

Investigation 1

ACE

Assignment Choices



Problem 1.1

Core 1

Other Connections 7–15

Problem 1.2

Core 2

Other Connections 16–22; unassigned choices from previous problems

Problem 1.3

Core 3–6

Other Connections 23–31; Extensions 32–34; unassigned choices from previous problems

Adapted For suggestions about adapting ACE exercises, see the *CMP Special Needs Handbook*.
Connecting to Prior Units 7, 8, 11: *Comparing and Scaling*; 9: *Covering and Surrounding*; 10: *Covering and Surrounding, Bits and Pieces II, Comparing and Scaling*; 12–26: *Moving Straight Ahead*; 27–29: *Accentuate the Negative*; 30, 31: *Filling and Wrapping*

Applications

1. a. Bridge-Thickness Experiment

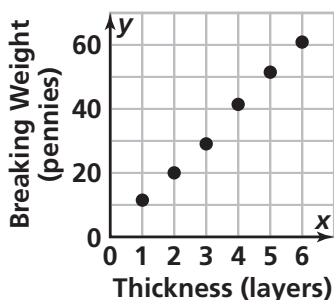


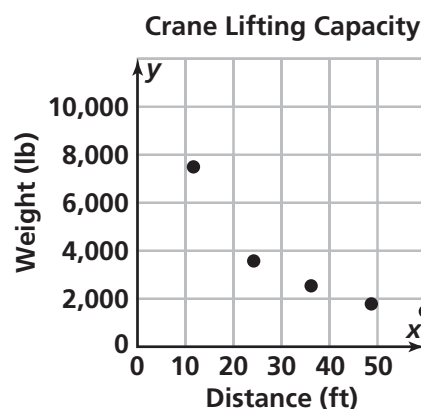
Figure 6

Costs of CSP Beams

Beam Length (ft)	1	2	3	4	5	6	7	8
Number of Rods	3	7	11	15	19	23	27	31
Cost of Beam	\$56.75	\$65.75	\$74.75	\$83.75	\$92.75	\$101.75	\$110.75	\$119.75

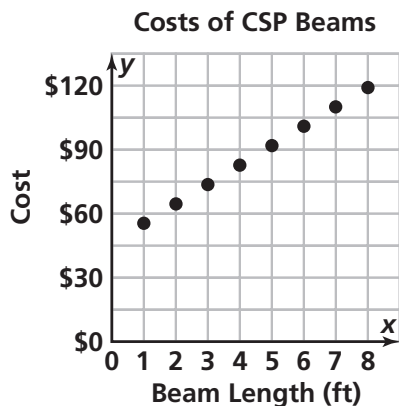
The data are very close to linear. Each time the class adds a layer, the bridge can hold approximately ten more pennies.

- Possible answer: 35 pennies. The breaking weight is about 10 pennies per layer. So, for 3.5 layers, it would be 35.
 - Possible answer: 80. The breaking weight is about 10 pennies per layer. So, for 8 layers, it would be about 80.
- As distance increases, weight decreases. The decrease is sharper at shorter distances. (The product of distance and weight is always 90,000.)
 - The graph shows that as distance increases, weight decreases—sharply at first, and then more gradually.



- $\approx 5,000$ lb; $\approx 3,000$ lb; $\approx 1,250$ lb
 - The graph's shape is similar to that for the bridge-length experiment because the values of the dependent variable decrease at a decreasing rate. (**Note:** You may want to revisit this problem after Problem 3.2 so that students can check their estimates.)
- (Figure 6)

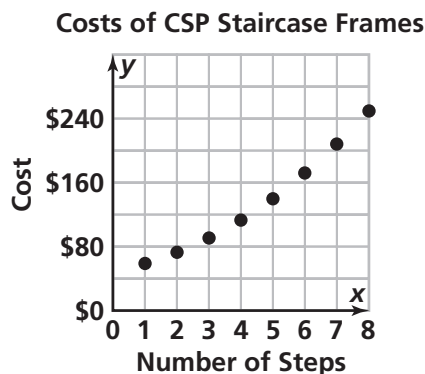
b.



c. This is a linear relationship. As beam length increases by 1 unit, cost increases by \$9.

d. (Figure 7)

e.



f. This is not a linear relationship. As the number of steps increases by 1, the cost increases at an increasing rate.

4. a. linear

b. not linear

c. linear

d. not linear

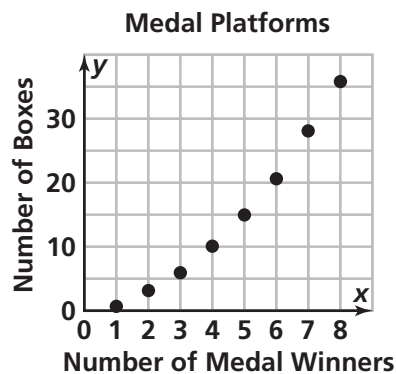
e. linear

f. nonlinear

g. The relationships in parts (b) and (f) are increasing, but at different rates. The relationship in part (d) is decreasing.

5. a. (Figure 8)

b.



c. This is not a linear relationship. In the table, when we add the second medal winner, we add 2 boxes. When we add a third medal winner, we add 3 more boxes. To add a 29th medal winner, we add 29 boxes to a 28-step platform. The change is increasing at each step. We see this in the graph because the graph rises more and more sharply as we move left to right along the x -axis.

Figure 7

Costs of CSP Staircase Frames

Number of Steps	1	2	3	4	5	6	7	8
Number of Rods	4	10	18	28	40	54	70	88
Cost of Frame	\$59	\$72.50	\$90.50	\$113	\$140	\$171.50	\$207.50	\$248

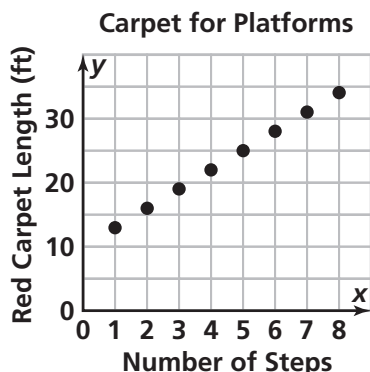
Figure 8

Medal Platforms

Number of Medalists	1	2	3	4	5	6	7	8
Number of Boxes	1	3	6	10	15	21	28	36

d. (Figure 9)

e.



f. The pattern in the data illustrates a linear relationship because, with every new step, the length of the red carpet increases by exactly 3 feet. This constant rate of change is different than the pattern in the number of boxes, which has an increasing rate of change.

6. a. (Figure 10)

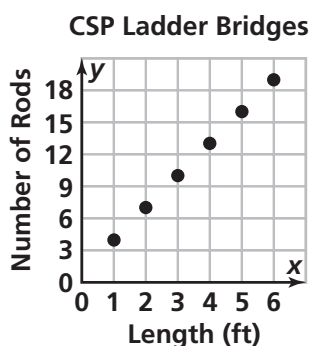


Figure 9

Carpet for Platforms

Number of Steps	1	2	3	4	5	6	7	8
Carpet Length (ft)	13	16	19	22	25	28	31	34

Figure 10

CSP Ladder Bridges

Bridge Length (ft)	1	2	3	4	5	6
Number of Rods	4	7	10	13	16	19

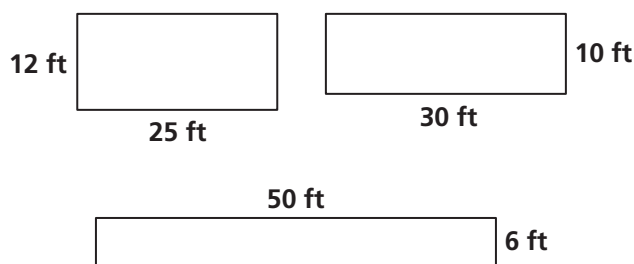
b. This is an increasing linear relationship like the relationship between beam length and number of rods. Although the relationship between number of steps and number of rods in a staircase frame is also increasing, it is not linear.

Connections

7. D ($\frac{2}{3}$ of 600)

8. H ($\frac{2}{3}$ of 450)

9. a. Possible rectangles:

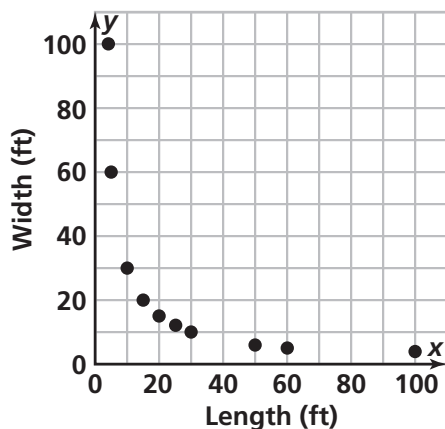


b. 300 feet; 150 feet; 100 feet

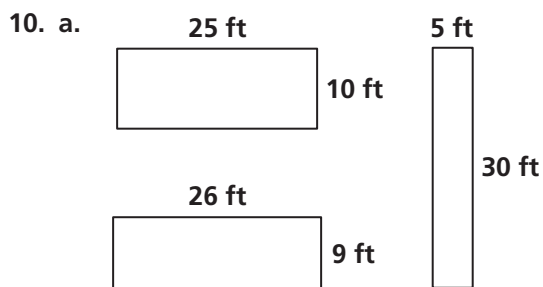
c. $\frac{300}{L}$ feet (Note: Some students may not be able to use symbols to describe this relationship. They will work more with the relationship between area, length, and width in Investigation 3.)

d. The width decreases, but not linearly.

e. Rectangles With an Area of 300 ft^2

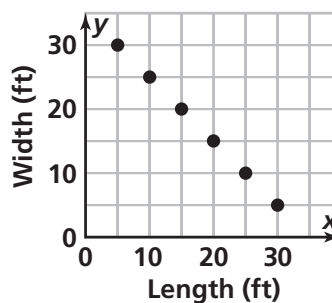


The graph decreases very sharply at first and then more gradually.



- b. 34 ft; 33 ft; $35 - L$ ft, or $0.5(70 - 2L)$ ft
 c. 34.5 feet, 33.5 feet
 d. 15 ft by 20 ft; about 15.4 ft by 19.25 ft; 17.5 ft by 17.5 ft
 e. It decreases linearly.

f. Rectangles With a Perimeter of 70 ft



We see a linear decrease in the graph.

11. a. See the “Probable Sales” row of Figure 11.
 b. See the “Income” row of Figure 11.
 c. \$2.50
 12. Graph C
 13. Graph A
 14. Graph D
 15. Graph B
 16. 2 coins; possible method: Take 3 coins from each side to get 3 pouches equals 6 coins. Because each pouch contains the same number of coins, there must be 2 coins in each pouch.
 17. 3 coins; possible method: Take 1 coin from each side to get 4 pouches equals 2 pouches and 6 coins. Now take 2 pouches from each side to get 2 pouches equals 6 coins. Because each pouch contains the same number of coins, there must be 3 coins in each pouch.

Figure 11

Predicted Ticket Sales for Whole School

Ticket Price	\$1.00	\$1.50	\$2.00	\$2.50	\$3.00	\$3.50	\$4.00	\$4.50
Probable Sales	400	400	360	300	240	200	160	140
Income	\$400	\$600	\$720	\$750	\$720	\$700	\$640	\$630

18. a. $3x + 3 = 9$ and $4x + 1 = 2x + 7$

b. Possible solution for $3x + 3 = 9$:

$$3x + 3 = 9$$

$$3x = 6 \quad \text{Subtract 3 from each side.}$$

$$x = 2 \quad \text{Divide each side by 3.}$$

Possible solution for $4x + 1 = 2x + 7$:

$$4x + 1 = 2x + 7$$

$$4x = 2x + 6 \quad \text{Subtract 1 from each side.}$$

$$2x = 6 \quad \text{Subtract 2x from each side.}$$

$$x = 3 \quad \text{Divide both sides by 2.}$$

c. Possible answer: The strategies were the same, but in part (b), symbols were used instead of objects.

19. $x = 2$

20. $x = 4$

21. $x = \frac{14}{6}$ or an equivalent form

22. $x = \frac{-2}{3}$

23. $x = 2\frac{1}{8}$ or $x = 2.125$

24. $x = -2$

25. $x = -3$

26. $x = 4$

27. False, because $42 < 50$

28. True, because $11 > 6$

29. False, because $-10 < 0$

30. a-c. The "wrap" part of the cylinder has the same area (8.5×11) for each cylinder, but the circular bases are larger for the cylinder with the 8.5-inch height.

d. The shorter cylinder. The base area depends on the radius. If the smaller dimension of the paper is used for the height of the cylinder, then the base area will have a larger radius.

31. Answers will vary. The only criterion is that $r^2h = 28$. Possible answers: $r = 2, h = 7$; $r = \sqrt{7}, h = 4$; $r = \sqrt{8}, h = 3.5$.

Extensions

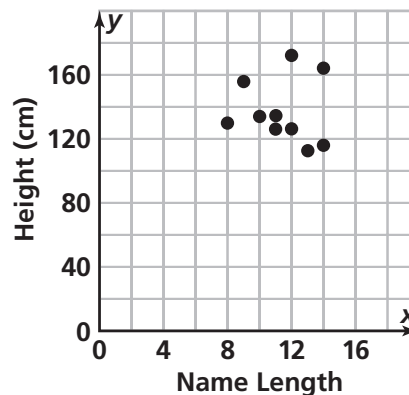
32. a.

x	p	q	y	z
1	1	1	2	1
2	4	8	4	$\frac{1}{2}$
3	9	27	8	$\frac{1}{3}$
4	16	64	16	$\frac{1}{4}$
5	25	125	32	$\frac{1}{5}$
6	36	216	64	$\frac{1}{6}$
10	100	1,000	1,024	$\frac{1}{10}$
11	121	1,331	2,048	$\frac{1}{11}$
12	144	1,728	4,096	$\frac{1}{12}$
n	n^2	n^3	2^n	$\frac{1}{n}$

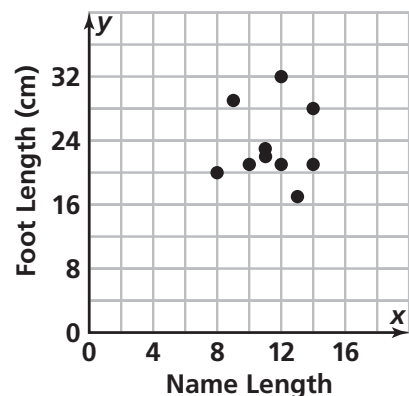
b. None of the patterns are linear because a constant change in x does not yield a constant change in y .

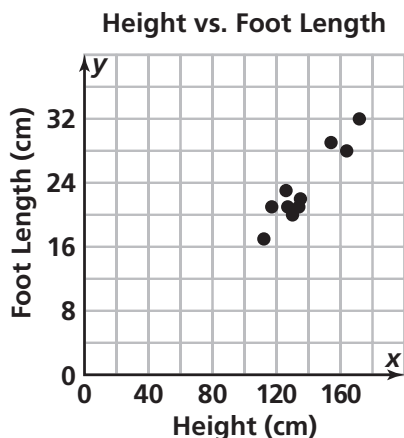
33. a.

Name Length vs. Height



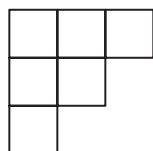
Name Length vs. Foot Length





- b. Only the (*height, foot length*) graph looks linear.
- c. Approximately 6 : 1; The average student is 6 “feet” tall.
- d. Shoshana White; Tonya Stewart

34. a. Orientation of base will vary; 6 sq. units



b.

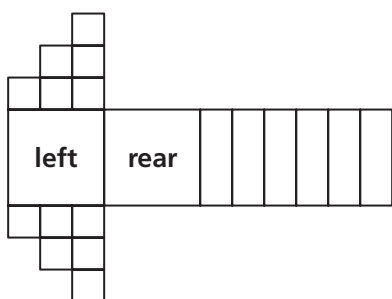
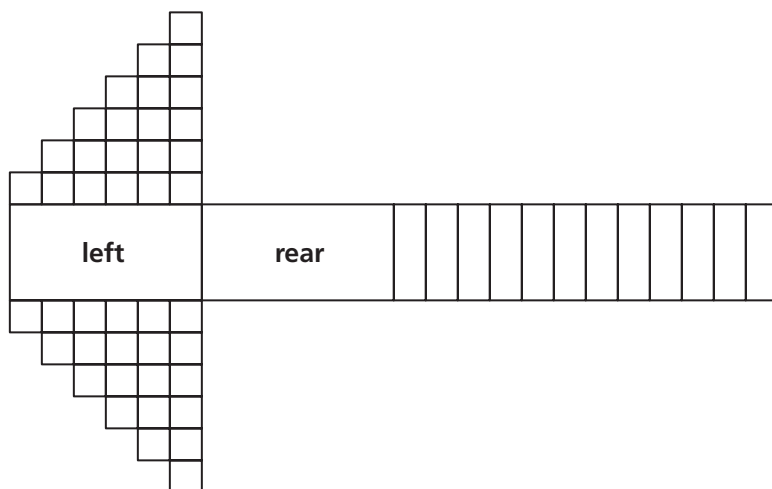


Figure 12



Surface area = 6 (top) + 6 (bottom) + 9 (left) + 9 (rear) + (3 + 3) + (3 + 3) + (3 + 3) = 48 sq. units.

- c. New surface area: 21 (top) + 21 (bottom) + 18 (left) + 18 (rear) + (3 + 3) + (3 + 3) + (3 + 3) + (3 + 3) + (3 + 3) + (3 + 3) = 114 sq. units. The top and bottom areas more than doubled. The left and rear areas exactly doubled (but they are no longer squares). The “stair” area doubles. So the total area is more than twice the original. A flat pattern is shown in Figure 12.

Possible Answers to Mathematical Reflections

- One advantage to a graph is that it gives a visual representation of the situation. A disadvantage is that a graph can be difficult to read for precise answers. An advantage of words is that they can communicate a situation easily without much effort on the part of the communicator, but sometimes words are not very precise. Some situations are just too complicated to explain easily in words. Tables give you precise information about certain coordinates, but they often leave you guessing about the information between the given coordinates.
- If constant change in the independent variable in the table yields constant change in the dependent variable, then the relationship is linear.
- If a graph is a straight line, the relationship is linear.