

# Investigation 3

## ACE

### Assignment Choices



#### Problem 3.1

Core 1, 4–7, 30, 31, 49

Other Applications 2, 3; Connections 29, 32; Extensions 47, 48

#### Problem 3.2

Core 8–15, 34

Other Applications 16, 17; Connections 33, 35–37, Extensions 50, 53; and unassigned choices from previous problems

#### Problem 3.3

Core 18–20

Other Connections 38–41, Extensions 51–52; and unassigned choices from previous problems

#### Problem 3.4

Core 21–26, 43

Other Connections 27–28, 42, 44–46; Extensions 54–57; and unassigned choices from previous problems

**Adapted** For suggestions about adapting Exercise 8 and other ACE exercises, see the CMP *Special Needs Handbook*.

**Connecting to Prior Units** 29, 30: *Moving Straight Ahead*; 31, 32: *Accentuate the Negative*; 33, 42–46: *Frogs, Fleas and Painted Cubes*; 36, 37, 39, 40: *Covering and Surrounding*; 38: *Looking for Pythagoras*; 41: *Filling and Wrapping and Looking for Pythagoras*

## Applications

1. a. Income will be  
 $1,000 + 5(40) + 15(30) = \$1,650$ .  
 Expenses will be  
 $500 + 250 + 6(40) + 8.50(30) = \$1,245$ .  
 The profit is thus  $\$1,650 - \$1,245 = \$405$ .  
 b.  $P = (1,000 + 5c + 15a) - (750 + 6c + 8.50a)$

$$\begin{aligned} \text{c. } P &= 1,000 + 5c + 15a - 750 - 6c - 8.50a \\ P &= 250 - c + 6.50a \end{aligned}$$

$$\begin{aligned} \text{d. } P &= 250 - 40 + 6.50(30) \\ P &= 250 - 40 + 195 \\ P &= 210 + 195 \\ P &= \$405 \end{aligned}$$

The answer is the same as in part(a): \$405 profit.

$$\begin{aligned} \text{e. } P &= 250 - c + 6.50a \\ \$1,099 &= 250 - 100 + 6.50a \\ 1,099 &= 150 + 6.50a \\ 1,099 - 150 &= 150 - 150 + 6.50a \\ 949 &= 6.50a \\ \frac{949}{6.50} &= \frac{6.50a}{6.50} \\ 146 &= a \end{aligned}$$

There are about 146 adults registered for the event.

2. a. When evaluating the second set of parentheses, Marcel distributed the minus sign to the 500, but not to the other three terms.  
 b. Kirsten combined  $5c - 6c$  and got  $c$  instead of  $-c$ .
3.  $1,000 = 200 + 50(T - 70)$   
 $1,000 = 200 + 50T - 3500$   
 $1,000 = 50T - 3300$   
 $4,300 = 50T$   
 $86 = T$

Other logical arguments are possible. Students might choose to solve this problem with a table or graph.

$$\begin{aligned} \text{4. } -4; 10 + 2(3 + 2x) &= 0 \\ 10 + 6 + 4x &= 0 \\ 16 + 4x &= 0 \\ 16 + 4x - 16 &= 0 - 16 \\ 4x &= -16 \\ x &= -4 \end{aligned}$$

5.  $1; 10 - 2(3 + 2x) = 0$

$$10 - 6 - 4x = 0$$

$$4 - 4x = 0$$

$$4 - 4x - 4 = 0 - 4$$

$$-4x = -4$$

$$x = 1$$

6.  $4; 10 + 2(3 - 2x) = 0$

$$10 + 6 - 4x = 0$$

$$16 - 4x = 0$$

$$16 - 4x - 16 = 0 - 16$$

$$-4x = -16$$

$$x = 4$$

7.  $-1; 10 - 2(3 - 2x) = 0$

$$10 - 6 + 4x = 0$$

$$4 + 4x = 0$$

$$4 + 4x - 4 = 0 - 4$$

$$4x = -4, \text{ so } x = -1$$

8. a. 17;

$$750 + 22(N - 12) = 650 + 30(N - 10)$$

$$750 + 22N - 264 = 650 + 30N - 300$$

$$486 + 22N = 350 + 30N$$

$$486 - 350 + 22N = 350 - 350 + 30N$$

$$136 + 22N = 30N$$

$$136 + 22N - 22N = 30N - 22N$$

$$\frac{136}{8} = \frac{8N}{8}$$

$$17 = N$$

NOTE: for this context to make sense,  
 $N \geq 12$ .

b. One possible way to check that the solution is correct is to put the value for  $N$ , 17, into each equation, solve for cost and see if both equations have the same value.

c. The point on the graphs at which the two lines intersect is the number of tiles for which the cost estimates are equal. Using a table one would look for the number of tiles for which both companies have the same cost values.

d. *Tile and Beyond* and *Cover and Surround It* have equal cost estimates when the number of tiles is 17.

e.  $C = 650 + 30(N - 10)$

$$C = 650 + 30N - 300$$

$$C = 30N + 350$$

The 30 means that each tile costs 30 dollars and the 350 is the start-up cost.

9. 80 boxes; Students may graph the two equations and find the  $x$ -coordinate of the intersection point. Or they may make a table for each equation and find for which  $x$ -coordinate the profits are equal. If students solve symbolically:

$$4s - 2(10 + s) = 5s - (100 + 2s)$$

$$4s - 20 - 2s = 5s - 100 - 2s$$

$$2s - 20 = 3s - 100$$

$$2s - 20 + 100 = 3s - 100 + 100$$

$$2s + 80 = 3s$$

$$2s + 80 - 2s = 3s - 2s$$

$$80 = s$$

10. One possible answer method:

$$8x + 16 = 6x$$

$$8x + 16 - 8x = 6x - 8x$$

$$16 = -2x$$

$$-8 = x$$

Check:

$$8(-8) + 16 = 6(-8)$$

$$-64 + 16 = -48$$

$$-48 = -48$$

11. One possible answer method:

$$8(x + 2) = 6x$$

$$8x + 16 = 6x$$

$$8x + 16 - 8x = 6x - 8x$$

$$16 = -2x$$

$$-8 = x$$

Check:

$$8(-8 + 2) = 6(-8)$$

$$8(-6) = -48$$

$$-48 = -48$$

12. One possible answer method:

$$6 + 8(x + 2) = 6x$$

$$6 + 8x + 16 = 6x$$

$$22 + 8x = 6x$$

$$22 + 8x - 8x = 6x - 8x$$

$$22 = -2x$$

$$-11 = x$$

Check:

$$6 + 8(x + 2) = 6x$$

$$6 + 8(-11 + 2) = 6(-11)$$

$$6 + 8(-9) = -66$$

$$6 + (-72) = -66$$

$$-66 = -66$$

13. One possible answer method:

$$\begin{aligned}4 + 5(x + 2) &= 7x \\4 + 5x + 10 &= 7x \\14 + 5x - 5x &= 7x - 5x \\14 &= 2x \\7 &= x\end{aligned}$$

Check:

$$\begin{aligned}4 + 5(7 + 2) &= 7(7) \\4 + 5(9) &= 49 \\49 &= 49\end{aligned}$$

14. One possible answer:

$$\begin{aligned}2x - 3(x + 6) &= -4(x - 1) \\2x - 3x - 18 &= -4x + 4 \\-x - 18 &= -4x + 4 \\-x - 18 - 4 &= -4x + 4 - 4 \\-x - 22 &= -4x \\-x - 22 + x &= -4x + x \\-22 &= -3x \\\frac{22}{3} &= x \text{ or } x = 7\frac{1}{3}\end{aligned}$$

Check:

$$\begin{aligned}2\left(\frac{22}{3}\right) - 3\left(\frac{22}{3} + 6\right) &= -4\left(\frac{22}{3} - 1\right) \\\frac{44}{3} - 22 - 18 &= -\frac{88}{3} + 4 \\\frac{44}{3} - 40 &= -\frac{88}{3} + \frac{12}{3} \\\frac{44}{3} - \frac{120}{3} &= -\frac{88}{3} + \frac{12}{3} \\-\frac{76}{3} &= -\frac{76}{3}\end{aligned}$$

15. One possible answer:

$$\begin{aligned}2 - 3(x + 4) &= 9 - (3 + 2x) \\2 - 3x - 12 &= 9 - 3 - 2x \\-3x - 10 &= 6 - 2x \\-3x - 10 - 6 &= 6 - 2x - 6 \\-3x - 16 &= -2x \\-3x - 16 + 3x &= -2x + 3x \\-16 &= x\end{aligned}$$

Check:

$$\begin{aligned}2 - 3(-16 + 4) &= 9 - (3 + 2(-16)) \\2 - 3(-12) &= 9 - (3 - 32) \\2 + 36 &= 9 - (-29) \\38 &= 38\end{aligned}$$

16. One possible answer:

$$\begin{aligned}2.75 - 7.75(5 - 2x) &= 26 \\2.75 - 38.75 + 15.5x &= 26 \\-36 + 15.5x &= 26 \\-36 + 36 + 15.5x &= 26 + 36 \\15.5x &= 62 \\x &= 4\end{aligned}$$

Check:

$$\begin{aligned}2.75 - 7.75(5 - 2(4)) &= 26 \\2.75 - 7.75(5 - 8) &= 26 \\2.75 - 7.75(-3) &= 26 \\2.75 + 23.25 &= 26\end{aligned}$$

$$\begin{aligned}17. \quad \frac{1}{2}x + 4 &= \frac{2}{3}x \\\frac{1}{2}x + 4 - \frac{1}{2}x &= \frac{2}{3}x - \frac{1}{2}x \\4 &= \frac{1}{6}x \\4 \div \frac{1}{6} &= x \\24 &= x\end{aligned}$$

Check:

$$\begin{aligned}\frac{1}{2}(24) + 4 &= \frac{2}{3}(24) \\12 + 4 &= 16 \\16 &= 16\end{aligned}$$

$$\begin{array}{ll}18. \text{ a. } x^2 - 4 & \text{ b. } x^2 - 25 \\ \text{ c. } x^2 - 16 & \text{ d. } x^2 - 144\end{array}$$

$$\begin{array}{ll}19. \text{ a. } (x + 1)(x + 4) & \text{ b. } (x + 2)(x + 4) \\ \text{ c. } (x - 5)(x - 2) & \text{ d. } x(x + 7) \\ \text{ e. } (x - 1)(x + 6) & \text{ f. } (2x + 3)(x - 4) \\ \text{ g. } (x + 1)(x - 8) & \text{ h. } x(x - 5)\end{array}$$

$$\begin{array}{ll}20. \text{ a. } (x + 4)(x - 4) & \text{ b. } (x + 6)(x - 6) \\ \text{ c. } (x - 7)(x + 7) & \text{ d. } (x + 20)(x - 20) \\ \text{ e. } (x - 8)(x + 8) & \text{ f. } (x - 12)(x + 12)\end{array}$$

$$\begin{aligned}21. \quad x^2 + 1.5x &= 0 \\x(x + 1.5) &= 0 \\x = 0 \text{ or } x &= -1.5\end{aligned}$$

$$\begin{aligned}22. \quad x^2 + 6x + 8 &= 0 \\(x + 2)(x + 4) &= 0 \\x = -2 \text{ or } x &= -4\end{aligned}$$

$$\begin{aligned}23. \quad 8x - x^2 &= 0 \\x(8 - x) &= 0 \\x = 0 \text{ or } x &= 8\end{aligned}$$

24. a. The jump equation is quadratic.

b.  $-8t(2t - 1)$  or  $8t(-2t + 1)$ . Some students may write an equivalent form like  $2(-8t^2 + 4t)$  for example, which is equivalent; however this will not help them when they try to solve symbolically in part (c), since there is still a quadratic factor in this expression.

c.  $\frac{1}{2}$ ; The flea lands on the ground when the height is 0 ft. So by solving the equation  $0 = -16t^2 + 8t$ , which is the same as solving  $0 = 8t(-2t + 1)$ , students should get  $8t = 0$  or  $-2t + 1 = 0$ . So  $t = 0$  which is when the flea starts the jump or  $t = \frac{1}{2}$  which is when the flea lands back on the ground.

25. a.

$x$	$x^2$	$3x$
5	$5x$	15
	$x$	3

$x^2 + 8x + 15$

b.

$3$		
$x$	$x(x - 3)$	$3(x - 3)$
	$x$	3

$x(x - 3) + 3(x - 3) = x^2 - 9$

c.

1	$x$	$x$	3
$x$	$x^2$	$x^2$	$3x$
	$x$	$x$	3

$2x^2 + 5x + 3$

NOTE: there are other area models that use  $2x + 3$  and  $x + 1$ .

26. a.  $x^2 + 8x + 15 = 0$   
 $(x + 5)(x + 3) = 0$   
 $x + 5 = 0$  or  $x + 3 = 0$   
 $x = -5$  or  $x = -3$

b.  $x^2 - 9 = 0$   
 $(x - 3)(x + 3) = 0$   
 OR  
 $x^2 - 9 = 0$   
 $x^2 = 9$   
 $x = 3$  or  $-3$

c.  $2x^2 + 5x + 3 = 0$   
 $2x^2 + 2x + 3x + 3 = 0$   
 $2x(x + 1) + 3(x + 1) = 0$   
 $(2x + 3)(x + 1) = 0$   
 $x = -1$  OR  $x = -\frac{3}{2}$

27.  $6x^2 - x = 1$   
 Solution:

$6x^2 - x - 1 = 0$   
 $(3x + 1)(2x - 1) = 0$   
 $3x + 1 = 0$  OR  $2x - 1 = 0$   
 $x = -\frac{1}{3}$  OR  $x = \frac{1}{2}$

- The factors the student gave were  $(3x - 1)(2x + 1)$ , which are the wrong factors, since when you use the Distributive Property you get  $6x^2 - 2x + 3x - 1$  or  $6x^2 + x - 1$ , and you need the middle term to be  $-1x$  not  $1x$ . With trial and error, students may find that if the signs are switched, the correct factorization can be found. So the correct factorization is

$(3x + 1)(2x - 1)$  with solutions of  $\frac{1}{2}$  and  $-\frac{1}{3}$ .

- To help the student, first I would tell them to check their answers in the original equation so they can see if they got the answer right. When factoring  $6x^2 - x - 1$ , students may suggest an alteration of Jaime's method that they found in Problem 3.3, or they can make an area model.

28. • The mistake is that the student divided each side of the equation by  $n$ . By doing this, the solution of  $n = 0$  disappears so the student is only partially correct.

- To help the student, first I would tell them to factor the expression  $24n^2 - 16n$  by factoring out  $8n$  and getting the new equation  $8n(3n - 2) = 0$  to solve. By setting the expression equal to zero, you get  $n = 0$  or  $n = \frac{2}{3}$ .

## Connections

29. a. Substitute  $11n$  (total number of boxes sold based on the number of choir members) for  $s$ , the number of boxes sold, in the equation  $P = 5s - (100 + 2s)$  and you will get:  
 $P = 5(11n) - [100 + 2(11n)]$

b. You can simplify the new equation for profit in part (a) by multiplying inside the parentheses, multiplying outside the parentheses, then applying the Distributive Property by multiplying  $-1$  by the numbers inside the brackets, and then combining like terms.

$$P = 5(11n) - [100 + 22n]$$

$$P = 55n - 100 - 22n$$

$$P = 33n - 100$$

c. If the number of choir members is 47, you would substitute it for  $n$  in the equation and solve for  $P$ :

$$P = 33(47) - 100$$

$$P = 1551 - 100$$

$$P = \$1451$$

d. Substitute \$1,088 for  $P$  and solve for  $n$ :

$$\$1,088 = 33n - 100$$

$$\$1,088 + 100 = 33n - 100 + 100$$

$$\$1,188 = 33n$$

$$\frac{\$1,188}{33} = \frac{33n}{33}$$

$$36 = n$$

There are 36 choir members when the profit is \$1,088.

e. The number of boxes  $x$  is 33 times the number of choir members,  $x = 33n$ . Therefore, when there are 108 choir members, there are  $33(108) = 3564$  boxes.

30. a. 11;  $48 = 2s + 2(s + 2)$

$$48 = 2s + 2s + 4$$

$$44 = 4s$$

$$11 = s$$

b. 11;  $48 = 4(s + 2) - 4$

$$48 = 4s + 8 - 4$$

$$48 = 4s + 4$$

$$48 - 4 = 4s + 4 - 4$$

$$44 = 4s$$

$$s = 11$$

c. The answers are the same since the expressions are equivalent expressions. So for any  $N$ -value, the corresponding  $s$ -value for both equations is the same.

31. D; Since  $\frac{3}{4}(20 - 4) = \frac{3}{4}(16) = 12$ .

32. H; Since  $5^2(7 - 5) + 1 = 25(2) + 1 = 50 + 1 = 51$

33. The largest square pool that can be built for the 40 tiles is a 9 tile by 9 tile pool because that would use  $(9 \text{ tiles} \times 4 \text{ sides}) +$  the 4 corner tiles would be 40 tiles.

34.  $16x + 22$ ;

$$2(9x + 15) - (8 + 2x) = 18x + 30 - 8 - 2x = 16x + 22$$

35.  $x - 32$ ;

$$(7x - 12) - 2(3x + 10) = 7x - 12 - 6x - 20 = x - 32$$

36. Approximately 4.899 meters; since  $x^2 = 24$  so  $x = \sqrt{24} \approx 4.899$

37. Approximately 2.76 meters; Using the formula  $A = \pi x^2$  we get  $24 = \pi x^2$  and so  $x^2 = \frac{24}{\pi}$  so  $x \approx 2.76$

38.  $x \approx 7.44$  meters; The height can be found using the Pythagorean theorem. When the height (altitude) is drawn, the resulting triangle has a side of length  $\frac{1}{2}x$  and a hypotenuse of  $x$ . Plug these into the Pythagorean theorem to solve  $(\frac{1}{2}x)^2 + h^2 = x^2$  for the height,  $h$ . So the height is  $\frac{\sqrt{3}}{2}x$ . Since the area is 24, you can use

the equation  $24 = \frac{1}{2}(x)(\frac{\sqrt{3}}{2}x)$  to find the value of  $x$ . Simplifying the right side gives

$24 = \frac{\sqrt{3}}{4}x^2$ . Dividing both sides by  $\frac{\sqrt{3}}{4}$ , you get that  $x^2 \approx 55.4256$  so  $x \approx 7.44$ .

39. The triangle has base 12 and height 4; since

$$24 = \frac{1}{2}(3x)(x) = \frac{1}{2}(3x^2) = \frac{3}{2}x^2 \text{ and}$$

$24 = \frac{3}{2}x^2$  simplifies to  $16 = x^2$ , the dimensions of the triangle are 4 and  $3 \times 4 = 12$ .

40. 3 and 8 meters; Since  $(x + 5)x = 24$  we need two factors of 24 which are 5 apart. These are 8 and 3 so  $x = 3$ . The dimensions of the rectangle are 3 and 8.

**41.** About 1.06 meters; The volume of the sphere is  $V = \frac{4}{3} \pi (\frac{3}{2})^3 = \frac{108}{24} \pi$  and the volume of the cylinder is  $V = \pi x^2 4$ , so we need  $x^2 4 = \frac{108}{24}$ . Dividing each side by 4 we get that  $x^2 = \frac{108}{96}$ . So the radius,  $x$ , of the cylindrical tank must be  $\sqrt{\frac{108}{96}}$ , or about 1.06 meters.  
(NOTE to teacher: This is an opportunity to see how students deal with irrational numbers involving fractions.)

**42.a.** Some possible answers:  $y = x^2 + 4x + 4$ ,  
 $y = x^2 + 6x + 9$ ,  $y = x^2 - 4x + 4$

**b.** Some possible answers:  $y = x^2 - 16$ ,  $y = x^2 + 7x + 10$  and  $y = x^2 + 6x - 7$

**43. a.** 3 seconds; to find out how long the ball is in the air, students could find out at what time the ball hits the ground or when height is zero. Students can do this by looking at a table or a graph for the value of  $t$  for which  $h$  is zero. Alternatively, they can solve the equation  $0 = 48t - 16t^2$  for  $t$ . Since solving  $0 = 48t - 16t^2$  is the same as solving  $16t(3 - t) = 0$ , we have  $t = 0$  or  $t = 3$ . So since the ball is back on the ground after 3 seconds, it is in the air for 3 seconds.

**b.** Yes; The maximum height of the ball will occur at  $t = 1.5$ . At this time the height of the ball will be 36, which can be found on a graph, on a table, or by substituting 1.5 into the equation to get  $48(1.5) - 16(1.5)^2 = 36$ , which is greater than 30 feet.

**44. a.** If the coefficient of the  $x^2$  term is positive, then the graph has a minimum point. If the coefficient of the  $x^2$  term is negative, then the graph has a maximum point. Either form can be used, but the coefficient of  $x^2$  is immediately available in the expanded form. In the factored form, some mental calculation may have to be done to find the coefficient.

**b.** The  $y$ -intercept can be read directly from the expanded form (the constant term), while the  $x$ -intercepts can be determined easily from the factored form (the values that make the factors zero).

**c.** The line of symmetry is a vertical line perpendicular to the  $x$ -axis through a point with an  $x$ -coordinate half way between the  $x$ -intercepts. The factored form can be used to find this point.

**d.** The  $x$ -coordinate of the maximum/minimum point lies on the line of symmetry. The factored form can be used to find the  $x$ -coordinate. To find the  $y$ -coordinate, substitute the value of  $x$  into either form to calculate the  $y$ -value.

**45. a.** If  $g = n^2 - n$ , then  $g = n(n - 1)$ . The  $x$ -intercepts are  $n = 0$  or  $n = 1$ . This means that for a league with 0 teams or 1 team, there are no league games.

**46. a.** 50,000 feet; the  $x$ -intercepts are 0 and 1,000. (The equation  $0.2x(1,000 - x) = 0$  has solutions  $x = 0$  and  $x = 1,000$ .) The axis of symmetry would be  $x = 500$ , so the maximum occurs when the  $x$ -coordinate is 500. This makes the height  $h = 0.2(500)(1,000 - 500) = 0.2(250,000) = 50,000$  feet.

## Extensions

**47.**  $c = -1$ ; when  $x = 3$  for  $3x + c = 2x - 2c$

$$3(3) + c = 2(3) - 2c$$

$$9 + c = 6 - 2c$$

$$9 - 6 + c = 6 - 6 - 2c$$

$$3 + c = -2c$$

$$3 + c - c = -2c - c$$

$$\frac{3}{-3} = \frac{-3c}{-3}$$

$$-1 = c$$

**48.**  $c = 5.5$ ; when  $x = 3$  for  $3x + c = cx - 2$

$$3(3) + c = c(3) - 2$$

$$9 + c = 3c - 2$$

$$9 + 2 + c = 3c - 2 + 2$$

$$11 + c = 3c$$

$$11 + c - c = 3c - c$$

$$\frac{11}{2} = \frac{2c}{2}$$

$$5.5 = c$$

**49.** Some possible answers:  $2x = 6$ ,  $9 - x = 6$ ,  $15 = 2x + 9$  or  $10 = 5(x - 1)$ . Not everyone will have the same equation.

50. The parentheses should be around  $(2 - 2x)$ :

$$\begin{aligned} 13 &= 3 + 5x - (2 - 2x) + 5 \\ 13 &= 3 + 5(1) - (2 - 2(1)) + 5 \\ 13 &= 3 + 5(1) - (2 - 2) + 5 \\ 13 &= 3 + 5(1) - 0 + 5 \\ 13 &= 3 + 5 - 0 + 5 \\ 13 &= 8 - 0 + 5 \\ 13 &= 8 + 5 \\ 13 &= 13 \end{aligned}$$

51. a.  $x^2 - 4$

b.  $x^2 - 156.25$

c.  $x^2 - 5$

d.  $x^2 - 2$

52. a.  $(x - 10)(x + 10)$

b.  $(x - 1.2)(x + 1.2)$

c.  $(x - \sqrt{7})(x + \sqrt{7})$

d.  $(x - \sqrt{24})(x + \sqrt{24})$

53. a. The equation of line 1 is  $y = -2x + 15$ . The equation of line 2 is  $y = 1.5x + 6$ .

- b.  $x \approx 2.6$ ;  $y \approx 10$ . (Previous discussion about the  $x$ -scale and the  $y$ -scale will be helpful here.)

- c. Solve  $1.5x + 6 = -2x + 15$  to get  $x \approx 2.571$ ,  $y \approx 9.857$ .

- d. Values of  $x$  that satisfy  $1.5x + 6 < -2x + 15$  are those values of  $x$  such that  $x < 2.571$ . The graph of  $y = 1.5x + 6$  is below the graph of  $y = -2x + 15$  for values of  $x < 2.571$ , i.e., to the left of the line  $x = 2.571$ .

- e.  $1.5x + 6 > -2x + 15$  when  $x > 2.571$ . The graph of  $y = 1.5x + 6$  is above the graph of  $y = -2x + 15$  for values of  $x > 2.571$ , i.e., to the right of the line  $x = 2.571$ .

54. a. The coordinates of the points where this graph crosses the  $x$ -axis are  $(0, 0)$  and  $(9, 0)$ .

- b. Rewrite the equation in factored form as  $y = x(x - 9)$ . The desired points are those whose  $x$ -coordinates are the  $x$ -intercepts for the given equation (the values of  $x$  for which  $y = 0$ ).

- c. The values of  $x$  that satisfy  $x^2 - 9x < 0$  are  $x$  such that  $0 < x < 9$ . The portion of the graph below the  $x$ -axis shows that.

- d. The values of  $x$  that satisfy  $x^2 - 9x > 0$  are  $x$  such that  $x > 9$  or  $x < 0$ . The portion of the graph to the right of the line  $x = 9$  or to the left of the line  $x = 0$  show that.

- e. The minimum value of  $y$  occurs when  $x = 4.5$ . That minimum value is  $-20.25$ .

55. a.  $a = 1$ ,  $b = -6$  and  $c = 8$ , so using the quadratic formula:

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(8)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 32}}{2}$$

$$x = \frac{6 \pm \sqrt{(4)}}{2} = \frac{6 \pm 2}{2}$$

$$x = 4 \text{ or } x = 2$$

- b.  $a = -1$ ,  $b = -1$  and  $c = 6$ , so using the quadratic formula:

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(-1)(6)}}{2(-1)}$$

$$x = \frac{1 \pm \sqrt{1 - (-24)}}{-2}$$

$$= \frac{1 \pm \sqrt{25}}{-2} = \frac{1 \pm 5}{-2}$$

$$x = -3 \text{ or } x = 2$$

- c.  $a = 1$ ,  $b = -7$  and  $c = 10$ , so using the quadratic formula:

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{49 - 40}}{2}$$

$$x = \frac{7 \pm \sqrt{9}}{2} = \frac{7 \pm 3}{2}$$

$$x = 5 \text{ or } x = 2$$

- d.  $a = 4$ ,  $b = -1$  and  $c = 0$ , so using the quadratic formula:

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(0)}}{2(4)}$$

$$x = \frac{1 \pm \sqrt{1 - 0}}{8}$$

$$x = \frac{1 \pm 1}{8}$$

$$x = 0 \text{ or } x = 0.25$$

e.  $a = 2$ ,  $b = -12$  and  $c = 18$ , so using the quadratic formula:

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(2)(18)}}{2(2)}$$

$$x = \frac{12 \pm \sqrt{144 - 144}}{4} = \frac{12 \pm 0}{4}$$

$$x = 3$$

f.  $a = 1$ ,  $b = 3$  and  $c = -4$ , so using the quadratic formula:

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{-(3) \pm \sqrt{(9) + 16}}{2}$$

$$x = \frac{-(3) \pm \sqrt{25}}{2} = \frac{-(3) \pm 5}{2}$$

$$x = 1 \text{ or } x = -4$$

56.  $x^2 + 5x + 7 = 1$   
 $x^2 + 5x + 7 - 1 = 1 - 1$   
 $x^2 + 5x + 6 = 0$   
 $(x + 2)(x + 3) = 0$   
 $x = -2 \text{ or } x = -3$

Check: If  $x = -2$ , then  
 $(-2)^2 + 5(-2) + 7 = 1$   
 $4 - 10 + 7 = 1$   
 $1 = 1$

57.  $x^2 + 6x + 15 = 6$   
 $x^2 + 6x + 15 - 6 = 6 - 6$   
 $x^2 + 6x + 9 = 0$   
 $(x + 3)(x + 3) = 0$   
 $x = -3$

Check: If  $x = -3$ , then  
 $(-3)^2 + 6(-3) + 15 = 6$   
 $9 - 18 + 15 = 6$   
 $6 = 6$

## Possible Answers to Mathematical Reflections

1. Linear equations can be solved by solving the equation to get  $x$  on one side and a constant on the other. The goal is to undo the operation on either side of the equation by adding, subtracting, multiplying, or dividing both sides by the same quantity. Some choices for proceeding are more helpful than others.

Adding or subtracting first does not always lead to a simpler equation. When equations include parentheses, you want to replace the side of the equation with the parentheses with an equivalent expression that is expanded.

Order of operations is used when you are simplifying expressions on one side of the equal sign. For example, you can use order of operations to solve the equation:

$42 = 6 + 2(x + 5)$ . You can either subtract 6 from both sides or you can simplify the right side and replace it with the expression  $6 + 2x + 10$ , getting  $42 = 6 + 2x + 10$ . Then you can replace the right side again with the expression  $16 + 2x$ , using the Commutative Property of Addition. Then you have  $42 = 16 + 2x$ . Subtracting 16 from both sides, you have  $26 = 2x$ , so  $x = 13$ . Alternatively, students can make a table to find the value of  $x$  for which the expressions on either side of the equation are equivalent, or they can graph the two related equations to find the solution.

2. You can make a table to find the value of  $x$  for which the expressions on either side of the equation are equivalent and a graph of the equation. You can also use graphs to find the solution to an equation. You may be able to factor the quadratic expression by applying the Distributive Property or by drawing an area model to find the factors and then setting each factor equal to 0 and solving for  $x$ . For example, in  $x^2 + 5x + 6 = 0$ , the Distributive Property helps you factor the expression to obtain  $(x + 3)(x + 2) = 0$ , so the solutions are  $x = -3$  or  $x = -2$ .
3. The solution or roots of a linear equation  $0 = mx + b$  are the  $x$ -intercepts of the graph of the associated linear equation,  $y = mx + b$ . Similarly the solutions, or roots, to a quadratic equation,  $0 = ax^2 + bx + c$ , are the  $x$ -intercepts of the graph of the associated graph of  $y = ax^2 + bx + c$ . To find the  $x$ -intercept of a linear equation or a quadratic equation you could substitute 0 for  $y$  and solve for  $x$ .