

Investigation 4

ACE

Assignment Choices



Problem 4.1

Core 1–2, 21–23

Other Applications 3, 4; Connections 20; Extensions 40

Problem 4.2

Core 5, 7, 26–29, 33–35

Other Applications 6; Connections 24, 25, 30–32; Extensions 41; and unassigned choices from previous problems

Problem 4.3

Core 11–18, 36, 37

Other Applications 8–10, 19; Connections 38–39; Extensions 42–43; and unassigned choices from previous problems

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.

Connecting to Prior Units 20: *Covering and Surrounding*; 21: *Covering and Surrounding and Filling and Wrapping*; 22, 34: *Moving Straight Ahead*; 23–24, 33, 36–37: *Frogs, Fleas and Painted Cubes*; 25, 35: *Growing, Growing, Growing*; 26–31: *Say it With Symbols*, Investigation 1; 32: *Say it With Symbols*, Investigation 2; 38: *Looking for Pythagoras*; 39: *Filling and Wrapping*

Applications

1. a. 275 gallons are being pumped out each hour; Students may make a table and notice the constant rate of change, which is -275 , or they may recognize that -275 is the coefficient of t in a linear relationship between w and t .
- b. 1,925 gallons. This can be found using a table and finding the value for and when $t = 0$ or by substituting into the equation $t = 0$, then solving for w .

- c. 3 hours. Students can find in their table the corresponding t value when w is 1,100 or they can solve the equation $w = -275t + 1,925$ for t if $w = 1,100$.
- d. 7 hours. Students can find in their table the corresponding t value when w is 0 or they can solve the equation $w = -275t + 1,925$ when $w = 0$.
- e. $w = -275(t - 7)$. The original equation tells us that before the pump started working, there were 1,925 gallons of water in the pool, and that every hour the pump emptied the pool of 275 gallons of water. In this equation it is easy to see that when $t = 7$, the amount of water is 0, or the tank is empty.
- f. The relationship is a linear relationship because there is a constant rate of change.
2. a. 550 gallons. Students may use a table and notice the constant rate of change which is -550 , after multiplying -275 by 2 or they may recognize that -550 is the coefficient of t in a linear relationship between w and t .
 $w = -275(2t - 7)$
 $w = -550t + 1,925$
- b. 1,925 gallons of water were in the pool when the pump started; This can be found using a table and finding the value for w when $t = 0$ or by substituting into the equation $t = 0$ and solving for w .
- c. 1.68 hours or 1 hour, 41 minutes. Students can estimate from their table the corresponding t value when w is 1,000; they can solve the equation $w = -275(2t - 7)$ for t if $w = 1,000$ by applying the Distributive Property and then solving for t :

$$\begin{aligned} 1,000 &= -550t + 1,925 \\ 1,000 - 1,925 &= -550t \\ \frac{-925}{-550} &= \frac{-550t}{-550} \\ 1.68 &\approx t \end{aligned}$$

The pump has been running for about 1.68 hours.

- d. 3.5 hours; Students can find in their table the corresponding t value when w is 0, or they can solve the equation

$$\begin{aligned} w &= -275(2t - 7) \text{ for } t \text{ when } w = 0. \\ 0 &= -550t + 1,925 \\ -1,925 &= -550t \\ \frac{-1,925}{-550} &= \frac{-550t}{-550} \\ 3.5 &= t \end{aligned}$$

The pool will be empty in 3.5 hours.

- e. $w = -550t + 1,925$. This equation tells us that before the pump started working there were 1,925 gallons of water in the pool, and that every hour the pump emptied the pool of 550 gallons of water.
3. a. 25 gallons. Let $m = 0$, as 0 miles have been driven since the last fill-up. From the equation, $g = 25$, meaning the tank holds 25 gallons of gas.
- b. 21.7 gallons. $g = 25 - \frac{50}{15} = 21.7$ gallons
- c. Substituting into the equation, $5 = 25 - \frac{1}{15}m$, so $m = 300$ miles. (NOTE: A graph and a table would also show that 5 gallons remain after 300 miles.)
- d. 375 miles. Students may use their table to find the value of m that corresponds to $g = 0$ or solve the equation $g = 25 - \frac{1}{15}m$ for m when g equals zero. Since m has a coefficient of $-\frac{1}{15}$, students may have a difficult time deciding how to apply the properties of equality. They may multiply by 15 (or they could also divide by $\frac{1}{15}$).
- e. The tank holds 25 gallons, so $g = 25 - 1 = 24$ when 1 gallon has been used. Therefore, solving $24 = 25 - \frac{1}{15}m$, $m = 15$. The driver would have to travel 15 miles to use 1 gallon of gas.
- f. 25 is the number of gallons of fuel in the tank after a fill-up, and $\frac{1}{15}$ indicates that the truck uses $\frac{1}{15}$ gallon of gas every 1 mile.
4. The variable y represents how much money they still need to pay for the printing bill, depending on the number of books sold. N represents the number of books sold or given away; 2,500 is the amount they owe for

printing at the start of the project; 15 is the price they charge for each book; and 8 represents the free copies they gave to the yearbook advisor and staff.

5. a. For Species 1, the 10,000 is the starting population. The 100 is the rate at which the population grows every year. So every year the population increases by 100 animals. The P_1 is the total population after x years.
- For Species 2, the 10 is the starting population, and the 3^x means that the population triples every year. The P_2 is the total population after x years.
- For Species 3, the 800 is the starting population. The P_3 is the total population after x years.
- b. The pattern of growth for Species 1 is linear. The pattern of growth for Species 2 is exponential. The pattern of growth for Species 3 is quadratic. After a certain time Species 2 will surpass both, since exponential growth patterns increase at an increasing rate.
- c. Answers will vary, however, any two populations will be the same at some value for x .
- Species 1 and 2 are the same when x is between 6.3 and 6.4.
- Species 2 and 3 are the same when x is between 4.1 and 4.2.
- Species 3 and 1 are the same when x is about 35.74 and -25.74 .
- One way to find these values for x is to use a graphing calculator. If you use the table function and set the increments to 0.1 or 0.01, you can get close estimates for the values for which the equations are equal.
6. a. **Table 1** is quadratic with a second difference of 1. **Table 2** is linear with a constant rate of change of 30. **Table 3** is exponential with a growth factor of 3.
- b. Possible answers: **Table 2:** Let N be the number of deer and y be the year. Then $N = 1,000 + 30(y - 2,000)$. Let N be the number of deer and x be the number of years after 2000 (so when $x = 1$ the year is 2001); then the equation is $N = 1,000 + 30x$.

Table 3: Let N be the number of deer and y be the year. Then $N = 1000(3)^{y-2000}$. Or if N is the number of deer and x is the number of years after 2000 (so $x = 1$ represents the year 2001), then the equation is $N = 1000(3)^x$.

- c. Table 3 shows the deer population growing at a rate of 300% per year.

7. a. Answers will vary.

- b. 70. Students may draw the next two figures and count the number of toothpicks. OR Make a table of values and use the pattern in the table to find the number of toothpicks in the 7th figure.

Figure	Toothpicks
1	4
2	10
3	18
4	28
5	40
6	54
7	70

- c. Linear. Possible answer: The figure number times 4 equals the perimeter. The figure number equals the number of the toothpicks on the bottom and the number of toothpicks going up (height). If you double the figure number, you get the number of toothpicks that make up the “stairs” on the left side of the figure giving $n + n + 2n = 4n$. This pattern shows that the data will go up at a constant rate. The graph will be a straight line with a slope of 4.

- d. Quadratic. Possible answer:

In the data table, as x increases by 1, the y -value has a second difference of 2.

- e. $P = 4N$. To find the perimeter you take the figure number and multiply it by 4.

- f. Possible answers:

$T = N^2 + 3N$, where T is total number of toothpicks and N is the figure number. If you work back on the table, you find that the y -intercept is 0. This means that in the quadratic equation form of $y = ax^2 + bx + c$, $c = 0$. Because the second difference is 2, the value of $a = 1$ ($a = \text{Half of the second difference}$). So far, we know that our equation is $y = 1x^2 + bx + 0 = x^2 + bx$. A table can be used to find b . See below.

Figure Number	x^2	bx	Total Toothpicks
0	0	$+ 0 (3 \times 0)$	0
1	1	$+ 3 (3 \times 1)$	4
2	4	$+ 6 (3 \times 2)$	10
3	9	$+ 9 (3 \times 3)$	18
4	16	$+ 12 (3 \times 4)$	28
x	x^2	$3x$	$x^2 + 3x$

$T = N(N + 3)$, where T is total number of toothpicks and N is figure number.

Figure (N)	$N + 3$	Total Toothpicks $N(N + 3)$
1	4	4
2	5	10
3	6	18
4	7	28

If you divide the total toothpicks by the figure number, the result is the second column of numbers. This number is the figure number plus 3. To get the total number of toothpicks, you multiply the $N + 3$ and the figure number.

8. Graph 1 is linear since it is a straight line with a constant rate of change of 3. Graph 2 is exponential since it has an increasing graph with a growth factor of 3. Graph 3 is quadratic since it has an upside down U-shape and a second difference of -2 . Graph 4 is none of the above. Graph 4 was studied in *Thinking With Mathematical Models* and is an inverse variation graph.

9.

Graph 1	
X	Y
1	1
2	4
3	7
4	10
5	13
6	16

Graph 2	
X	Y
1	3
2	9
3	27
4	81
5	243
6	729

Graph 3	
X	Y
1	2
2	2
3	0
4	-4
5	-10
6	-18

10. **Graph 1:** $y = 3x - 2$. To find this equation you need to find the y -intercept and the slope or rate of change. Students may use the formula $m = \frac{\Delta y}{\Delta x}$, i.e. $m = \frac{y_2 - y_1}{x_2 - x_1}$, to find slope by using two of their points in the table. They may look at the constant rate of change, which is 3. To find the y -intercept, they may look at the graph and see that it is -2 .

Graph 2: $y = 3^x$. To find this equation students need the starting point and the growth factor. By looking at the table it is easy to see that each y -value increases by a growth factor of 3. The starting point can be found by dividing the y -value for $x = 1$ by 3 in order to get the y -value associated with $x = 0$. Doing this, you get $(0, 1)$ for the starting point. So the equation is $y = 1(3)^x$ or $y = 3^x$.

Graph 3: Since the x -intercepts are zero and 3, the factors could be $x(3 - x)$, and the equation may be $y = x(3 - x)$. By checking the point $(1, 2)$ in this equation, we see that this is correct, since three points [the x -intercepts and the point $(1, 2)$] determine a parabola. NOTE: The equation for Graph 4 is $y = \frac{1}{x}$ or equivalently $xy = 1$.

11. G

12. D

13. B

14. F

15. E

16. C

17. A

18. a. Linear: Equations 2 and 4.

Quadratic: Equations 1, 5, 8 and 9.

Exponential: Equations 3, 6, and 7.

b. Equations 2 and 4 represent the same function, Equations 1 and 5 represent the same function and Equations 3 and 6 represent the same function.

c. The graph of equations 2 and 4 is a line with a starting point of $(0, 0)$, a rate of change of 2, and an increasing pattern from left to right. The graph of equations 1 and 5 is a parabola that opens up with a y -intercept of $(0, 0)$, x -intercepts of $(0, 0)$ and $(-8, 0)$ and a minimum point at $(-4, -16)$. The graphs of Equations 3 and 6 are increasing curves with y -intercept $(0, 1)$ and a growth factor of 4.

19. Answers will vary. Possible answer for the linear equation: $y = 2x$. You get 2 dollars for every kilometer you walk where x is the number of kilometers walked and y is the total amount of money collected. Possible answer for the quadratic equation: $y = x^2 + 8x$. This represents the number of handshakes between two teams if one team has x members and the other team has $x + 8$ members. Possible answer for the exponential equation: $y = 4^{x-1}$. y is the number of rubas on the x th square of a checkerboard if the King puts 1 on the first square, 4 on the second, and 16 on the third and then continues to quadruple the number of rubas for each successive square.

Connections

20. a. 394 tiles. Students will calculate this in various ways, for example:
 $6(45) + 4(30) + 4$, or
 $4(45 + 1) + 2(45) + 4(30)$, or
 $4(45) + (45 + 30 + 45) + 4 + 3(30)$.
- b. 6,300 ft². Students will calculate this in various ways, for example:
 $2(45^2) + 30(30 + 45)$, or
 $2(45^2) + 30^2 + 30(45)$, or
 $45(45 + 30 + 45) + 30^2$.

- c. $30,600 \text{ ft}^3$. Students will calculate this in various ways, for example:
 $4(2)(45^2) + 4(45)(30) + 10(30^2)$, or
 $4(45^2) + 4(45)(30) + 4(45^2) + 10(30^2)$ or
 $4(45)(45 + 30 + 45) + 10(30^2)$.
- d. 51 hours. It will take $30,600 \text{ ft}^3 \div 600 \text{ ft}^3/\text{h} = 51$ hours to fill the pool.
21. a. $C = 2\pi r$ or $C = \pi d$
 b. $A = \pi r^2$
 c. $V = \pi r^2 h$
 d. The equation in part (a) is a linear relationship between C and r . If students are confused about what the variables are, remind them that π is a constant.
22. A line has a slope of 1.5 and goes through the point (2, 5).
 a. Possible answers include points that satisfy the equation $y = 1.5x + 2$. For example, (0, 2), (1, 3.5), and (-1, 0.5). Students may make a graph with the given conditions or a table like the one shown below, starting from the point (2, 5) and using the constant rate of change to find y -values for x -values greater than and less than 2.

X	Y
1	3.5
2	5
3	6.5
4	8
5	9.5

Another way students may solve this problem is by finding the equation of the line right away. The equation of a line that has a slope of 1.5 and that passes through the point (2, 5) would be $y = 1.5x + 2$.

- b. (0, 2). The y -intercept can be found by subtracting the rate of change of 1.5 from 5 twice to get the value of 2 for y when $x = 0$ or by solving the equation $5 = 1.5(2) + b$ for b .
- c. -4. Solving the equation $y = 1.5(-4) + 2$ for y gives -4.
- d. $y = 1.5x + 2$

23. a. First Tara distributed $(x + 2)$ to x and to 3. Second she distributed x to x and to 2. Third she distributed 3 to x and to 2. Then she applied the Distributive Property when she said that $2x + 3x = x(2 + 3)$. To get the term $5x$, the Commutative Property was used: $x(2 + 3) = (2 + 3)x = 5x$.
- b. Finding the area of the left-hand column of the table, which is $(x + 2)x$, and adding it to the right hand column area $(x + 2)3$, is the same as her first step. Her second step is just expressing the two column's areas as a sum of the parts that make them up. The last two steps are just to combine the two x -terms and are not represented in the area model.
- c. i. $x^2 + 8x + 15$
 $(x + 5)(x + 3) =$
 $(x + 5)x + (x + 5)3 =$
 $x^2 + 5x + 3x + 15 =$
 $x^2 + x(5 + 3) + 15 = x^2 + 8x + 15$
- ii. $x^2 + 5x + 4$
 $(x + 4)(x + 1) = (x + 4)x + (x + 4)1 =$
 $x^2 + 4x + x + 4 =$
 $x^2 + x(4 + 1) + 4 = x^2 + 5x + 4$
- iii. $x^2 + 2x - 8$
 $(x - 2)(x + 4) = (x - 2)x + (x - 2)4 =$
 $x^2 - 2x + 4x - 8 =$
 $x^2 + x(-2 + 4) - 8 = x^2 + 2x - 8$
24. a. 7.94 feet;
 $d = -16(0.1^2) + 16(0.1) + 6.5 = 7.94$
- b. 9.86 feet;
 $d = -16(0.3^2) + 16(0.3) + 6.5 = 9.86$
- c. 6.5 feet; $d = -16(1^2) + 16(1) + 6.5 = 6.5$
- d. The operations are exponentiation, multiplication, and addition; the exponentiation is done first, then the multiplication and then the addition. (NOTE: the multiplication of numbers not involving exponents could be done before the exponentiation.)
- 25.a. 40,000 and 160,000; To find the population after 3 hours, substitute 3 into the equation $b = 5000(2^t)$ for t . Then $b = 5,000(8) = 40,000$. To find the population after 5 hours, substitute 5 into the equation for t . Then $b = 5000(32) = 160,000$.

b. First perform the repeated multiplication defined by the 2^i in the parentheses; then take this product and multiply it by 5,000.

26. Possible answers: $-59 - 6x$, $1 - 6x - 60$, or $5 - 6x - 60 - 4$.

27. Possible answers: $-4x + 9$;
 $-3x + 12 - (x + 3)$; $-3x + 12 - x - 3$;
 $-3(x - 4) - x - 3$; $-4x + 12 - 3$;
 $-3x + 9 - x$

28. Possible answers: $x^2 + 2x - 5x + 6$;
 $x^2 - 3x + 6$;

29. Possible answers:
 $6x^2 + 5x^2 - 50x + 10$
 $11x^2 - 50x + 10$

30. Possible answers:

$$\frac{3}{4}x^2 + x^2 + 3x;$$

$$\frac{7}{4}x^2 + 3x$$

$$x(\frac{7}{4}x + 3)$$

$$\frac{7}{4}x(x + \frac{12}{7})$$

31. Possible answers:

$$7x^2 - 2.75x - 8;$$

$$3.5x(2x - 1) + 0.25(3x - 32)$$

32. a. $y = 6(2z - 7) + 10$ or equivalently
 $y = 12z - 42 + 10$ or $y = 12z - 32$

b. $P = (12 - n)n - 6n$ or equivalently
 $P = 12n - n^2 - 6n$ or $P = 6n - n^2$ or
 $P = n(6 - n)$

c. $A = (15 - w)w$ or equivalently
 $A = 15w - w^2$.

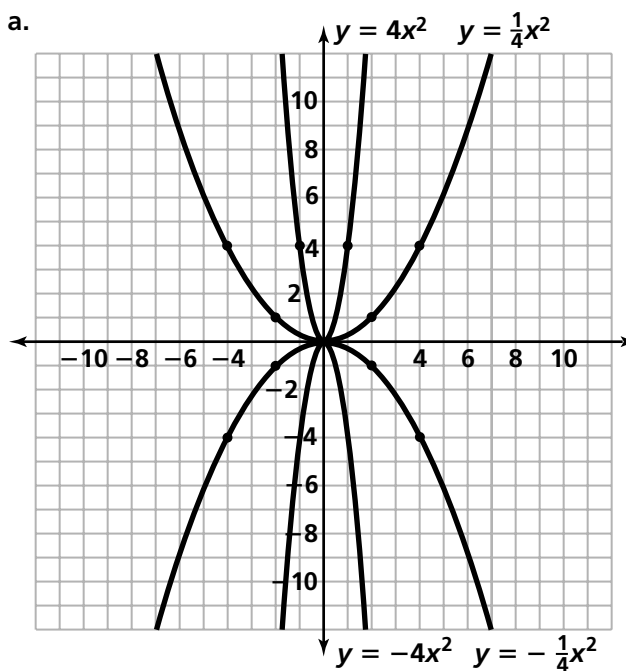
33. Possible answer:

$y = (x + 3)(x - 2) = x^2 + x - 6$. Some students may have equations that are quadratics with a factored form of $a(x + 3)(x - 2)$, where a is a non-zero real number. As long as the linear factors have -3 and 2 as their solutions for x when the factor is set equal to zero, the answer is valid. Also, equations which are not of the form $a(x + 3)(x - 2)$ may work, too. For example, $3(\frac{4}{3}x + 4)(x - 2)$, which expands to $y = 4x^2 + 4x - 24$, is a possible answer.

34. $y = -4x + 8$ is the only possible equation unless the student writes another equation that is equivalent to this.

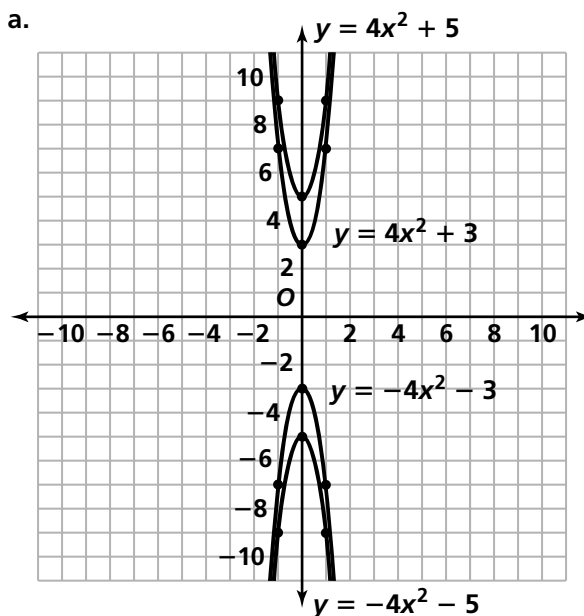
35. Possible answers: $y = 1.25^x$, $y = a(1.25)^x$, where a is a real number.

36. a.



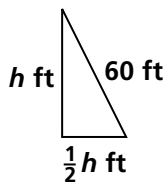
b. If a is positive, then the parabola opens up and if a is negative, then the parabola opens down. As $|a|$ increases the parabola becomes thinner and as $|a|$ decreases the parabola becomes wider.

37. a.



- b. The c -value is the y -intercept so changes in the c -value move the parabola up or down. So if c is 0, the y -intercept is at the origin, and when c increases, the parabola moves up, since the y -intercept value is increasing. As c decreases, the parabola moves down, since the y -intercept value is decreasing.

38. About 53.67. To find h in the diagram below using the Pythagorean theorem, we need to solve the equation $h^2 + (\frac{1}{2}h)^2 = 60^2$, which is the same as solving the equation $h^2 + \frac{1}{4}h^2 = 3,600$, or $\frac{5}{4}h^2 = 3,600$. Students may either divide both sides by $\frac{5}{4}$ and obtain the equation $h^2 = 2,880$, or they may look at the table of $y = \frac{5}{4}h^2$ in the graphing calculator and find h when $y = 3,600$.

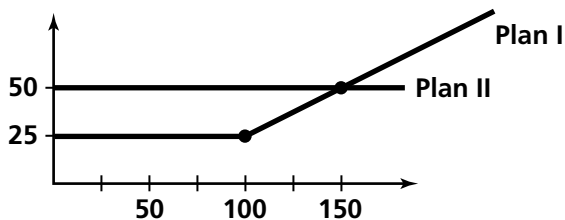


39. Cone 2. The base area of the first cylinder is πx^2 . The volume of cylinder 1 is $\pi x^2(3)$, so the volume of cone 1 is a third of that, or $\frac{1}{3}\pi x^2$. The base area of cylinder 2 is $\pi(3x)^2 = 9\pi x^2$, so the volume of this cylinder is $9\pi x^2(1)$ and the volume of cone 2 is a third of that, or $3\pi x^2$ cubic units.

Extensions

40. a. Plan II. If Caley uses Plan 1, she will owe $25 + 0.50 \times 100 = \$75$. If she uses Plan II, she will owe \$50. So Plan II is the better choice.
- b. The equations are the same when you use 150 minutes. In order to pay fifty dollars for Plan I, you would pay 25 for up to 100 minutes and then 25 dollars more for 50 more minutes for a total of $100 + 50 = 150$ minutes.

- c. Suppose that M_1 and M_2 are the monthly bill amounts and n is the number of minutes used. Plan I's equation is $M_1 = 25$ when n is from 0 to 100 minutes and $M_1 = 25 + (100 - n)0.5$ for more than 100 minutes. Plan II's equation is $M_2 = 50$. The growth pattern for M_2 is linear and M_1 is linear in pieces.



- d. Plan I is a horizontal line, and then after 100 minutes it has a positive slope. Plan II remains a horizontal line no matter how many minutes you talk. So Plan I will always cost more money if you talk more than 150 minutes.

41. a. 122.5 feet; A car should be $S = \frac{44^2}{32} + 44 + 18$ or 122.5 feet away.
- b. 30 mph; 44 ft/sec is $44 \times 60 = 2,640$ feet per minute and $2,640 \times 60 = 158,400$ feet per hour. Then $158,400/5,260 \approx 30$ miles per hour
- c. About 37.5 or 38 ft/sec (25.57 or 25.9 miles/hr). If a car is trailing 100 feet behind a car, its safe speed would be 37.5 or 38 feet per second, which is about 25.57 or 25.9 miles per hour. Students can find these values by putting the equation $S = \frac{v^2}{32} + v + 18$ into a graphing calculator and going to the table to find the value for v when S is 100.
42. a. Graph $y = x^2 + 4$. It is not possible to find x when y is zero, since there are no x -intercepts. Also, solving the equation $0 = x^2 + 4$ means finding a number that when you square it and add 4 gives you zero, which is impossible, since any real number whether negative or positive squared is positive, and adding four results in a positive number. Therefore the result cannot be zero.

b. Answers will vary. Some possible answers: $y = x^2 + 4$, $y = x^2 + 1x + 8$. For the teacher: Any answer $y = ax^2 + bx + c$ in which the value of $b^2 - 4ac$ is a negative number is a possible answer. This is because in the quadratic formula given in the Did You Know? in the SE after Problem 3.4, if $b^2 - 4ac$ is negative, the result is a negative value under the radical in the formula. This results in roots that are not in the real number system. When this happens the parabola does not cross the x -axis in the coordinate (real) plane.

c. Answers will vary. Some possible answers: $x^2 + 4x + 4$, $x^2 + 8x + 16$; any quadratic that can be factored into something of the form $a(yx + z)^2$ where a , y , and z are real numbers.

d. Answers will vary. Some possible answers: $x^2 - 4$, $x^2 + 6x + 8$.

43. a. Solutions of $(x + 2)(x - 1)(x - 5) = 0$ are $x = -2$, $x = 1$, or $x = 5$. Those solutions are shown on the graph by the points where the graph crosses the x -axis.

b. The values of x that satisfy $(x + 2)(x - 1)(x - 5) < 0$ are those such that $x < -2$ or $1 < x < 5$. That can be seen on the graph when portions of it are below the x -axis.

c. Using only the equation and answers to part (a), one can find answers to part (b) by “plugging in” a number for x that is less than -2 into the expression $(x + 2)(x - 1)(x - 5)$ and asking if the result is positive or negative. Repeat the process for a number between -2 and 1 , for a number between 1 and 5 , and for a number greater than 5 . When the result is a negative number for the chosen interval, the x -values in that interval satisfy the given inequality $(x + 2)(x - 1)(x - 5) < 0$.

Possible Answers to Mathematical Reflections

1. If an equation can be put in the form $y = mx + b$, then it is linear. You may have to apply the Distributive or Commutative properties in order to get the equation in this form. The highest exponent of the independent and dependent variable is 1. If an equation is of the form, $y = a(b)^x$ or is equivalent to an equation of this form then it is exponential. If an equation can be written in the form $y = ax^2 + bx + c$ for real numbers a , b , and c , the highest exponent of the independent variable is 2, and the exponent of the dependent variable is 1, then it is quadratic. In factored form, a quadratic will have exactly two linear factors and both factors must contain x^1 . For example, the quadratic expression $3(x + 3)(2x - 1)$ has exactly two linear factors $x + 3$ and $2x - 1$, each containing x^1 .

2. Looking at a linear equation $y = mx + b$, the value of b gives the y -intercept, the value of m gives the slope and tells you whether it is rising from left to right (when m is positive) or falling (when m is negative). The m also tells you how steep the slope is.

Looking at an exponential equation $y = a(b)^x$ the a tells you the y -intercept and the b tells you how fast the exponential will grow; it is the growth factor for the function.

Looking at the quadratic equation $y = ax^2 + bx + c$, the c tells you the y -intercept and the a tells you whether the parabola opens up (a is positive) or down (a is negative) and that the constant second difference is $2a$. The factored form of a quadratic expression makes it easier to see the x -intercepts. For example, if the quadratic equation is $y = (x + 3)(x - 1)$ the x -intercepts are the values for x that make each factor equal to 0 so they are -3 and 1 . The value for x between these two x -intercepts is the x -value of the maximum or minimum of the parabola. In the case of $y = (x + 3)(x - 1)$ the point $(-1, -4)$ is a maximum since the parabola opens up [the a will be positive when the equation $y = (x + 3)(x - 1)$ is expanded].