

# Investigation 3

## ACE

### Assignment Choices



#### Problem 3.1

Core 1, 2, 5, 6, 8–12

Other Applications 3, 4, 7, 13, 14

#### Problem 3.2

Core 23, 26

Other Connections 18–22; unassigned choices from earlier problems

#### Problem 3.3

Core 24

Other Applications 27–35; unassigned choices from earlier problems

#### Problem 3.4

Core 15–17

Other Connections 25; unassigned choices from earlier problems

**Adapted** For suggestions about adapting Exercises 8–11 and other ACE exercises, see the CMP *Special Needs Handbook*.

**Connecting to Prior Units** 18–22, 25, 26: *Filling and Wrapping*; 23: *Accentuate the Negative*

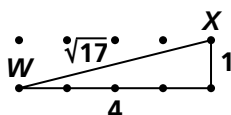
## Applications

1. a.  $5^2 + 12^2 = 169 \text{ in.}^2$

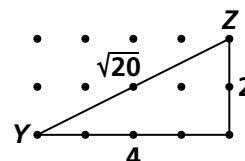
b. 13 in.

2.  $c^2 = 3^2 + 6^2 = 45$ ,  $c = \sqrt{45} \text{ cm}$ , or about 6.7 cm.

3. WX is the hypotenuse of a right triangle with legs of length 4 units and 1 unit. Because  $4^2 + 1^2 = 17$ , the length of segment WX is  $\sqrt{17}$  units. Therefore, W and X are  $\sqrt{17}$  units apart.



4. YZ is the hypotenuse of a right triangle with legs of length 4 units and 2 unit. Because  $4^2 + 2^2 = 20$ , the length of segment YZ is  $\sqrt{20}$  units. Therefore, the distance between Y and Z is  $\sqrt{20}$  units.



**Note:** There are many triangles with a hypotenuse length of  $\sqrt{20}$  units (for example, one with legs 3 and  $\sqrt{11}$ ). However, in this case, we want to use integer lengths so we can draw the triangle on dot paper.

5.  $h^2 = 4^2 + 3^2 = 25$ , so  $h = \sqrt{25} \text{ in.} = 5 \text{ in.}$
6.  $k^2 = 3^2 + 8^2 = 73$ , so  $k = \sqrt{73} \text{ cm} \approx 8.5 \text{ cm.}$
7.  $x^2 = 7^2 - 4^2 = 33$ , so  $x = \sqrt{33} \text{ m} \approx 5.7 \text{ m.}$   
 $y^2 = 21^2 - 4^2 = 425$ , so  $y = \sqrt{425} \text{ m} \approx 20.6 \text{ m.}$
8. Because  $4^2 + 3^2 = 25$ , the distance is 5 blocks.
9. Because  $6^2 + 5^2 = 61$ , the distance is  $\sqrt{61}$  blocks  $\approx 7.8$  blocks.
10. The distance is 4 blocks.
11. Because  $4^2 + 4^2 = 32$ , the distance is  $\sqrt{32} \approx 5.7$  blocks.
12. D
13. a. 2 units, 2 units, 4 units  
 b. The side lengths are  $\sqrt{2}$  units,  $\sqrt{2}$  units, and 2 units, and  $(\sqrt{2})^2 + (\sqrt{2})^2 = 2^2$  (that is,  $2 + 2 = 4$ ), so the side lengths satisfy the Pythagorean Theorem.
14. The sides have lengths  $\sqrt{5}$  units,  $\sqrt{5}$  units, and  $\sqrt{10}$  units and, because  $(\sqrt{5})^2 + (\sqrt{5})^2 = (\sqrt{10})^2$  (that is,  $5 + 5 = 10$ ), the triangle satisfies the Pythagorean Theorem.

**Note:** This is a nice place to remind students that  $\sqrt{5} + \sqrt{5} \neq \sqrt{10}$ , even though  $(\sqrt{5})^2 + (\sqrt{5})^2 = (\sqrt{10})^2$ . They can use the diagram to show  $\sqrt{5} + \sqrt{5} > \sqrt{10}$  or they can use estimation.

15. F

16. This is a right triangle.  $10^2 + 10^2 = (\sqrt{200})^2$

17. This is not a right triangle.  $9^2 + 16^2 \neq 25^2$ .

**For the Teacher** In fact, these side lengths will not form a triangle of any kind. As in Exercise 16, watch for students who incorrectly write that  $\sqrt{9} + \sqrt{16} = \sqrt{25}$ .

## Connections

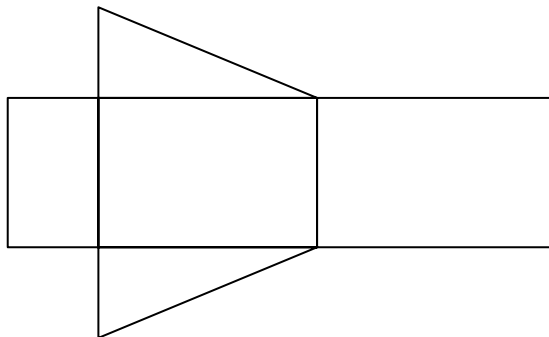
18. a. 6.5 cm

b. You do not need to know the value of  $a$  to find the volume, but it is needed to find the surface area. To find the volume, you multiply 4 by the area of the triangular face, which you can find using only the given base and height. To find the surface area, you need to find the areas of the rectangular faces. For one of these faces, you need to know the value of  $a$ .

c.  $30 \text{ cm}^3$ ;  $0.5(6 \cdot 2.5) \cdot 4 = 30$

d.  $75 \text{ cm}^2$ ;  $(2.5 \cdot 4) + 2[0.5(6 \cdot 2.5)] + (6 \cdot 4) + (6.5 \cdot 4) = 10 + 15 + 24 + 26 = 75$

e. Possible sketch:



19. B

20. H

21. B

22. H

23. a. 4 blocks

b.  $\sqrt{10}$  blocks. Find the length of the segment connecting the points. It is the hypotenuse of a right triangle with leg lengths 1 and 3. The leg lengths are the vertical and horizontal distances between the two points  $[(5 - 2) \text{ units and } -3 - (-4) \text{ units}]$   
 $3^2 + 1^2 = 10$ , so the distance is  $\sqrt{10}$  blocks.

24. Points  $A$  and  $B$  are 5 units apart. Point  $F$  is also 5 units from point  $A$ .

25. a. Using the Pythagorean Theorem,  $2^2 + h^2 = 29$ , so the height  $h$  of the cone is 5 units.

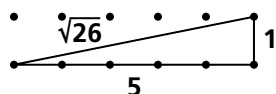
b. The volume of the cylinder is  $\pi(2)^2(5) = 20\pi \text{ units}^3$ . So the volume of the cone is  $\frac{20\pi}{3} \text{ units}^3$ , or about  $20.94 \text{ units}^3$ .

26. a. 72 cubic units. The volume of the cube is  $6 \cdot 6 \cdot 6 = 216 \text{ units}^3$ . The volume of the pyramid is  $\frac{1}{3}$  of the cube's volume, or  $72 \text{ units}^3$ .

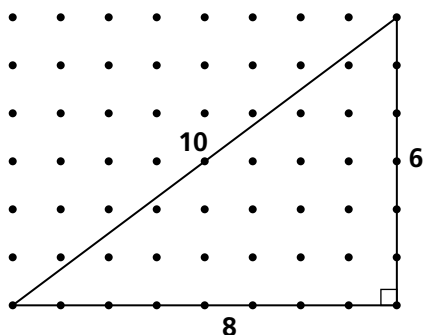
b.  $\frac{1}{3}x^3$ . The cube has volume  $x^3$ . The volume of this pyramid is one-third the volume of the cube, so it is  $\frac{1}{3}x^3$ .

## Extensions

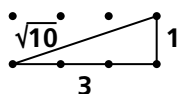
27. a. (Figure 2)      b. i. 1 and 9  
c. 9 and 16      d. 25 and 64  
e.  $1 + 25 = 26$ , so a triangle with leg lengths of 1 unit and 5 units has a hypotenuse of length  $\sqrt{26}$  units.



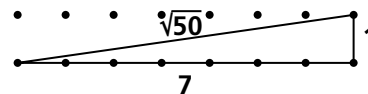
- f.  $36 + 64 = 100$ , so a triangle with leg lengths of 6 units and 8 units has a hypotenuse of length 10 units.



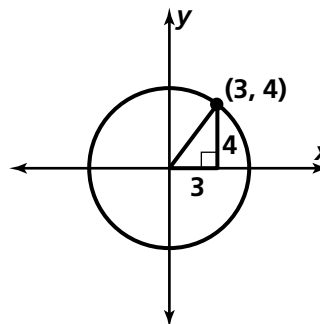
- g.  $9 + 1 = 10$ , so a triangle with leg lengths of 3 units and 1 unit has a hypotenuse of length  $\sqrt{10}$  units.



- h.  $49 + 1 = 50$ , so a triangle with leg lengths of 7 units and 1 unit has a hypotenuse of length  $\sqrt{50}$  units.



28. Yes.  $\sqrt{2}$  units is the length of the hypotenuse of a right triangle with leg lengths of 1 unit.  
29. No. 3 is not the sum of two square numbers.  
30. Yes.  $\sqrt{4} = 2$ , so just draw a horizontal or vertical segment with length 2 units.  
31. Yes.  $\sqrt{5}$  units is the length of the hypotenuse of a right triangle with leg lengths of 2 units and 1 unit.  
32. No. 6 is not the sum of two square numbers.  
33. No. 7 is not the sum of two square numbers.  
34. a. Possible answer: Draw a right triangle as shown below, and use the Pythagorean Theorem to find the hypotenuse, which is the radius.



- b. 5 units

Figure 2

+	1	4	9	16	25	36	49	64
1	2	5	10	17	26	37	50	65
4	5	8	13	20	29	40	53	68
9	10	13	18	25	34	45	58	73
16	17	20	25	32	41	52	65	80
25	26	29	34	41	50	61	74	89
36	37	40	45	52	61	72	85	100
49	50	53	58	65	74	85	98	113
64	65	68	73	80	89	100	113	128

35. a.  $J(1, 1); K(4, 7)$

- b. About 6.7 units. You can draw a right triangle with hypotenuse  $JK$ . The length of one leg is the positive difference of the  $x$ -coordinates, which is  $4 - 1$ , or 3. The length of the other leg is the positive difference of the  $y$ -coordinates, which is  $7 - 1 = 6$ . So the length of  $JK$  is  $\sqrt{9 + 36} = \sqrt{45} \approx 6.7$  units.

**Note:** In high school, students will see the distance formula,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$

or  $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ . The distance formula follows directly from the Pythagorean Theorem. If you use the segment between two points as the hypotenuse of a right triangle, the length of the horizontal leg will be  $|x_2 - x_1|$  and the length of the vertical side will be  $|y_2 - y_1|$ , so the distance between the points, which is the length of the hypotenuse, is  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ .

- c. 2.8 units.  $\sqrt{(7 - 5)^2 + (10 - 8)^2} = \sqrt{4 + 4} = \sqrt{8} \approx 2.8$

**Note:** You can give additional extension problems to interested students. For example, you might ask students to find the length of a diagonal of a square with side length  $a$ . Or, you could ask them to draw a square of side  $a$  inscribed in a circle and then to find the radius and area of the circle in terms of  $a$ .

## Possible Answers to Mathematical Reflections

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1. If we know the lengths of the legs, the length of the hypotenuse can be found by taking the square root of the sum of the squares of the leg lengths. If we know the lengths of one leg and the hypotenuse, we can find the length of the other leg by subtracting the square of the given leg length from the square of the hypotenuse length; this is the square of the missing leg length. Take the square root of that difference to get the missing leg length.
2. Think of the segment between the two points as the hypotenuse of a right triangle. Find the lengths of the legs of the right triangle (which lie on a vertical line and a horizontal line). Apply the Pythagorean Theorem by adding the squares of these two lengths and taking the square root of that sum.
3. Check whether the side lengths satisfy the relationship  $a^2 + b^2 = c^2$ , where  $a$  and  $b$  are the lengths of the shorter sides, and  $c$  is the length of the longest side. If they do, then the triangle is a right triangle.