

# Investigation 5

## ACE

### Assignment Choices



#### Problem 5.1

Core 1, 2, 13, 14

Other unassigned choices from previous problems

#### Problem 5.2

Core 3, 4

Other 15; unassigned choices from previous problems

#### Problem 5.3

Core 5–8

Other unassigned choices from previous problems

#### Problem 5.4

Core 9–12

Other 16–20; unassigned choices from previous problems

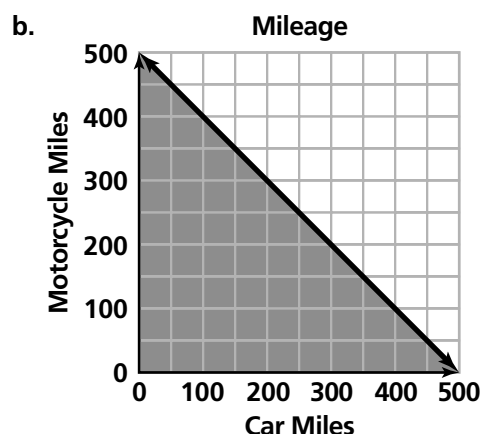
Connecting 16: *Shapes and Designs*

**Adapted** For suggestions about adapting Exercise 1 and other ACE exercises, see the *CMP Special Needs Handbook*.

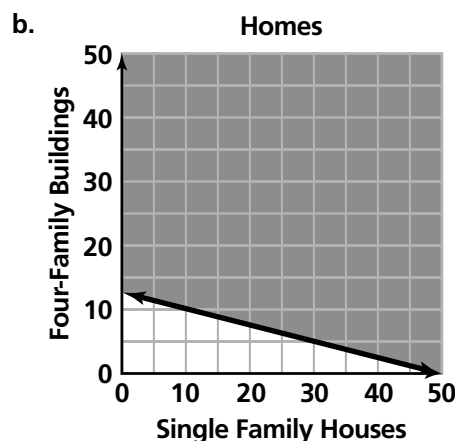
**Connecting to Prior Units** 16: *Shapes and Designs*

## Applications

1. a. Let  $x$  be the number of miles on the car and  $y$  be the number of miles on the motorcycle. Then the inequality is  $x + y \leq 500$ . Students may include  $x \geq 0, y \geq 0$ .

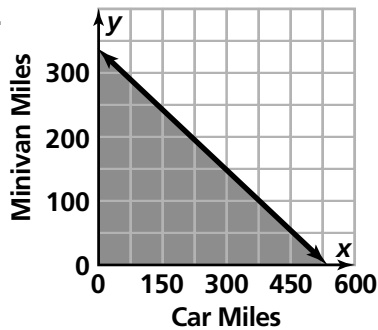


- c. Answers may vary. One possibility: first, I drew the line  $x + y = 500$ . Then I shaded the region of solutions.
2. a. Let  $x$  be the number of single-family houses and  $y$  be the number of four-family apartment buildings. Then the inequality is  $x + 4y \geq 50$ . Students may include  $x \geq 0, y \geq 0$ .



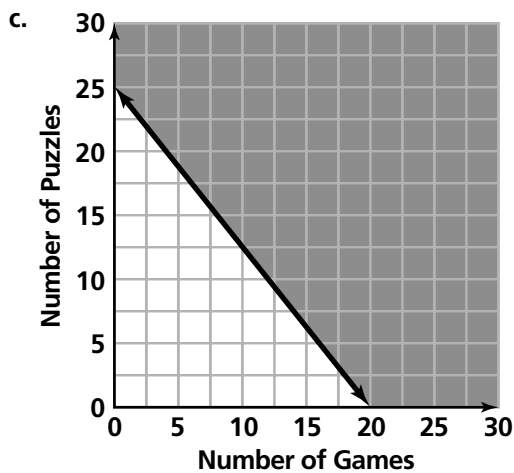
3. a.  $0.75x + 1.2y \leq 400$

b.



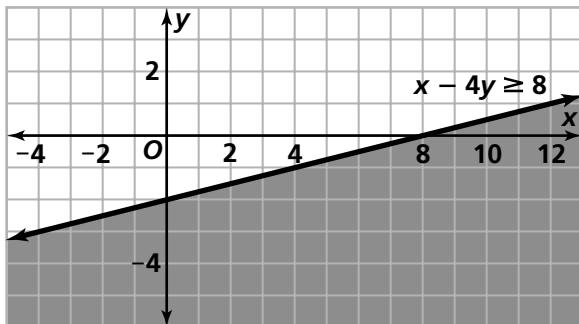
4. a. Answers may vary. Some possibilities: 20 games and 5 puzzles. 10 games and 16 puzzles.

b. Let  $g$  be the number of games the math club sells and  $p$  be the number of puzzles they sell. Then the inequality is  $10g + 8p \geq 200$ .



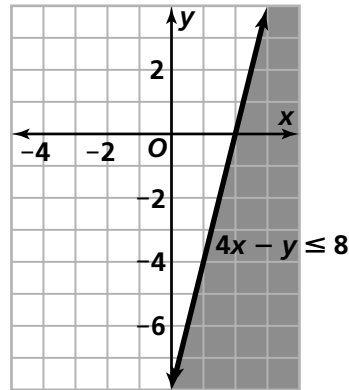
5. Answers will vary. Possible answers include (10, 0), (32, 2), and (8, 0).

Answers will vary. Possible answers include (10, 1), (20, 5), and (0, -1).



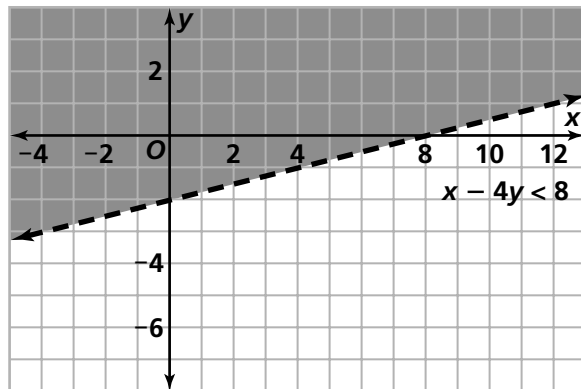
6. Answers will vary. Possible answers include (0, 0), (1, 2), (2, 8).

Answers will vary. Possible answers include (0, -10), (2, -1), (3, 3).

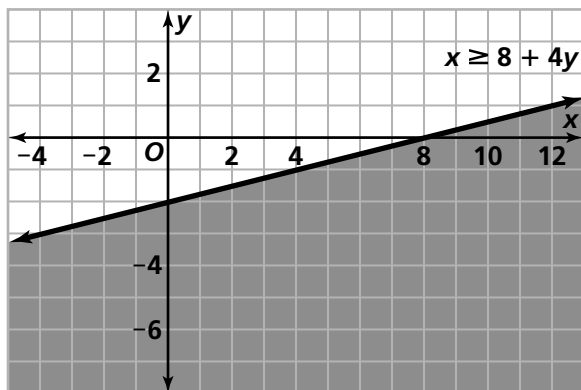


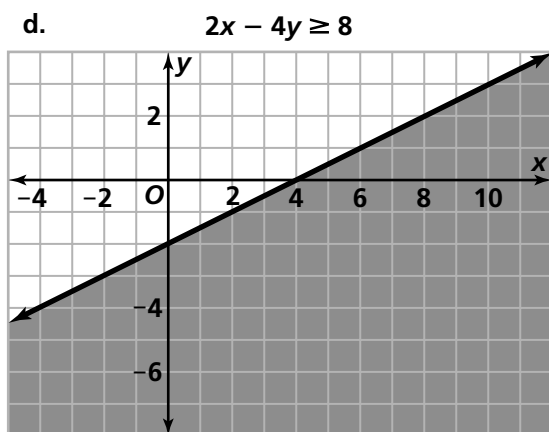
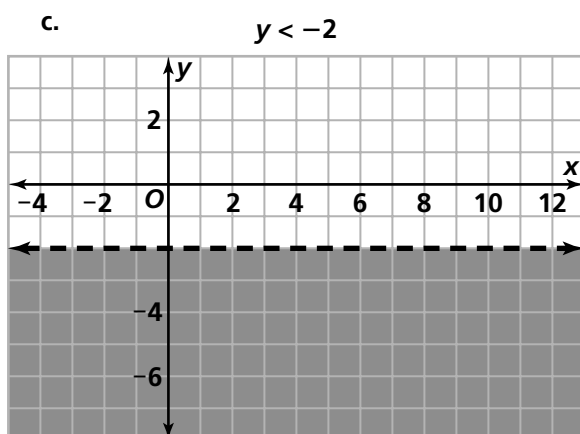
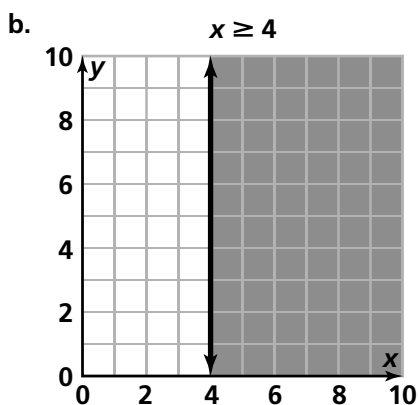
7. Answers will vary. Possible answers include (0, 0), (1, 2), (2, 8).

Answers will vary. Possible answers include (8, 0), (16, 2), (4, -1).



8. a.



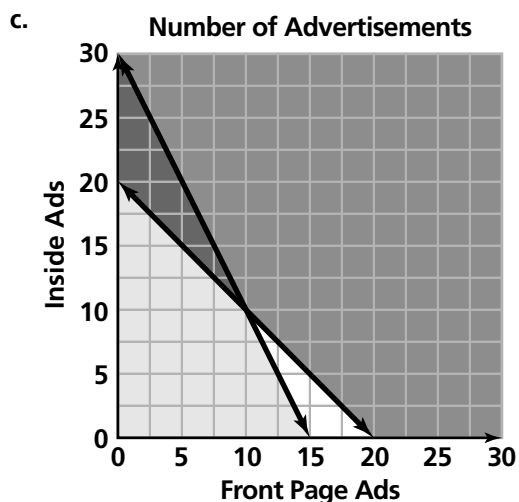


e. Answers will vary. One possible answer: I drew the line first, then decided whether or not it should be dotted. Then I chose test points to decide on which side of the line to shade.

9. a. Answers may vary, but they can advertise 20 times inside the paper and 5 times on the front page; or they can advertise 10 times on the front page and 10 times inside the paper; or they can advertise 30 times on the inside of the paper and never on the front page.

b. Let  $f$  be the number of times they advertise on the front page and let  $p$  be the number of times they advertise inside the paper. We know  $f \geq 0$  and  $p \geq 0$ . Also:

$$\begin{aligned} 2f + p &\leq 30 \\ f + p &\geq 20 \end{aligned}$$



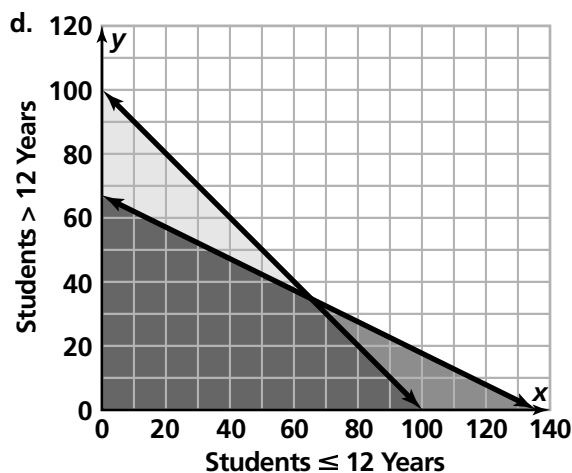
Note that students may choose different axes for their graphs.

10. a. If no students older than 12 go on the trip, then as many as 100 students can go. This is within the budget of \$400, and they are limited by the number of chaperones.

b. If no students 12 or younger go, then 66 students can go. This is because they are limited by the \$400 budget.

c. Let  $x$  be the number of students 12 and younger going on the trip. Let  $y$  be the number of students older than 12 going on the trip. Then  $x \geq 0$  and  $y \geq 0$ . Also:

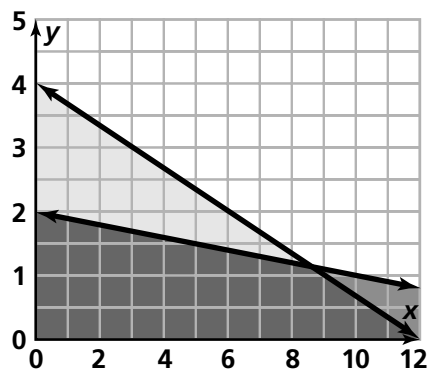
$$\begin{aligned} 3x + 6y &\leq 400 \\ x + y &\leq 100 \end{aligned}$$



Note: students may choose different axes for their graphs.

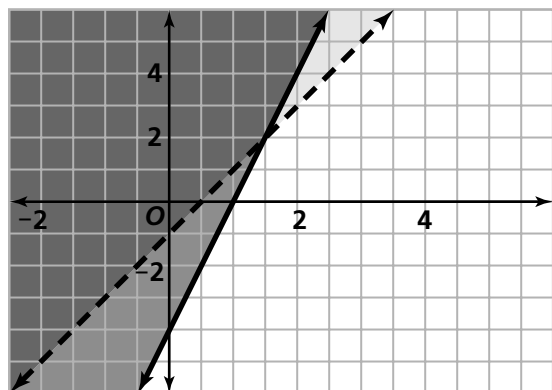
11. Answers will vary. Possible solutions include  $(0, 0)$ ,  $(1, 1)$ ,  $(-1, -1)$ .

Answers will vary. Possible non-solutions include  $(1, 2)$ ,  $(2, 4)$ ,  $(3, 6)$ .



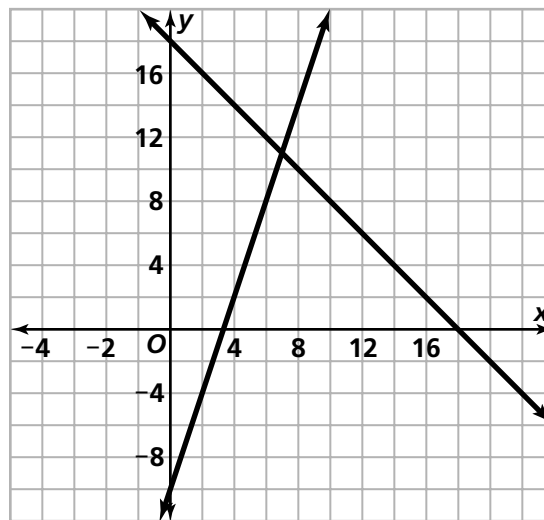
12. Answers will vary. Possible solutions include  $(0, 3)$ ,  $(-1, 1)$ ,  $(-2, 0)$ .

Answers will vary. Possible non-solutions include  $(3, 0)$ ,  $(0, -2)$ ,  $(1, -3)$ .

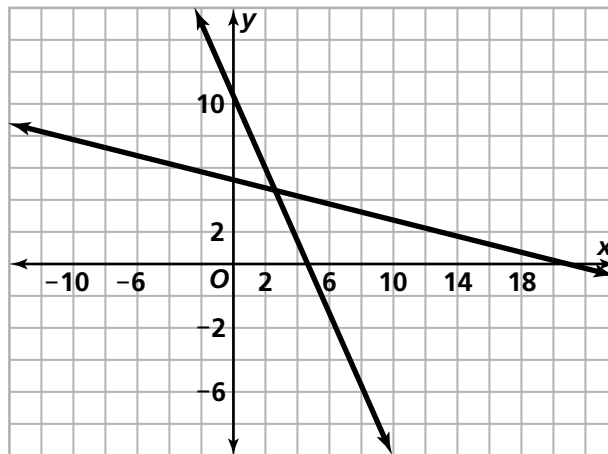


## Connections

13. The solution is  $(7, 11)$ .



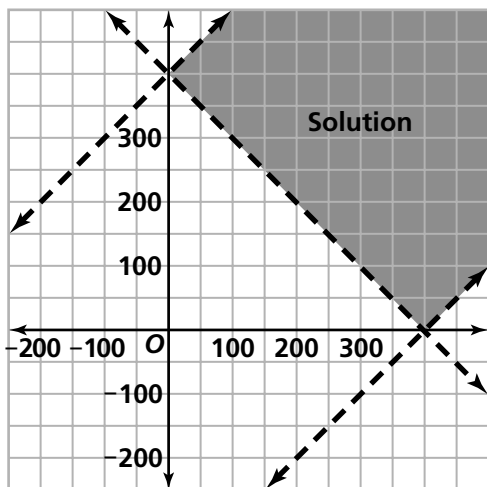
14. Possible estimate for solution:  $(2.7, 4.6)$ .  
Students can use trace or table features of their graphing calculators.



Answers will vary. One possibility: I graphed the equations of the lines and looked for the point of intersection.

15. B
16. a. Some possibilities for the lengths of the other side are 300 and 300, or 200 and 500. We know that the sum of any two sides must be greater than the third.
- b. The city planner needs all three inequalities because the sum of any of the two side lengths of a triangle must be greater than the third side.

c.



Answers describing the region that represents the possibilities will vary, but one possibility is that it looks like a rectangle that has been infinitely extended on one side.

d. Answers will vary. One possibility is (400, 100). You can see this in the graph because it's from the solution region for all three inequalities.

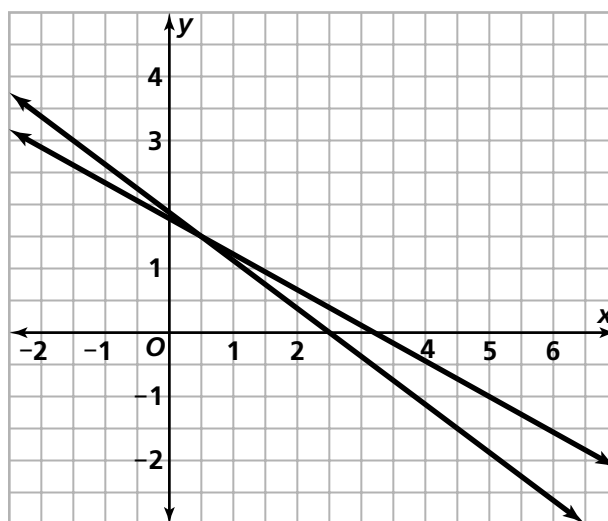
e. Answers will vary. One possible answer is (100, 100), because it is not in the solution region for all three inequalities.

17. a. Let  $m$  represent the number of cups of milk Robin uses and  $y$  stand for the number of cups of yogurt. Then the system of equations is:

$$100m + 190y = 335$$

$$9m + 13y = 24.$$

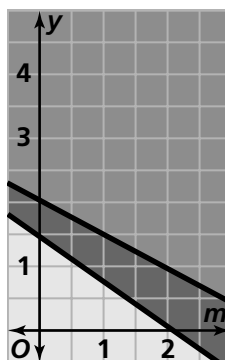
b.



c. She needs to use half a cup of milk and 1.5 cups of yogurt, because the graphs of the two equations intersect where  $m = 0.5$  and  $y = 1.5$ .

18. a. 
$$\begin{cases} 100m + 190y \leq 400 \\ 9m + 13y \geq 20 \end{cases}$$

b.



c. Some possibilities are 1 cup of milk and 1.5 cups of yogurt; or 0.7 cups of milk and 1.4 cups of yogurt; or a cup of yogurt and a cup of milk.

## Extensions

### 19. systems with three inequalities

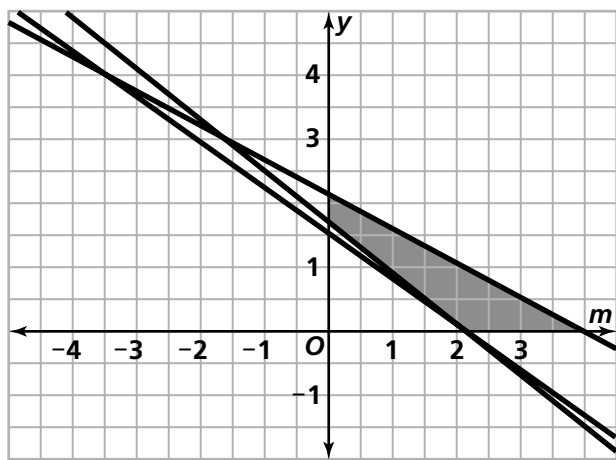
- a. Let  $m$  represent the number of cups of milk and  $y$  represent the number of cups of yogurt. We know  $m \geq 0$  and  $y \geq 0$ . The system is

$$100m + 190y \leq 400$$

$$9m + 13y \geq 20$$

$$306m + 415y \geq 700$$

b.



- c. Some possibilities are  $(3, 0)$ ,  $(0, 2)$ ,  $(1.5, 1)$

20. Answers will vary.

## Possible Answers to Mathematical Reflections

- If the inequality has the form  $ax + by \leq c$  or  $ax + by \geq c$ , first graph the equation  $ax + by = c$ , and then figure out which region represented the solution set of the inequality by testing points.
- To find the solution to a system of linear inequalities, first find the regions of solution to each inequality and then find the intersection of those regions, which is the solution of the system.

## Answers to Looking Back and Looking Ahead

1. a. The circle on the left has equation  $(x + 3)^2 + y^2 = 25$ ; the circle on the right has the equation  $(x - 3)^2 + y^2 = 25$ . This can be confirmed by checking to see that particular points on each circle satisfy the equations. For example,  $(-3, 5)$  satisfies the first equation and  $(3, 5)$  satisfies the second equation.

This match of coordinate equations and geometric shapes is always counter-intuitive for students. They expect  $(x - 3)^2 + y^2 = 25$  to be the one that represents a circle shifted to the left from the standard form centered at the origin. Full and convincing explanation of this translation idea is better left to a future course. The essential idea is that replacing  $x$  by  $(x + 3)$  has the effect of making a point act like its counterpart 3 units to the right in the standard equation  $x^2 + y^2 = 5^2$ . Thus the image of this new circle will make it look as if the original has been shifted left 3 units.

- b. The circles intersect at  $(0, 4)$  and  $(0, -4)$ . The  $(x, y)$  coordinates of those points satisfy both equations:  $(0 + 3)^2 + 4^2 = 25$  and  $(0 - 3)^2 + (4)^2 = 25$  and  $(0 - 3)^2 + (-4)^2 = 25$  and  $(0 + 3)^2 + (-4)^2 = 25$ .
- c.  $(-7 + 3)^2 + 3^2 = 25$

The next diagram shows a right triangle connecting the center of the circle to the point  $(-7, 3)$ . Using the Pythagorean theorem, we see that  $4^2 + 3^2 = 5^2$ .

