

# Investigation 2

## ACE

### Assignment Choices



#### Problem 2.1

Core 1, 2, 3

Other Connections 35, 36

#### Problem 2.2

Core 5–17

Other Applications 4, 18, 19; Connections 37–42; Extensions 57–63; unassigned choices from previous problems

#### Problem 2.3

Core 20–24

Other Connections 43; unassigned choices from previous problems

#### Problem 2.4

Core 25–28, 33, 34, 44–52

Other Applications 29–32; Connections 53–56; unassigned choices from previous problems

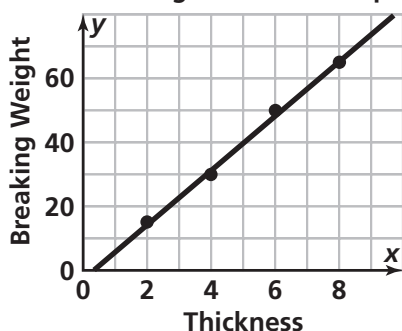
**Adapted** For suggestions about adapting ACE exercises, see the *CMP Special Needs Handbook*.

**Connecting to Prior Units** 35, 36: *Moving Straight Ahead*; 44–52: *Accentuate the Negative*; 43, 55: *Stretching and Shrinking*; 54: *Variables and Patterns*

## Applications

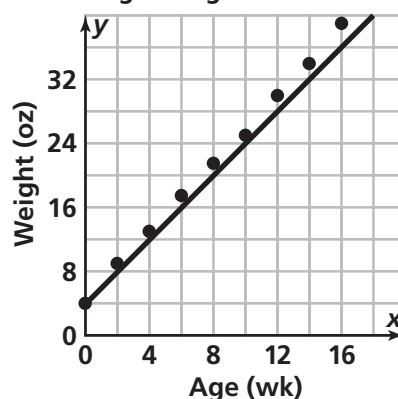
- a. Accept any line that approximates the data. Here is one possibility:

Arkansas Bridge–Thickness Experiment



- Possible answer:  $y = 8.5x - 2.5$ . Students might come up with a simpler model with a  $y$ -intercept of 0, such as  $y = 8x$  (because 0 thickness should suggest 0 strength).
  - Answers depend on the equation. Using the preceding equation, the breaking weights are 23, 40, and 57.
- Student 1's line is a better fit. Overall, the data points are closer to Student 1's line. Also, the slope of Student 1's line seems to better match the rate of change in the data points.
  - Lines and strategies will vary. Note that Labsheet 2ACE Exercise 3 includes Graphs A–C, plus six additional graphs. When you look at Graphs A, B, C, D, and F from left to right, you see a definite linear trend in the data; these data can be modeled with a straight line. The points in Graph E might represent a linear relationship, but they might be better modeled with a curve, especially toward the right part of the graph. The relationships in Graphs G, H, and I are definitely not linear. The points in G and H show nonlinear trend. The points in I do not seem to show any trend; they bounce all around.
  - a. Lines will vary. Here is one possibility:

Average Weight for Chihuahuas

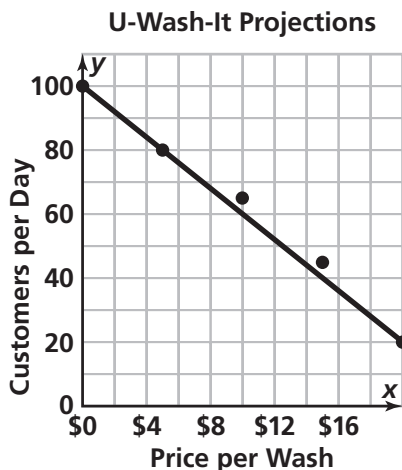


- b. Possible answer:  $y = 2.2x + 4$ . The value of  $m$  indicates that on average, a Chihuahua grows 2 oz per wk. To find this value, students may notice that as the age changes by 2 wk, the weight usually changes by about 4 oz. The value of  $b$  indicates that the weight of a Chihuahua at birth is 4 oz.
- c. The model  $y = 2.2x + 4$  gives the following estimates:

Age (wk)	Weight (oz)
1	6
3	10.5
5	15
7	19.5
9	23.5
11	28
13	32.5
15	37

- d. Answers depend on the model from part (b). The model  $y = 2.2x + 4$  predicts a weight of 321 oz or 20 lb for a 36-mo-old Chihuahua. In reality, a Chihuahua of this age is full grown and typically weighs only 4 lb. This error of prediction illustrates the danger of using a data-based model to make predictions far beyond the data on which the model was based.

5. a. Lines will vary. Possible answer:



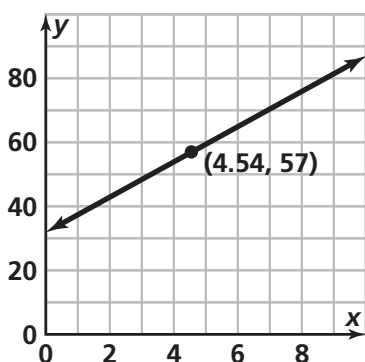
- b. Possible equation:  $y = 100 - 4x$ . The 100 means that if the car wash were free, there

would be about 100 customers per day.

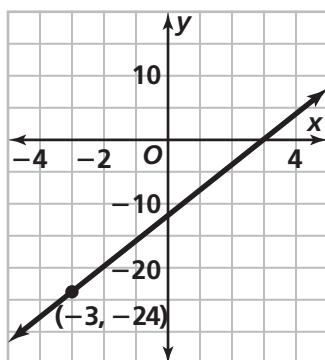
The  $-4$  means that for each \$1 increase in price, there would be a decrease of about 4 customers per day.

- c. \$2.50: 90; \$7.50: 70; \$12.50: 50
6. Note: Many students find parts (c) and (d) challenging. You may want to discuss these.
- a. Slope: 0.5, y-intercept: 3;  $y = 0.5x + 3$
- b. Slope:  $-3$ , y-intercept: 24;  $y = -3x + 24$
- c. Slope: 0, y-intercept: 9;  $y = 9$
- d. Slope: undefined, y-intercept: none;  $x = 6$ .
7. a.  $w = 8 + 1.5a$
- b. No. A person would weigh 1,448 pounds when he reaches 80, according to the model.
8.  $v = 20 - 0.15t$
9.  $d = 1500 - 60t$       10.  $d = 28g$
11. a.  $y = 2x + 2$       b.  $y = -4x + 20$
- c.  $y = 1.5x + 2$       d.  $y = -3x + 20$
12.  $y = 4.2x + 3.4$       13.  $y = \frac{2}{3}x + 5$
14.  $y = 2x + 4$
15.  $y = -\frac{12}{5}x + 15$  or  $y = -2.4x + 15$
16.  $y = -\frac{6}{7}x + \frac{2}{7}$       17.  $y = -2x + 6$
18.  $\ell_1: y = x + 2$ ;  $\ell_2: 0.5x - 1$ ;  $\ell_3: -1.5x + 3$
19. a. The graph that has y-intercept  $(0, 0)$  shows Anchee's earnings because her father does not pay her any money at the beginning of the summer. The graph that has y-intercept  $(0, 20)$  shows Jonah's earnings because he gets \$20 at the beginning of the summer. On the graph, we can see that Anchee's earnings increase by \$5 per week and Jonah's by \$3 per week.
- b. Anchee:  $y = 5x$   
Jonah:  $y = 20 + 3x$
- c. The value of  $m$  in each case tells the rate at which earnings increase per week. The value of  $b$  is the amount of money each student received at the beginning of the summer.
- d. The value of  $b$  is the y-coordinate of the point where the graph crosses the y-axis. That is, it is the y-intercept. The value of  $m$  is the slope of the line.

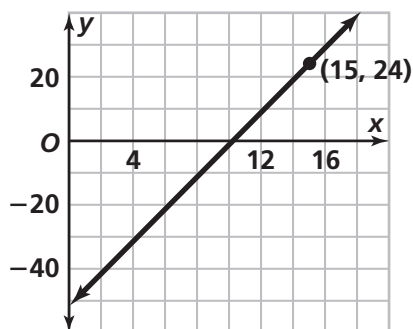
20.  $x \approx 4.54$



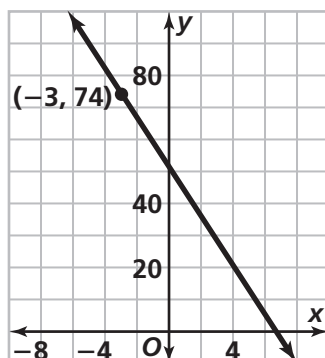
21.  $x = -3$



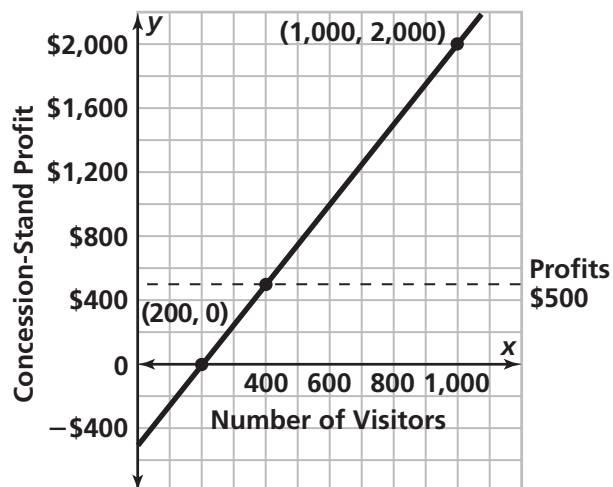
22.  $x = 15$



23.  $x = -3$

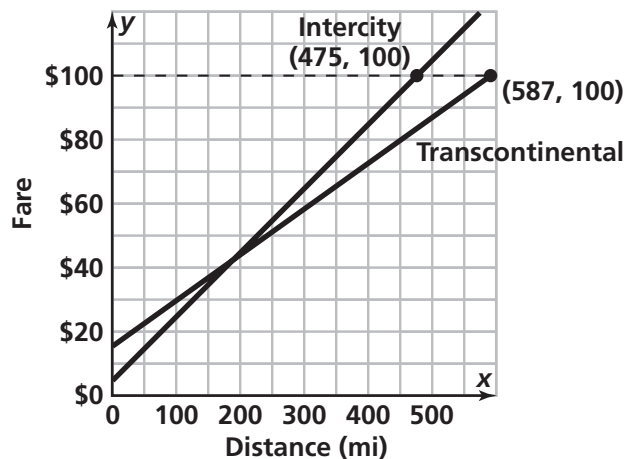


24.



- About 1,000 visitors;  $2,000 = 2.5v - 500$ , so  $v = 1,000$  visitors
- About \$0;  $p = 2.50(200) - 500 = 0$
- $v \geq 400$ ; on the graph, the profit is greater than \$500 for all points on or above the dashed line; solve  $2.50v - 500 \geq 500$ .

25.



- 580 mi;  $99 = 0.15d + 12$ , so  $d = 580$  mi
- 470 mi;  $99 = 5 + 0.20d$ , so  $d = 470$  mi
- Yes; about 140 mi and \$33;  
 $0.15d + 12 = 5 + 0.20d$ , so  $d = 140$

Students can use a table or a graphing calculator to help them solve the equations in Exercises 26–28.

26.  $x = -6$

$$5x + 7 = 3x - 5$$

$$5x = 3x - 12$$

$$2x = -12$$

$$x = -6$$

27.  $x = 6$

$$7 + 3x = 5x - 5$$

$$12 + 3x = 5x$$

$$12 = 2x$$

$$6 = x$$

28.  $x = -8$

$$2.5x - 8 = 5x + 12$$

$$2.5x - 20 = 5x$$

$$-20 = 2.5x$$

$$-8 = x$$

In Exercises 29–31, students should list three values in the given interval.

29.  $x \leq 3$

30.  $x < 6$

31.  $x \leq 2$

32.  $x \leq 9$

33. Possible answers given.

a.  $d = 15g$

b. 150 mi

c. About 16.7 gal

d. The value of  $m$  is the number of miles a bus travels on 1 gal of gasoline. The value of  $b$  is the number of miles covered on 0 gal of gas, which should be 0 mi.

34. Possible answers given.

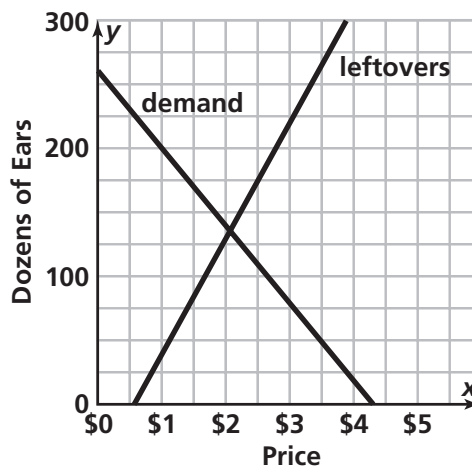
a. As price goes up, some customers will decide it is too expensive and will not buy corn.

b. As price goes up, fewer people will buy the corn, so the farmers have a larger supply of leftover corn.

c.  $d = 260 - 60p$

d.  $\ell = 85p - 45$

e. The graph follows. The leftovers and demand are equal at the point of intersection. This happens at a price of just over \$2. To find the answer symbolically, solve  $260 - 60p = 90p - 50$ . The solution is \$2.07.



**Note to the Teacher:** Because models are approximations, the price here is also an approximation. In the table, the leftovers and demand are not shown as equal for a price of \$2.

## Connections

35. a. Linear; as  $x$  increases by 2 units,  $y$  increases by 1 unit.

b. Not linear; as  $x$  increases in 1-unit steps,  $y$  increases by increasing amounts.

c. Linear; for every 1-unit increase in  $x$ ,  $y$  decreases by 1.

36. a. Linear; the rate of change is constant.

$x$	-5	-2	1	4
$y$	7	-2	-11	-20

b. Linear; the rate of change is constant.

$x$	-3	0	3	6
$y$	-34	-22	-10	2

- c. Not linear; the rate of change is not constant.

x	-3	0	3	6
y	21	0	33	120

- d. Linear; the rate of change is constant.

x	-3	0	3	10
y	13	4	-5	-26

37.  $-5 < 3$                       38.  $\frac{2}{3} > \frac{1}{2}$   
 39.  $\frac{9}{12} = \frac{3}{4}$                       40.  $3.009 < 3.1$   
 41.  $\frac{-2}{3} < \frac{-1}{2}$                       42.  $-4.25 < 2.45$   
 43.  $a = 1.5b$   
 44. -22                      45. -22                      46. 8  
 47. -8                      48. -4                      49. 4  
 50. -5                      51. 8                      52. 50  
 53. a.  $\frac{-4}{-2} = 2 = 200\%$  and  $1.5 = 150\%$ .

- b. 200% is the greatest; 60% is the least

54. a. Story 1 goes with Graph A. The first part of the graph shows height increasing rapidly as the plane ascends. The middle section shows height remaining constant as the plane holds its altitude. The last part shows height decreasing, rapidly at first and then more slowly, as the parachutist descends. The graph stops at a point just above the  $x$ -axis, where the parachutist got caught in the tree. The  $x$ -axis label could be "Time," and the  $y$ -axis label could be "Height."

Story 2 goes with Graph D. The graph shows the account balance increasing, slowly at first, and then more rapidly as time passes. The last section of the graph shows the balance dropping to half the amount, without any time passing, as half the money is removed. The  $x$ -axis label could be "Time," and the  $y$ -axis label could be "Account Balance."

Story 3 goes with Graph B. The graph shows the volume of the pile of gravel decreasing rapidly at first, and then less rapidly. Gerry removes half of the pile each day, but because the pile is smaller every day, the amount he removes is less and less every

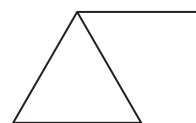
day. The  $x$ -axis label could be "Day," and the  $y$ -axis label could be "Volume of Gravel."

- b. Possible story: Graph C shows how a student travels during an afternoon. First he runs from his home to a friend's house at a constant speed, until he gets tired and walks. He meets his friend, and they travel back to his home in a car, gradually accelerating.

55. a. 2 in.                      b. 2                      c.  $\frac{1}{2}$   
 d. They are reciprocals.

## Extensions

56. a. \$24,000; \$42,000; \$78,000  
 b.  $B = (5,000 + 150L) + 0.20(5,000 + 150L)$   
 or  $B = 6,000 + 180L$   
 c. \$33,000, \$60,000, \$87,000  
 d.  $B = (5,000 + 150L) + 0.15(5,000 + 150L)$   
 or  $B = 5,750 + 172.5L$   
 57. a.  $r = \ell + (\ell - 1) + 2\ell$ ,  $r = 4(\ell - 1) + 3$ , and  $r = 4\ell - 1$   
 b. Possible explanations:  
 $r = \ell + (\ell - 1) + 2\ell$ : There are  $\ell$  rods along the bottom,  $\ell - 1$  rods along the top, and 2 additional diagonal rods for every foot.  
 $r = 4(\ell - 1) + 3$ : We start with 3 rods and then add 4 for each additional foot.  
 $r = 4\ell - 1$ : Look at each 1-foot segment, except the last, as a triangle with a rod extending from the top like the one below. The last foot does not require the top segment, so we need to subtract one.



58.  $r = n^2 + 3n$ ,  $r = n(n + 3)$ , and  $r = (n + 3)n$   
 59. 6                      60. D                      61. H  
 62. a. Sid's formula will not work because it does not account for the fact that the cost for the first 10 min is fixed at \$5.  
 b. Tina's formula works for any time beyond 10 min, but it does not work for times under 10 min.  
 c. Yes.



63. a. The Bluebird Taxi rule has two parts. For distances less than 2 mi,  $f = 1.50$ ; for distances of 2 mi or more,  $f = 1.2(d - 2) + 1.5$ , because  $d$  is the distance in mi and  $f$  is the fare.
- b. The parking charge rule needs two parts—one for times of 30 min or less and one for times greater than 30 min (or 0.5 hr). Furthermore, the way these charge schemes usually work, the charge for any time between 0.5 hr and 1.5 hr will be \$2. Then the charge for any time between 1.5 hr and 2.5 hr will be \$4, and so on. This is hard to express as a simple algebraic rule, but one could use  $p = 2(t - 0.5)$ , where  $p$  is price and  $t$  is time, and then round the result up to the nearest \$2. For example, if  $t = 4.7$ ,  $p = 2(4.7 - 0.5) = 2(4.2) = 8.4$ , which would round up to \$10.
- c.  $p = 6.50n - 750$ , where  $p$  is the profit in dollars and  $n$  is the number of tickets sold.
- d.  $c = 50 + 50t$ , where  $t$  is the repair time required and  $c$  is the cost for the repair.

## Possible Answers to Mathematical Reflections

- A model allows you to summarize the overall trend in the data with a simple line or equation, and it lets you predict data values between and beyond the data points.
- Figure out which variable is the independent variable and which is the dependent variable. Then find the rate of change in the dependent variable as the independent variable changes by a fixed amount. This is the slope  $m$ . Then you need to find the starting value, or the value of the dependent variable when the independent variable is 0. This is the  $y$ -intercept  $b$ . Plug the values of  $m$  and  $b$  into the equation  $y = mx + b$ .
  - Figure out the rate of change. This is the slope  $m$ . You can find the rate by dividing the difference in two  $y$ -values by the difference in the corresponding two  $x$ -values. Find the  $y$ -intercept  $b$ . If this is not a value in the table, use the rate of change to extend the table to include a value for which the independent variable is 0. Plug the values of  $m$  and  $b$  into the equation  $y = mx + b$ .
  - Pick any two points on the line and use their coordinates to calculate the slope  $m$ . You do this by finding the ratio of differences in  $y$ -values to differences in the corresponding  $x$ -values, or  $m = \frac{y_2 - y_1}{x_2 - x_1}$ . Estimate the  $y$ -intercept  $b$  of the graph. You can also find the value of  $b$  by working backward from a known point, using the slope, or you can use the coordinates of one known point  $(x_0, y_0)$  on the line and the slope you have calculated to solve  $y_0 = mx_0 + b$  for  $b$ .
- You can graph the equation for the associated line and use the graph to estimate the solution. For example, to solve  $500 = 245 + 5x$ , you graph  $y = 245 + 5x$ , and then find the point on the line with  $y$ -coordinate 500. The  $x$ -coordinate of that point is the solution.  
If you have a graphing calculator, you can enter the equation for the associated line and trace a table or graph to find the solution.  
To solve the equation symbolically, apply the same operation to both sides to get  $x$  by itself on one side. For example, to solve  $500 = 245 + 5x$ , subtract 245 from both sides to get  $255 = 5x$ , and then divide both sides by 5 to get  $51 = x$ .
  - You can graph the two associated equations and find the point where the graphs intersect. For example, to solve  $500 + 3x = 245 + 5x$ , graph  $y = 500 + 3x$  and  $y = 245 + 5x$ . The  $x$ -coordinate of the point of intersection is the solution.  
If you have a graphing calculator, you can enter the two equations and trace a table or graph to find the  $x$ -value of the point of intersection.  
To solve the equation symbolically, apply the same operation to both sides to get  $x$  by itself on one side. For example, to solve  $500 + 3x = 245 + 5x$ , subtract 245 from both sides to get  $255 + 3x = 5x$ , then subtract  $3x$  from both sides to get  $255 = 2x$ , and then divide both sides by 2 to get  $127.5 = x$ .