

# Investigation 2

## ACE

### Assignment Choices



#### Problem 2.1

Core 2, 3–5, 13–15

Other Applications 1, Connections 16–18, Extensions 38

#### Problem 2.2

Core 6–8, 19–20

Other Applications 9, Connections 21–26; and unassigned choices from previous problems

#### Problem 2.3

Core 12, 27–29, 39

Other Applications 10, 11; Connections 30–37; and unassigned choices from previous problems

**Adapted** For suggestions about adapting Exercise 1 and other ACE exercises, see the CMP *Special Needs Handbook*.

**Connecting to Prior Units** 14, 27–29: *Frogs, Fleas and Painted Cubes*; 13, 15, 16: *Say it with Symbols*, Investigation 1; 17–26: *Moving Straight Ahead*; 30–35: *Growing, Growing, Growing*; 36: *Bits and Pieces III*; 37: *Filling and Wrapping*

## Applications

1. a.  $I = 12n + 150$   
b.  $E = 250 + 4.25n$   
c. \$675; If you plug in 100 T-shirts to the income equation you'll get  $12(100) + 150 = 1,350$  in income and if you plug in 100 to the expense equation, you'll get  $E = 250 + 4.25(100) = 675$ . So the profit is  $1,350 - 675 = 675$ .  
d. Possible answers:  
 $P = 12n + 150 - (250 + 4.25n)$ ;  
 $P = 12n + 150 - 4.25n - 250$  or  
 $P = 7.75n - 100$
2. a.  $E = 125n + 30n + 700$  or  $E = 155n + 700$   
b.  $I = 350n$

- c.  $P = 350n - (125n + 30n + 700)$  or  
 $P = 350n - 125n - 30n - 700$  or  
 $P = 195n - 700$
- d. \$4,175; Substituting 25 for  $n$  into the profit equation we get  
 $P = 195(25) - 700 = 4,175$
- e. 9 bikers; Substituting 1,055 in for  $P$  in the equation  $P = 195n - 700$  and solving for  $n$  gives that  $1,055 = 195n - 700$  or  $1,755 = 195n$ . Dividing both sides by 195 we get that the number of bikers is 9.
- f. The profit equation is a linear equation because it can be written in the form  $y = mx + b$ , it has a constant rate of change and a linear graph (any of these three are acceptable answers).
3. B
4. F
5. C
6. \$375; Since the probability of rain is 50% or 0.50, the number of predicted visitors is  $V = 600 - 500(0.50) = 350$ . Based on this number the profit will be  
 $P = 2.50(350) - 500 = 375$
7. 72%; If students use the combined equation which is  $P = -1,250R + 1,000$  and solve for  $R$  when  $P = 100$  they should get 72%. If students use both equations separately, then for  $P = 100$  the number of visitors would be 240 which can be found by solving the equation  $100 = 2.50n - 500$  for  $n$ . So to find the probability of rain we must solve the other equation  $240 = 600 - 500R$  for  $R$  and we get 0.72 or 72%.
8. a. \$325; Combining both equations into one results in the equation,  
 $B = 100 + 0.50(600 - 500R)$  or  
 $B = 400 - 250R$ . So if the probability of rain is 30%, the daily employee-bonus fund is \$325. Students may use both equations

separately to find the number of visitors,  $V = 600 - 500R$  when  $R = 30\%$  which is 450. Then substitute 450 into the equation  $B = 100 + 0.50V$  and solve for  $V$  getting \$325.

b.  $B = 100 + 0.50(600 - 500R)$  or  
 $B = 400 - 250R$

c. \$275;  $B = 400 - 250(0.5)$  gives  $B = \$275$

d. 10%; Solving the equation  $\$375 = 400 - 250R$  for  $R$  gives  $R = 0.1$  or 10%.

9. a.  $65^\circ$ ;  $V = 50(T - 45)$   
 $1,000 = 50(T - 45)$   
 $1,000 = 50T - 2250$   
 $1,000 + 2250 = 50T - 2250 + 2250$   
 $3,250 = 50T$   
 $\frac{3,250}{50} = \frac{50T}{50}$   
 $65 = T$

b. To find the profit based on the temperature, substitute  $50(T - 45)$  for  $V$  in the equation  $P = 4.25V - 300$  and get  $P = 4.25[50(T - 45)] - 300$ .

c.  $P = -9862.5 + 212.5T$ ; To simplify  $P = 4.25[50(T - 45)] - 300$  first distribute the 50 by multiplying it by  $T$  and  $-45$ , multiply each of those terms by 4.25, and then combine like terms:

$$P = 4.25[50(T - 45)] - 300$$

$$P = 4.25[50T - 2250] - 300$$

$$P = -9562.5 + 212.5T - 300$$

$$P = -9862.5 + 212.5T$$

The 212.50 represents the rate of change for the profit as the temperature increases 1 degree. The y-intercept is  $-9862.5$ . However,  $-9862.5$  does not have a physical meaning since  $T$  must be always greater than or equal to  $45^\circ$  to have a positive number of visitors.  $T$  represents the independent variable or the temperature and  $P$  represents the dependent variable or the profit, which depends on the temperature because it changes at the rate of \$212.50 per  $1^\circ$  change in temperature.

d. \$5,012.50; Students may choose to use either of their equations from parts (b) or (c).

10. a.  $A = \ell(120 - \ell)$ ; since  $A = \ell w$ , we need to write  $w$  in terms of  $\ell$ . The only thing we know about the situation is that the perimeter is 240. So using the equation  $240 = 2\ell + 2w$  solving for  $w$  we get  $w = 120 - \ell$  and thus  $A = \ell(120 - \ell)$ .

b. The maximum area is when  $\ell$  and  $w$  each equal 60.

c. If you graph the equation  $A = \ell(120 - \ell)$  you would get a parabola that opens down. To find the maximum area you look at the maximum point on the parabola or the vertex. The  $x$ -coordinate is the length of the rectangle with the largest area. To find the width plug this  $x$ -coordinate into the equation  $w = 120 - \ell$  and solve for  $w$ .

d. The equation is quadratic because it is the product of two linear factors that are in terms of  $\ell$ . The equation  $A = \ell(120 - \ell)$  can be written as  $A = 120\ell - \ell^2$  where the exponent on  $\ell$  is 2 and this is the highest exponent to which  $\ell$  is raised.

11. a.  $A = \ell(120 - 0.5\ell)$ . Since  $A = \ell w$  we need to write  $w$  in terms of  $\ell$ . The only thing we know about the situation is that the fencing is 240 meters. So using the equation  $240 = w + \ell + w$ , we get that  $w = 120 - 0.5\ell$ .

b. The length would be 120 meters and so the width could be found using the equation  $w = 120 - 0.5\ell$  so the width would be 60 meters.

c. The equation is quadratic because it is the product of two linear factors that are in terms of  $\ell$ . The equation  $A = \ell(120 - 0.5\ell)$  can be written as  $A = 120\ell - 0.5\ell^2$  where the exponent on  $\ell$  is 2 and this is the highest exponent to which  $\ell$  is raised.

12. a. First we need to write the equation  $n = 20 - x$  in terms of  $x = 20 - n$ . So substituting into  $P = xn - 6n$  we get that  $P = (20 - n)n - 6n$  or  $P = 20n - n^2 - 6n$  which is equivalent to  $P = 14n - n^2$ .

b. \$40; using the equation  $P = 14n - n^2$ , substitute 10 in for  $n$ . The profit is  $P = 14(10) - 10^2 = 40$ .

- c. The selling price can be found using the equation  $n = 20 - x$ . So when  $n = 10$  the selling price is \$10.
- d. \$49; the greatest profit can be found by making a table or graph for the profit equation  $P = 20n - n^2 - 6n = 14n - n^2$ . The greatest profit occurs when they sell 7 posters, which yields a value of  $P = 14(7) - 7^2 = 49$ .

## Connections

13. J; Students can try an example like  $a = 1$  and  $b = 2$  to check that J is false. The other letters are true: F and H are the Associative Property of Addition and Multiplication, respectively, and G is the Commutative Property of Multiplication.

14.  $x(x + 5) = x^2 + 5x$

	$x$	$5$
$x$	$x^2$	$5x$

15.  $(2 + x)(2 + 3x) = (2 + x)2 + (2 + x)3x = 4 + 2x + 6x + 3x^2 = 4 + 8x + 3x^2$

	$2$	$3x$
$2$	$4$	$6x$
$x$	$2x$	$3x^2$

16.  $(x + 2)(2x + 3) = (x + 2)2x + (x + 2)3 = 2x^2 + 4x + 3x + 6 = 2x^2 + 7x + 6$

	$2x$	$3$
$x$	$2x^2$	$3x$
$2$	$4x$	$6$

17.a.  $11x - 12 = 30 + 5x$   
 $11x - 12 + 12 = 30 + 12 + 5x$   
 $11x = 42 + 5x$   
 $11x - 5x = 42 + 5x - 5x$   
 $6x = 42$   
 $\frac{6x}{6} = \frac{42}{6}$   
 $x = 7$

- b. To check, substitute 7 into the original equation for  $x$  and see if the values on each side of the equal sign are equal to each other.

$11x - 12 = 30 + 5x$   
 $11(7) - 12 = 30 + 5(7)$   
 $77 - 12 = 30 + 35$   
 $65 = 65$

- c. To solve the equation using a graph, first graph each of the equations  $y = 11x - 12$  and  $y = 30 + 5x$  and use the  $x$ -value of their point of intersection for the solution. To solve the equation using a table, look on the tables for each equation and see for which value of  $x$  their  $y$ -values coincide.

18. a. \$1,000

b.

Number of Soccer Balls	Income from Soccer Balls	Expenses of Soccer Balls
500	500	1,250
1,000	1,000	1,500
3,000	3,000	2,500

- c. To find the profit of soccer balls, subtract the expenses from the income. See table below.

Number of Soccer Balls	Profit of Soccer Balls
500	$500 - 1,250 = -750$
1,000	$1,000 - 1,500 = -500$
3,000	$3,000 - 2,500 = 500$

- d. The break-even point is at 2,000 soccer balls; the income and expenses are both \$2,000.

- e. Income = 1 times the number of soccer balls or

$$I = 1n$$

- Expenses = 1,000 + 0.5 times number of soccer balls or

$$E = 1,000 + 0.5n$$

- Profit = Income - Expenses or

$$P = 1n - (1,000 + 0.5n) \text{ or}$$

$$P = n - 1,000 - 0.5n, \text{ or}$$

$$P = 0.5n - 1000$$

- f. \$-125 or a loss of \$125; (Figure 1).

- g. 22,000 ; Profit =

$$-1,000 + 0.5(\text{number of soccer balls})$$

$$\$10,000 = -1,000 + 0.5n$$

$$10,000 + 1,000 = 1,000 + 1,000 + 0.5n$$

$$11,000 = 0.5n$$

$$\frac{11,000}{0.5} = \frac{0.5n}{0.5}$$

$$22,000 = n$$

- The number of soccer balls produced and sold if the profit is \$10,000 is 22,000

19. One possible solution:

$$7x + 15 = 12x + 5$$

$$7x - 7x + 15 = 12x - 7x + 5$$

$$15 = 5x + 5$$

$$15 - 5 = 5x + 5 - 5$$

$$10 = 5x$$

$$\frac{10}{5} = \frac{5x}{5}$$

$$2 = x$$

Check:

$$7x + 15 = 12x + 5$$

$$7(2) + 15 = 12(2) + 5$$

$$14 + 15 = 24 + 5$$

$$29 = 29$$

20.  $x = 2$ ; The solution is the same as Exercise 19 because the Commutative Property does not change the value of the variables when solving an equation.

21. One possible solution:

$$-3x + 5 = 2x - 10$$

$$-3x - 2x + 5 = 2x - 2x - 10$$

$$-5x + 5 = -10$$

$$-5x + 5 - 5 = -10 - 5$$

$$-5x = -15$$

$$\frac{-5x}{-5} = \frac{-15}{-5}$$

$$x = 3$$

Check:

$$-3x + 5 = 2x - 10$$

$$-3(3) + 5 = 2(3) - 10$$

$$-9 + 5 = 6 - 10$$

$$-4 = -4$$

22. One possible method:

$$14 - 3x = 1.5x + 5$$

$$14 - 3x - 14 = 1.5x + 5 - 14$$

$$-3x = 1.5x - 9$$

$$-3x - 1.5x = 1.5x - 9 - 1.5x$$

$$(-3 - 1.5)x = -9$$

$$-4.5x = -9$$

$$x = 2$$

Check:

$$14 - 3(2) = 1.5(2) + 5$$

$$8 = 3 + 5$$

$$8 = 8$$

Figure 1

Number of Soccer Balls	Income = # of Soccer Balls $I = 1n$	Expenses = 1,000 + 0.5 (# of soccer balls)	Profit = Income - Expenses or $P = 0.5n - 1,000$
1,750	1,750	$1,000 + 0.5(1,750) = 1,000 + 875 = 1,875$	$1,750 - 1,875 = -125$ , or $0.5(1,750) - 1,000 = 875 - 1,000 = -125$

23. One possible solution:

$$\begin{aligned} 9 - 4x &= \frac{(3 + x)}{2} \\ 2(9 - 4x) &= 2 \times \frac{(3 + x)}{2} \\ 18 - 8x &= 3 + x \\ 18 - 8x - x &= 3 + x - x \\ 18 - 9x &= 3 \\ 18 - 9x - 18 &= 3 - 18 \\ -9x &= -15 \\ \frac{-9x}{-9} &= \frac{-15}{-9} \\ x &= 1\frac{2}{3} \end{aligned}$$

Check:

$$\begin{aligned} 9 - 4(1\frac{2}{3}) &= \frac{3 + (1\frac{2}{3})}{2} \\ 9 - 6\frac{2}{3} &= \frac{4\frac{2}{3}}{2} \\ 2\frac{1}{3} &= 2\frac{1}{3} \end{aligned}$$

24. One possible solution:

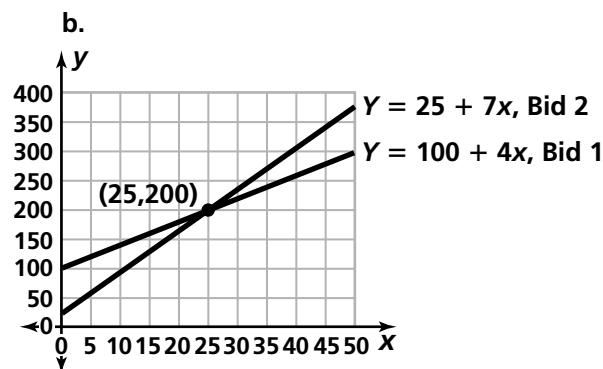
$$\begin{aligned} -3(x + 5) &= \frac{(2x - 10)}{3} \\ -3x - 15 &= \frac{(2x - 10)}{3} \\ 3(-3x - 15) &= 3 \times \frac{(2x - 10)}{3} \\ -9x - 45 &= 2x - 10 \\ -9x - 2x - 45 &= 2x - 2x - 10 \\ -11x - 45 &= -10 \\ -11x - 45 + 45 &= -10 + 45 \\ -11x &= 35 \\ \frac{-11x}{-11} &= \frac{35}{-11} \\ x &= -3\frac{2}{11} \end{aligned}$$

Check:

$$\begin{aligned} -3(-3\frac{2}{11} + 5) &= \frac{2x - 10}{3} \\ -3(1\frac{9}{11}) &= \frac{2(-3\frac{2}{11}) - 10}{3} \\ \frac{-60}{11} &= \frac{\frac{-70}{11} - 10}{3} \\ \frac{-60}{11} &= \frac{\frac{-180}{11}}{3} \\ \frac{-180}{11} &= \frac{-180}{11} \end{aligned}$$

25. a. The two bids are equal when the y-values for a common x-value are equal. This occurs when  $x = 25$  and  $y = 200$ , meaning the bids are both \$200 for 25 books.

$x$ (Number of books printed)	$Y = 100 + 4x$	$Y = 25 + 7x$
10	\$140	\$ 95
15	160	130
20	180	165
25	200	200
30	220	235



c. For 25 books, the bids are equal. The graph shows that for more than 25 books, Bid 1 is less than Bid 2 because the graph for Bid 1 is lower than the graph for Bid 2 for  $x > 25$ . For example, if the number of books is 26, Bid 1 is \$204 and Bid 2 is \$207. Since Bid 2 increases more with each book, if the number of books is greater than 25, Bid 1 is lower.

26. a. Bid 1:  $100 + 4(75) = \$400$ ,  
 Bid 2:  $25 + 7(75) = \$550$   
 (Students might also find these values from the table or the graph.)
- b. The greatest number of books that can be printed is 50 for Bid 1 and 39 for Bid 2. Explanations will vary. Students might extend their tables or graphs, use trial and error, or apply methods for solving linear equations.

- c. The related equations are  $y = 8x$  and  $y = 30 + 6x$ . The two bids are equal when  $x = 15$  and  $y = 120$ , meaning they are both \$120 for 15 books. Explanations will vary; students may use tables, graphs, or begin to see a pattern and solve the equations  $8x = 30 + 6x$  symbolically.

27. a. 210;  $(21 \times 20) \div 2 = 420 \div 2 = 210$

b.  $h = \frac{n(n-1)}{2}$

c.  $h = \frac{n^2 - n}{2}$  or  $h = \frac{1}{2}(n^2 - n)$

28. a.  $x^2 + 7x + 10$  or  $x^2 + 2x + 5x + 10$

- b. Answers will vary. Students may use an area model to justify that their expressions are equivalent. They may use a graph or a table to show that their expressions are equivalent. (Teacher: Note that if three points satisfy different quadratic expressions, then the expressions are equivalent.)

29. a. y-intercept is 10; Students may find this by looking at a graph or a table for when  $x = 0$ .

- b. The x-intercepts are  $-2$  and  $-5$ . Students can find the x-intercepts by looking at a graph or a table for when  $y = 0$ .

- c. The minimum is at  $x = -3.5$  where the value of  $y$  is  $-2.25$ . The students may use a table or graph. There is no maximum.

- d. The line of symmetry is the vertical line through the value  $x = -3.5$ .

X	Y
-5	0
-4	-2
-3	-2
-2	0
-1	4
0	10
1	18
2	28
3	40
4	54
5	70
6	88

30.  $x^5$

31.  $x^6$

32.  $x^4$

33.  $x^3$

34.  $\frac{1}{x^3}$

35.  $2x^3$

36. a. \$34,500; she gets  $30,000 \times 1.15 = 34,500$ .

b. \$33,600; since  $30,000 \times 1.12 = 33,600$ .

c. \$33,000; since  $30,000 \times 1.10 = 33,000$ .

37. a. Cylinder A is fatter and shorter than either of the other 2 cylinders. Cylinder C is the same height as Cylinder B but skinnier. Cylinders B and C are both twice as tall as Cylinder A.

- b. Cylinder A has the largest surface area. Students may count squares on the grid pattern to estimate the surface area. If they use formulas they will probably use the actual measurements. In general, Cylinder A has radius 2 and height 4 and surface area  $= 2(\pi(2)^2) + (\pi(4)(4)) = 24\pi$ . Cylinder B has radius 1 and height 8 and surface area  $= 2(\pi(1)^2) + (\pi)(2)(8) = 18\pi$ . (Cylinder C's surface area is smaller than Cylinder B since its rectangle and circles are smaller)

- c. Cylinder A; Since volume equals  $\pi r^2 h$  we need to find when  $r^2 h$  is the greatest. For Cylinder A,  $r^2 h = 16$ ; for Cylinder B,  $r^2 h = 8$ ; and for Cylinder C,  $r^2 h = 2$ .

## Extensions

38. a.  $P = 15x - 500 + 106x - x^2$

- b. The maximum profit is 3,160.25 which occurs between 60 and 61 people.

39. Completion Rating:

$$CR = 5\left(\frac{288}{474}\right) - 1.5 \approx 1.5$$

Yards Rating:

$$YR = \frac{\frac{3692}{474} - 3}{4} \approx 1.2$$

Touchdown Rating:

$$TR = 20\left(\frac{28}{474}\right) \approx 1.18$$

Interception Rating:

$$IR = 25(0.095 - \frac{14}{474}) \approx 1.64$$

Overall Rating:

$$\begin{aligned} OR &= 100 \frac{(CR + YR + TR + IR)}{6} \\ &\approx 100 \frac{(1.5 + 1.2 + 1.18 + 1.64)}{6} \\ &= 92 \end{aligned}$$

## Possible Answers to Mathematical Reflections

1. If you have two or more equations for the amount of money each person collects for walking  $n$  kilometers, you can add them to find the total amount  $t$  of money collected by the group. For example if  

$$M_{\text{Leanne}} = 16(10)$$

$$M_{\text{Gilberto}} = 7(2n)$$

$$M_{\text{Alana}} = 11(5 + 0.5n); \text{ then}$$

$$t = 16(10) + 7(2n) + 11(5 + 0.5n) = 19.5n + 215.$$
2. Answers will vary. If you have two equations and they have a variable in common like  $V$ , the number of visitors in Problem 2.2 where  $P = 2.50V - 500$  and  $V = 600 - 500R$ , you can combine the equations into one by taking the expression  $600 - 500R$  for  $V$  and substituting it into the equation,  $P = 2.50V - 500$ . The equation becomes  $P = 2.50(600 - 500R) - 500$ . By combining them into one equation, if you know the probability of rain and want to predict the profit, you only have to do one calculation instead of two separate calculations.
3. The advantage of working with one equation is that you only have to solve one equation. If you have to find more than one data point such as in the previous example—finding the profit when the probability of rain is 10%, 20%, 30%, etc, then you can graph the equation or make a table in your calculator and find all the profit values at once. A disadvantage of combining into one equation is that you may not be able to see the individual patterns that are involved in the separate equations and you may lose sense of the context of the problem. For example, the equation  $V = 600 - 500R$  tells you that when the probability of rain is 0 there will be 600 visitors and that as the probability of rain increases, the number of visitors decreases. If profit is written in terms of the probability of rain, you lose the information about the number of visitors.