-5}{4} \\ x = -\frac{5}{4} \end{array}$$

or

$$\begin{array}{r} 2x - 3 = 0 \\ +3 \quad +3 \\ \hline 2x = 3 \\ \frac{2x}{2} = \frac{3}{2} \\ x = \frac{3}{2} \end{array}$$

ok

Factor

$$\textcircled{1} \quad 3x^2 + 7x + 2 = (3x + 1)(x + 2)$$

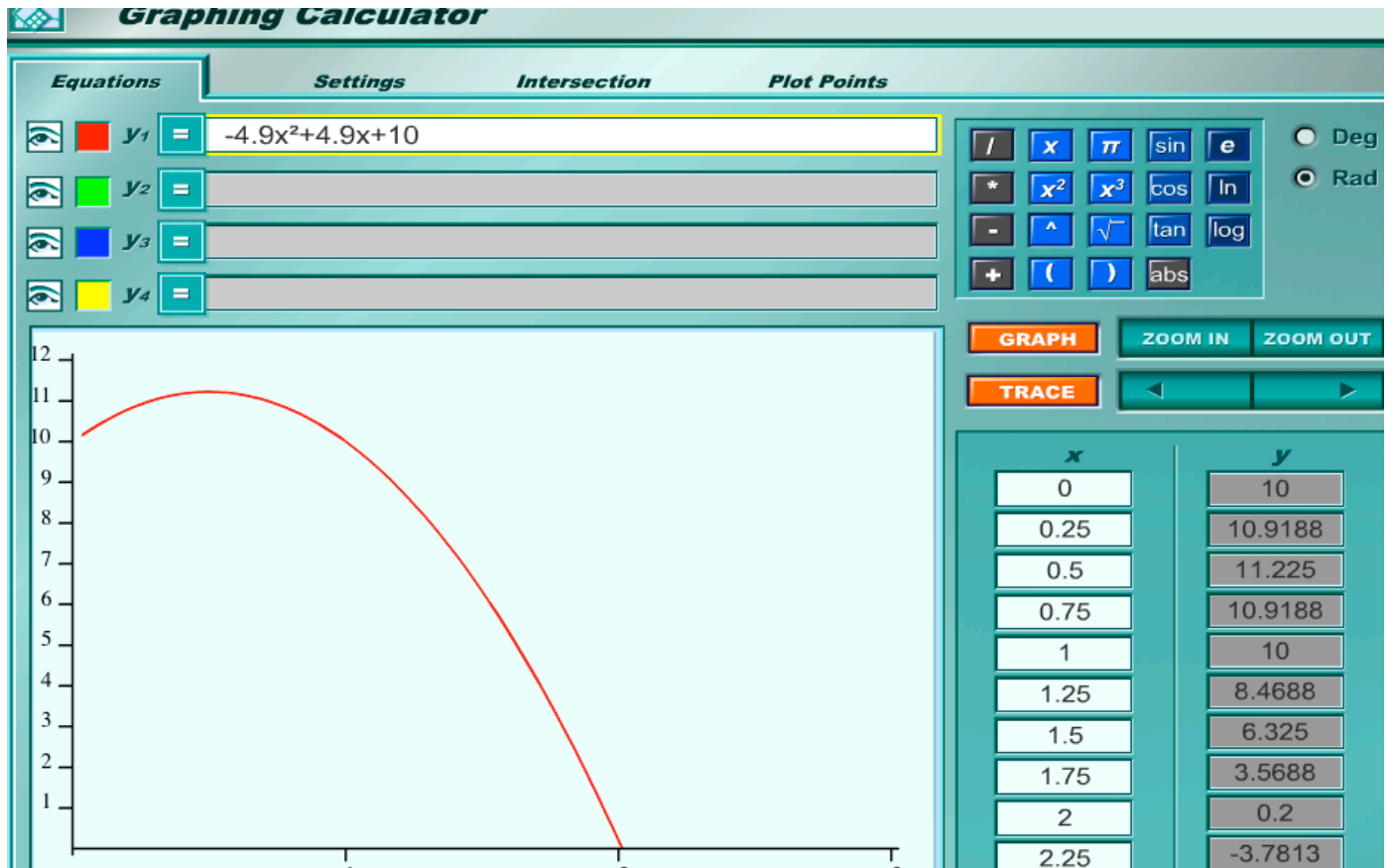
$3x^2 + 6x + 1x + 2$
 $3x^2 + 7x + 2$

$$\textcircled{2} \quad 2x^2 + 5x + 3 = (2x + 3)(x + 1)$$

$2x^2 + 2x + 3x + 3$
 $2x^2 + 5x + 3$

$$\textcircled{3} \quad 3x^2 - 16x + 5 = (3x - 1)(x - 5)$$

4. The highest dive in the Olympic Games is from a 10-meter platform. The height h in meters of a diver t seconds after leaving the platform can be estimated by the equation $h = 10 + 4.9t - 4.9t^2$.
- a. Make a table of the relationship between time and height.
 - b. Sketch a graph of the relationship between time and height.
 - c. When will the diver hit the water's surface? How can you find this answer by using your graph? How can you find this answer by using your table?
 - d. When will the diver be 5 meters above the water?
 - e. When is the diver falling at the fastest rate? How is this shown in the table? How is this shown in the graph?



In 1961, U.S. President John F. Kennedy had a challenge for NASA. The challenge was to land a man on the moon before the end of the decade (before 1970). The race to meet his goal would require the greatest technological achievement the world has ever seen. The first Apollo missions were spent getting ready for the moon landing. Apollo 8 and Apollo 10 even flew all the way to the moon, around it, and back to Earth. Finally, everything was ready. On July 16, 1969, Apollo 11 launched from Kennedy Space Center in Florida. They traveled to the moon and arrived in lunar orbit on July 19.



Image above: Astronauts Neil Armstrong, Michael Collins, and Buzz Aldrin. Credit: NASA

4.2 Measuring Jumps

Many animals are known for their jumping abilities. Most frogs can jump several times their body length. Fleas are tiny, but they can easily leap onto a dog or a cat. Some humans have amazing jumping ability as well. Many professional basketball players have vertical leaps of more than 3 feet!



Factor

$$\textcircled{1} \quad 7x^2 - 9x + 2 = (7x - 2)(x - 1)$$

$$\textcircled{2} \quad 6u^2 + 5u + 1 = (2u + 1)(3u + 1)$$

$$\textcircled{3} \quad 8u^2 - 9u + 1 = (8u - 1)(u - 1)$$

Problem 4.2 Comparing Quadratic Relationships

- A.** Suppose a frog, a flea, and a basketball player jump straight up. Their heights in feet after t seconds are modeled by these equations.

$$\text{Frog: } h = -16t^2 + 12t + 0.2$$

$$\text{Flea: } h = -16t^2 + 8t$$

$$\text{Basketball player: } h = -16t^2 + 16t + 6.5$$

1. Use your calculator to make tables and graphs of these three equations. Look at heights for time values between 0 seconds and 1 second. In your tables, use time intervals of 0.1 second.
2. What is the maximum height reached by each jumper? When is the maximum height reached?
3. How long does each jump last?
4. What do the constant terms 0.2 and 6.5 tell you about the frog and the basketball player? How is this information represented on the graph?
5. For each jumper, describe the pattern of change in the height over time. Explain how the pattern is reflected in the table and the graph.



- B.** A jewelry maker would like to increase his profit by raising the price

factor

$$\textcircled{1} 10x^2 + 17x + 3 \rightarrow (2x+3)(5x+1)$$

$$\textcircled{2} 9m^2 - 9m + 2 \rightarrow (3m-2)(3m-1)$$

$$\textcircled{3} 5j^2 + 14j + 6 \rightarrow (5j+6)(j+1)$$

$$\textcircled{1} 10x^2 + 17x + 3$$

guess and check

$$(5x + 3)(2x + 1)$$

$$~~10x^2 + 5x + 6x + 3~~$$

$$(5x + 1)(2x + 3)$$

$$10x^2 + 15x + 2x + 3$$

② $9m^2 - 9m + 2$

A rectangular box containing the factored expression $(3m-1)(3m-2)$. Four curved arrows illustrate the FOIL method: one from the first $3m$ to the first $3m$, one from the first $3m$ to the second -2 , one from the -1 to the first $3m$, and one from the -1 to the second -2 .

✓ $9m^2 - 6m - 3m + 2$

③ $5j^2 + 11j + 6$

A handwritten diagram showing the factoring of the quadratic expression $5j^2 + 11j + 6$. The expression is enclosed in a rectangular box. Inside the box, the factors $(5j + 6)$ and $(j + 1)$ are written. Arrows indicate the cross-multiplication process: a curved arrow from the 5 in the first binomial to the 1 in the second binomial, and another curved arrow from the 6 in the first binomial to the 1 in the second binomial. Below the box, the expanded form $5j^2 + 5j + 6j + 6$ is written, with a checkmark to its left.

✓ $5j^2 + 5j + 6j + 6$

Solving a quadratic equation using factoring
and the zero-product property

$$\textcircled{3} \quad 5j^2 + 14j + 6 = 0$$

$$(5j + 6)(j + 1) = 0$$

$$5j + 6 = 0 \quad \text{or}$$

$$j + 1 = 0$$

$$\frac{5j}{5} = \frac{-6}{5}$$

$$j = -\frac{6}{5}$$

$$-\frac{6}{5} \text{ or } -1$$

- B. A jewelry maker would like to increase his profit by raising the price of his jade earrings. However, he knows that if he raises the price too high, he won't sell as many earrings and his profit will decrease.

The jewelry maker's business consultant develops the equation $P = 50s - s^2$ to predict the monthly profit P for a sales price s .

1. Make a table and a graph for this equation.
2. What do the equation, table, and graph suggest about the relationship between sales price and profit?
3. What sales price will bring the greatest profit?
4. How does this equation compare with the equations in Question A? How does it compare with other equations in this unit?

ACB Homework starts on page 64.

$$y = ax^2 + bx + c$$

$$P = 50x - x^2$$

$$P = x(50 - x)$$

x-int

$$0 = x(50 - x)$$

$$x = 0$$

$$(0, 0)$$

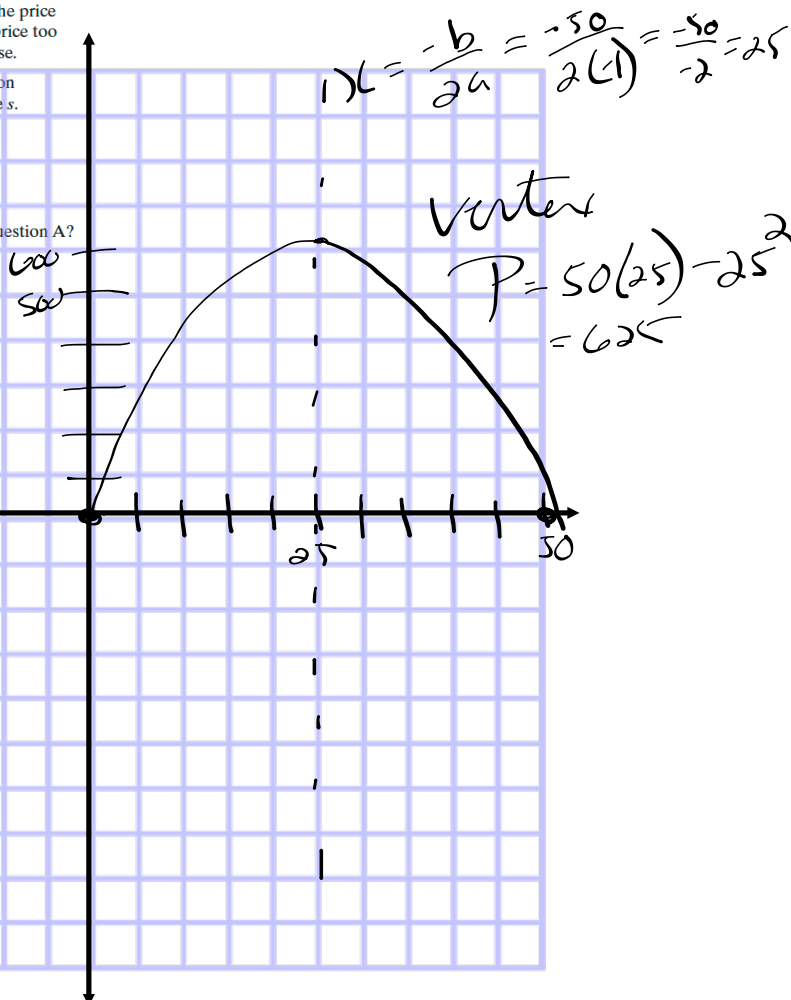
L.O.S

$$x = 25$$

$$50 - x = 0$$

$$x = 50$$

$$(50, 0)$$



$$y = ax^2 + bx + c$$

$$d = -16t^2 + 18t + 10$$

L.O.S

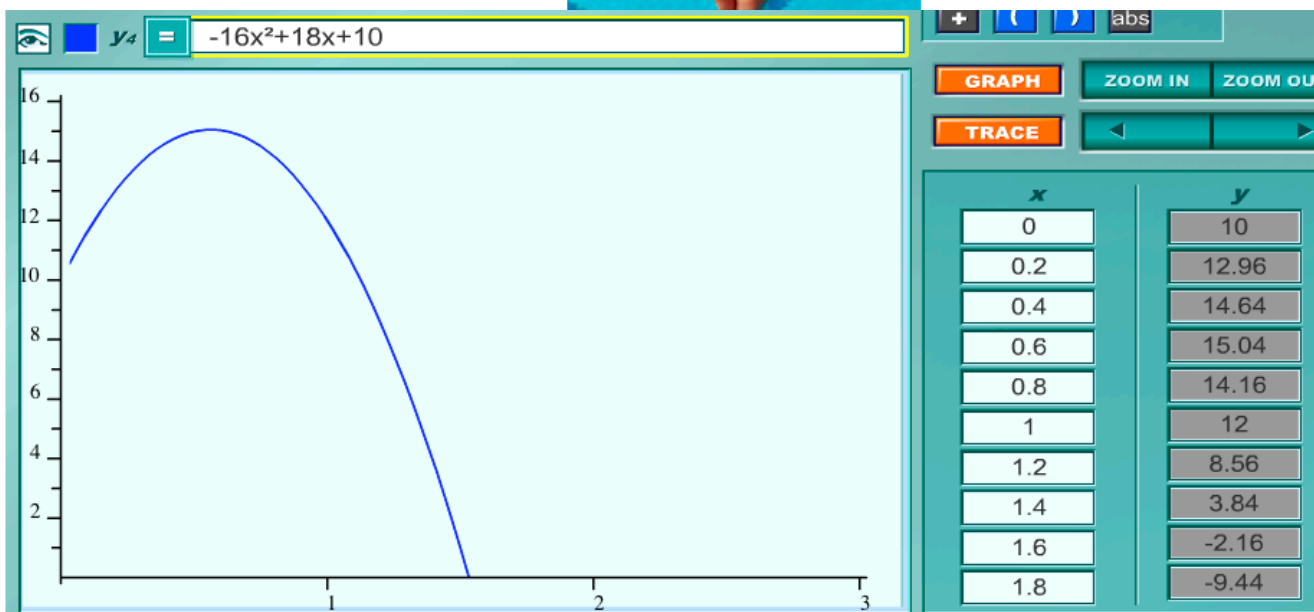
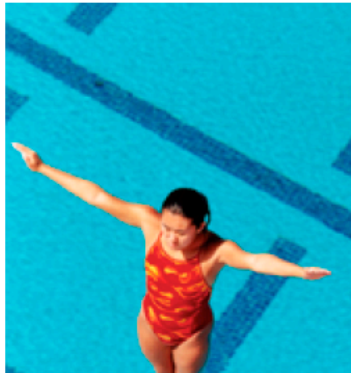
$$x = \frac{-b}{2a} = \frac{-18}{2(-16)} = \frac{-18}{-32} = \frac{9}{16}$$

Factor?

$$b^2 - 4ac = (18)^2 - 4(-16)(10)$$

$$\text{Can't factor} = 964$$

5. Kelsey jumps from a diving board, springing up into the air and then dropping feet-first. The distance d in feet from her feet to the pool's surface t seconds after she jumps is $d = -16t^2 + 18t + 10$.
- What is the maximum height of Kelsey's feet during this jump? When does the maximum height occur?
 - When do Kelsey's feet hit the water?
 - What does the constant term 10 in the equation tell you about Kelsey's jump?



5. Kelsey jumps from a diving board, springing up into the air and then dropping feet-first. The distance d in feet from her feet to the pool's surface t seconds after she jumps is $d = -16t^2 + 18t + 10$.

- a. What is the maximum height of Kelsey's feet during this jump? When does the maximum height occur?
- b. When do Kelsey's feet hit the water?
- c. What does the constant term 10 in the equation tell you about Kelsey's jump?

