

# Investigation 1

## ACE

### Assignment Choices



#### Problem 1.1

Core 1, 2, 18–20

Other *Connections* 21–24

#### Problem 1.2

Core 3, 4, 25, 26

Other *Connections* 27–32; *Extensions* 57, 58; unassigned choices from previous problems

#### Problem 1.3

Core 5, 6

Other *Connections* 33–50; unassigned choices from previous problems

#### Problem 1.4

Core 8, 9, 12–14, 51

Other *Applications* 7, 10, 11, 15–17;

*Connections* 52–56, *Extensions* 59; unassigned choices from previous problems

**Adapted** For suggestions about adapting Exercise 3 and other ACE exercises, see the *CMP Special Needs Handbook*.

**Connecting to Prior Units** 18–23, 27–32: *Frogs, Fleas, and Painted Cubes*; 24, 50: *Covering and Surrounding*; 25, 52–54: *Moving Straight Ahead*; 26, 37–45, 56: *Accentuate the Negative*; 33–36: *Bits and Pieces II*; 46–49: *Prime Time*; 55: *Filling and Wrapping*

## Applications

1. a.  $2(10) + 2(5) + 4 = 34$  tiles

b. Possible equations:

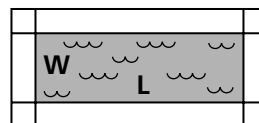
$$N = 2L + 2W + 4$$

$$N = 2(L + 1) + 2(W + 1)$$

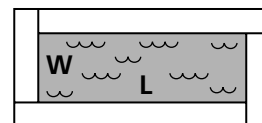
$$N = 2(L + 2) + 2W$$

$$N = 2L + 2(W + 2)$$

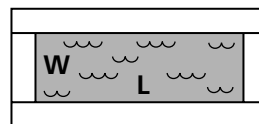
c. See part (b) for some equations; explanations will vary. Students might draw sketches. For example:



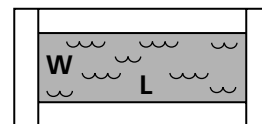
$$2L + 2W + 4$$



$$2(L + 1) + 2(W + 1)$$



$$2(L + 2) + 2W$$



$$2L + 2(W + 2)$$

They might substitute values for  $L$  and  $W$  in the equations; for example, when  $W = 2$  and  $L = 3$ :

$$N = 2L + 2W + 4 = 2(3) + 2(2) + 4 = 14$$

$$N = 2(L + 1) + 2(W + 1) = 2(4) + 2(3) = 14$$

$$N = 2(L + 2) + 2W = 2(5) + 2(2) = 14$$

$$N = 2L + 2(W + 2) = 2(3) + 2(4) = 14$$

2. a.  $4(7) + 4(0.5) = 30$  tiles

b. Possible answers:

$$N = 4s + 2$$

$$N = 4(s + 0.5)$$

$$N = 2s + 2(s + 1)$$

c. See equations in part (b). Students might substitute values for  $s$  [in this case 2 values ( $s, N$ ) are sufficient because these are linear relationships], generate tables for both equations, or make a geometric argument to show that the two equations are equivalent. They may also graph each equation.

d. The relationship is linear; students may say that this is because the graphs are straight lines; the table increases by a constant value of 4 for every increase of 1 ft in the side length.

3. a.  $2(30) + 2(20) + 2 = 102$  tiles

b. Possible answers:

$$N = 2L + 2W + 2$$

$$N = 2(L + 0.5) + 2(W + 0.5)$$

$$N = 2(W + 1) + 2L$$

- c. Students might substitute values for  $L$  and  $W$ , create tables or graphs, or make geometric arguments to show that their two equations are equivalent.

4. a. First equation:  $4\left(\frac{0}{2} + \frac{0}{4}\right) + 4 = 4(0) + 4 = 4$ ;

Second equation:  $2(0 + 0.5) + 2(0 + 1.5) = 2(0.5) + 2(1.5) = 1 + 3 = 4$ ;

Third equation:  $4\left[\frac{0 + (0 + 2)}{2}\right] = 4\left(\frac{2}{2}\right) = 4$

- b. You cannot determine whether the expressions are equivalent by checking them at one point, although students may think that they are equivalent since these expressions produced the same number of tiles for  $s = 0$ .

c. First equation:  $4\left(\frac{12}{2} + \frac{12}{4}\right) + 4 = 4(6 + 3) + 4 = 40$ ;

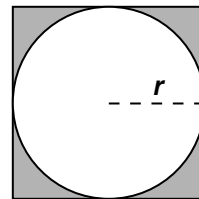
Second equation:  $2(12 + 0.5) + 2(12 + 1.5) = 2(12.5) + 2(13.5) = 52$ ;

Third equation:  $4\left[\frac{12 + (12 + 2)}{2}\right] = 4\left(\frac{26}{2}\right) = 4(13) = 52$

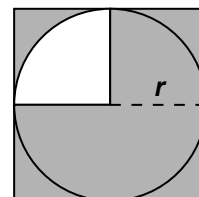
- d. Since you can determine non-equivalency of linear equations by checking one point, the first expression is not equal to the second and the third expressions because they did not produce the same number of tiles when you checked using the same side value.

In general, it is not enough to show that two expressions are equivalent when they have the same value at two different points, because you need to check all points, which is impossible. However, for linear equations such as those in this problem, checking only two values would be enough because only one line can pass through the two points. So linear expressions which agree on two values (two points) contain the same two points. So, the lines that they represent must be the same. Students will either need to check all points, which is impossible, or know that two points uniquely determine a line. (This topic was addressed on the Summary Transparency for Problem 1.2.)

5. a. The shape is the area between the circle and the square.



- b. The shape is all the area inside the square except a quarter of the area of the circle.



6. a. ii and iv

- b. i and iii

- c. For part (a), ii and iv are equivalent since:

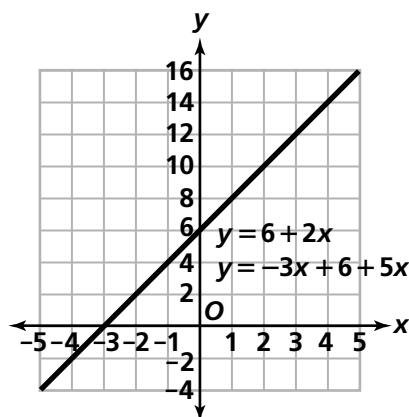
$$(s - 10)^2 = (s - 10)(s - 10) = s(s - 10) - 10(s - 10) = s^2 - 10s - 10s + 100 = s^2 - 20s + 100.$$

For part (b), i and iii are equivalent because they both represent the same part of the pool.

- d. Answers will vary, but must be equivalent to  $A = (s^2 - 20s + 100) + (3s^2 - 10s)$
- e. The equation in part (d) is a quadratic relationship.

7. a.

$x$	$-3x + 6 + 5x$	$6 + 2x$
-3	0	0
-2	2	2
-1	4	4
0	6	6
1	8	8
2	10	10
3	12	12

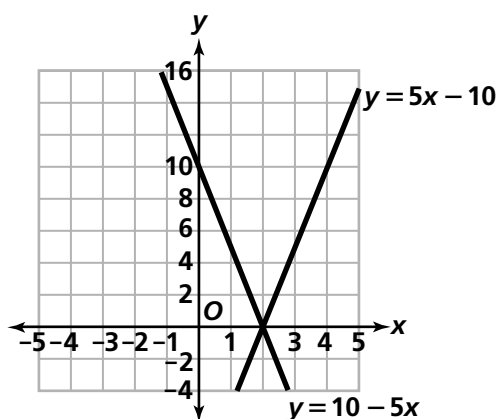


b. The expressions are equivalent because the table values are the same and the graph is a single line. NOTE: These are linear expressions so it is enough to show that they all pass through the same two points.

c.  $-3x + 6 + 5x = 6 + -3x + 5x =$   
 $6 + (-3 + 5)x = 6 + 2x$

8. a.

x	$10 - 5x$	$5x - 10$
-3	25	-25
-2	20	-20
-1	15	-15
0	10	-10
1	5	-5
2	0	0
3	-5	5

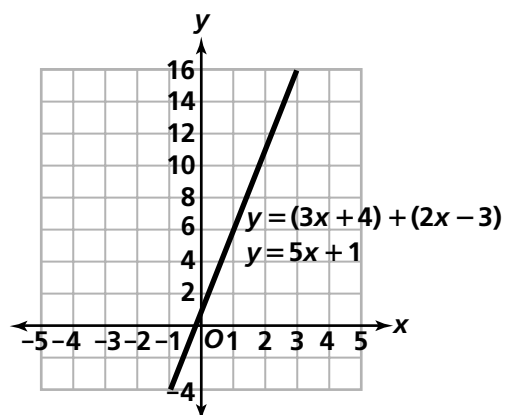


b. The expressions are not equivalent because the table values are different and the graphs are separate lines; one has a negative slope and one has a positive slope.

c.  $10 - 5x = -5x + 10 \neq 5x - 10$

9. a.

x	$(3x + 4) + (2x - 3)$	$5x + 1$
-3	-14	-14
-2	-9	-9
-1	-4	-4
0	1	1
1	6	6
2	11	11
3	16	16



b. The expressions are equivalent because the table values are the same and the graph is a single line. NOTE: These are linear expressions so it is enough to show that they all pass through the same two points.

c.  $(3x + 4) + (2x - 3) = 3x + 2x + 4 - 3 =$   
 $(3 + 2)x + 1 = 5x + 1$

10. a.  $3x + 21$

b.  $25 - 5x$

c.  $8x - 16$

d.  $x^2 + 4x + 2x + 8 = x^2 + x(4 + 2) + 8 =$   
 $x^2 + 6x + 8$

11. a. Possible answers:  $2(x - 5x)$  or  $x(2 - 10) =$   
 $-8x$

b.  $2(x + 3)$

c.  $7(2 - x)$

12. a. equal;  $3x + 7x = (3 + 7)x = 10x$

b. not equal;  $5x - 10x = (5 - 10)x =$   
 $-5x \neq 5x$

c. equal;  $4(1 + 2x) - 3x = 4 + 8x - 3x =$   
 $4 + 5x = 5x + 4$

Using the Commutative Property of Addition,  $5x + 4 = 4 + 5x$ .

d. equal;  $5 - 3(2 - 4x) = 5 - 6 + 12x =$   
 $-1 + 12x$

13. Step (1): Distributive Property  
 Step (2): Commutative Property  
 Step (3): Distributive Property  
 Step (4): Addition

14. Possible answers:  $3(2x + 1), x + 5x + 3,$   
 $2x + 2 + 4x + 1.$

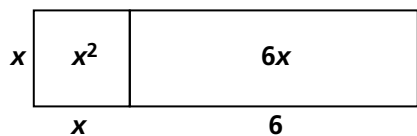
15.  $(7 + 5)p - p = 11p$

16.  $7 + 5(p - p) = 7$

17. Parentheses are not needed.

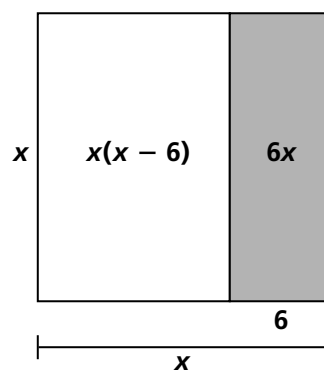
## Connections

18.



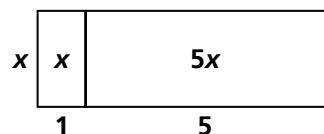
$$x(x + 6) = x^2 + 6x$$

19.



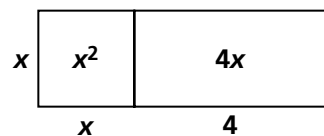
$$x(x - 6) = x^2 - 6x$$

20.



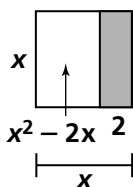
$$x(1 + 5) = x + 5x$$

21.



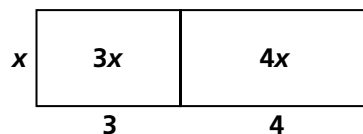
$$x^2 + 4x = x(x + 4)$$

22.



$$x^2 - 2x = x(x - 2)$$

23.



$$3x + 4x = x(3 + 4) \text{ or } 7x$$

24. a. Area of water =  $\pi(4)^2 = 16\pi \approx 50 \text{ ft}^2$

b. Area of border =  $\pi(5^2) - \pi(4^2) = 25\pi - 16\pi = 9\pi \approx 28 \text{ ft}^2$

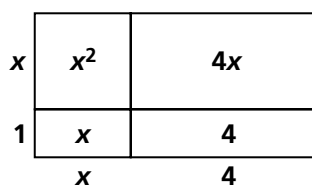
c. Area of water =  $\pi r^2$

d. Area of border =  $\pi(r + 1)^2 - \pi r^2$ , or  $2\pi r + \pi$

25. B

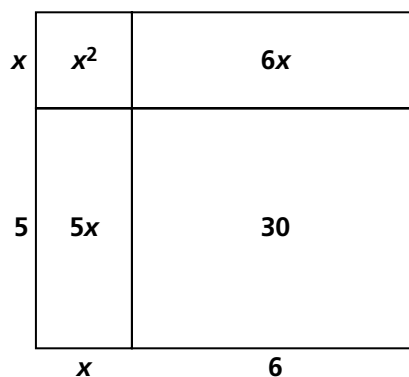
26. J

27.



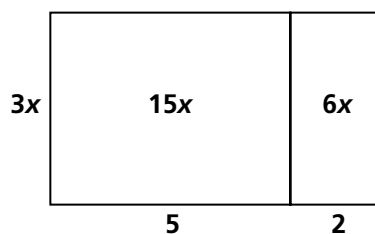
$$(x + 1)(x + 4) = x^2 + 1x + 4x + 4 \text{ or } x^2 + 5x + 4$$

28.



$$(x + 5)(x + 6) = x^2 + 5x + 6x + 30 \text{ or } x^2 + 11x + 30$$

29.



$$3x(5 + 2) = 15x + 6x \text{ or } 21x$$

30.

$x$	$x^2$	$2x$
1	$x$	2
	$x$	2

$$x^2 + x + 2x + 2 =$$

$$x^2 + 3x + 2 = (x + 1)(x + 2)$$

31.

$x$	$x^2$	$2x$
5	$5x$	10
	$x$	2

$$x^2 + 7x + 10 = (x + 5)(x + 2)$$

32.

$x$	$x^2$	$7x$
7	$7x$	49
	$x$	7

$$x^2 + 14x + 49 = (x + 7)(x + 7)$$

33.  $\frac{8}{21}$

34.  $\frac{17}{6}$

35.  $x$

36.  $\frac{1}{6}x$

37. 28

38. 12

39.  $\frac{1}{7}$

40. 66

41. 5

42. -5

43. -54x

44. 3x

45. -6x

46. 5

47. 12

48. 25

49. 3

50. a. Possible answers:  $(2x)(4x) + \pi(x)^2$  or  $\frac{1}{2}\pi(x)^2 + \frac{1}{2}\pi(x)^2 + 8x^2$

b. The fencing needed for the rectangular region is  $4x + 4x = 8x$  since you don't count the two shorter sides. The two half circles each have a perimeter of  $\frac{1}{2}\pi(2x)$ , which is half of the circumference  $\pi(2x)$ . So the perimeter is  $8x + 2[\frac{1}{2}\pi(2x)]$  or  $2\pi x + 8x$ .

c. Possible answers:  $\pi x + \pi x + 4x + 4x$  or  $(2\pi + 8)x$ .

51. a. Yes.  $8 + 4(s - 1) = 8 + 4s - 4 = 8 - 4 + 4s = 4 + 4s = 4s + 4$

b. Hank

52. a. Since the expression represents her money after one year, she would have the money she put in, which is  $D$ , plus the interest the account accrues in that year, which is 0.10 times  $D$ , so the expression  $D + 0.10D$  is correct.

b.  $D(1 + 0.10)$       c.  $\$1,500(1.1) = \$1,650$

53. a. Corey's estimate is correct:  
 $C = 200 + 10(50) = 200 + 500 = \$700$ .

b. Duncan performed the operations incorrectly by doing the addition first:  
 $C = (200 + 10)50 = \$10,500$ .

54. a.  $S = \frac{200 + 10(20)}{20} = \frac{200 + 200}{20} = \frac{400}{20} = \$20$

b.  $S = \frac{200 + 10(N)}{N}$

c.  $S = \frac{200 + 10(40)}{40} = \frac{200 + 400}{40} = \frac{600}{40} = \$15$

55. a. The volume of the prism is  $6 \times 6 \times 4$  cubic units = 144 cubic units. So the volume of the pyramid is  $\frac{144}{3} = 48$  cubic units.

b. A cube with edges 2 units would have volume 8 cubic units. A pyramid that fits inside of this cube would have the given volume.

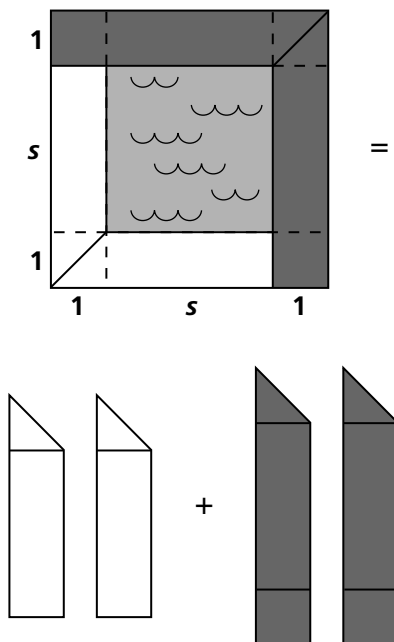
c. A cube with edge 3 units would have volume 27 cubic units, so a pyramid that fits inside this cube would have the given volume.

d. A cube with edge  $3x$  units would have volume  $27x^3$  cubic units. So a pyramid with base  $3x$  by  $3x$  and height  $3x$  units would have a volume of  $\frac{1}{3}(27x^3) = 9x^3$ .

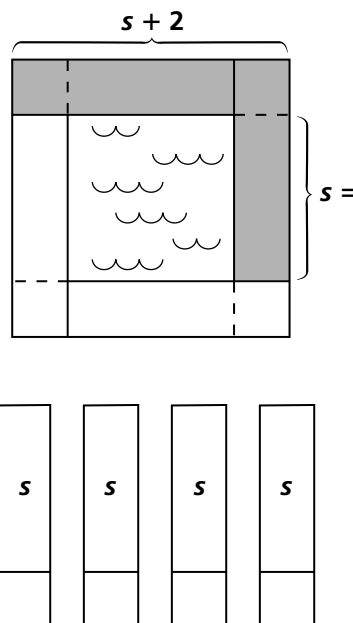
56. a. Sarah performed the calculations correctly.  
 b. Emily did not use the order of operations correctly. In the second line, she added 4 and 11 before the multiplication of 11 and 3. In the third line, she added 15 and 10 to get 25 instead of multiplying the 15 by 3.

## Extensions

57. a. For  $s = 1$ , 8 tiles are needed.  
 For  $s = 2$ ,  $8 + 4$  tiles are needed.  
 For  $s = 3$ ,  $8 + 4 + 4$  tiles are needed.  
 Thus, for any  $s$ , the number of tiles needed is equal to 8 plus  $(s - 1)$  fours, or  
 $N = 8 + 4(s - 1)$ .
- b. Percy's equation is equivalent to Stella's equation,  $4(s + 1)$ . Explanations will vary; they may be based on tables, graphs, the substitution of specific values of  $s$ , or the sameness of the expressions.
58. For  $2(s + 0.5) + 2(s + 1.5)$ , the picture should look like:



For  $4\left[\frac{s + (s + 2)}{2}\right]$ , the picture should look like:



where  $4\left[\frac{s + (s + 2)}{2}\right]$  is the area of half the shaded region multiplied by 4. Half the shaded region can be represented by one of the four rectangles to the right of the equal sign.

### 59. Puzzle 1:

- a.  $2(n - 3) + 4n + 6n + 1 = 12n - 5$   
 b.  $2(n - 3) + 4n + 6n + 1$   
 $= 2n - 6 + 4n + 6n + 1$  Distributive Prop.  
 $= 2n - 6 + (4 + 6)n + 1$  Distributive Prop.  
 $= 2n - 6 + 10n + 1$  Addition  
 $= 2n + 10n - 6 + 1$  Comm. Property  
 $= (2 + 10)n - 5$  Distributive Prop.  
 $= 12n - 5$  Addition

### Puzzle 2:

- a.  $2n - 3 + 4n + 6(n + 1) = 12n + 3$   
 b.  $2n - 3 + 4n + 6(n + 1)$   
 $= 2n - 3 + 4n + 6n + 6$  Distributive Prop.  
 $= 2n - 3 + (4 + 6)n + 6$  Distributive Prop.  
 $= 2n - 3 + 10n + 6$  Addition  
 $= 2n + 10n - 3 + 6$  Comm. Property  
 $= (2 + 10)n + 3$  Distributive Prop.  
 $= 12n + 3$  Addition

**Puzzle 3:**

a.  $2n - 3 + 4n + 6n + 1 = 12n - 2$ ;  
no need for parentheses

b.  $2n - 3 + 4n + 6n + 1$   
 $= 2n - 3 + (4 + 6)n + 1$  Distributive Prop.  
 $= 2n - 3 + 10n + 1$  Addition  
 $= 2n + 10n - 3 + 1$  Comm. Property  
 $= (2 + 10)n - 2$  Distributive Prop.  
 $= 12n - 2$  Addition

**Puzzle 4:**

a.  $2n - (3 + 4)n + 6n + 1 = n + 1$

b.  $2n - (3 + 4)n + 6n + 1$   
 $= 2n - 7n + 6n + 1$  Addition  
 $= (2 - 7 + 6)n + 1$  Distributive Prop.  
 $= n + 1$  Add. and Subtr.

## Possible Answers to the Mathematical Reflections

- Two expressions are equivalent when they are symbolic representations for the same situation. For all values of  $n$ , they should give the same result. The same table and the same graph can represent the expressions.

- The Distributive Property can be used to rewrite expressions as the product of two or more factors (factored form) or as the sum of two or more terms (expanded form). For example, the expression  $2x(x + 5)$  can be written as the sum of two terms using the Distributive Property:  $2x^2 + 10x$ . The expression  $6x^2 - 9x$  can be written in factored form using the Distributive Property:  $3x(2x - 3)$ . The Commutative Property states that we can change the order of addition or multiplication and still have equivalent expressions. For example,  $2x + 6 = 6 + 2x$  and  $2(x + 3) = (x + 3)2$ .
- To show that two expressions are equivalent, apply the Distributive and Commutative properties to one of the expressions until the original expression is identical to the second expression. If the two expressions are not equivalent, then this procedure will result in a contradiction. For example, the expressions  $2(x + 3)$  and  $2x + 5$  are not equivalent. If we apply the Distributive Property to the first expression, we get:  $2(x + 3) = 2x + 6$  and  $2x + 6 \neq 2x + 5$ .