

ACE Assignment Guide for Problem 4.3

**Differentiated
Instruction**
Solutions for All Learners

Core 18–22, 25, 26

Other Applications 23, 24; Connections 36–40;
Extensions 55; unassigned choices from previous
problems

Adapted For suggestions about adapting ACE
exercises, see the *CMP Special Needs Handbook*.

Answers to Problem 4.3

A. 1. a.

$$y = 2x(x + 3)$$

x	$2x(x + 3)$		First Differences	Second Differences
-5	20			
-4	8		-12	
-3	0		-8	4
-2	-4		-4	4
-1	-4		0	4
0	0		4	4
1	8		8	4
2	20		12	4
3	36		16	4
4	56		20	4
5	80		24	4

b.

$$y = 3x - x^2$$

x	$3x - x^2$		First Differences	Second Differences
-5	-40			
-4	-28		12	-2
-3	-18		10	-2
-2	-10		8	-2
-1	-4		6	-2
0	0		4	-2
1	2		2	-2
2	2		0	-2
3	0		-2	-2
4	-4		-4	-2
5	-10		-6	-2

c.

$$y = (x - 2)^2$$

x	$(x - 2)^2$		First Differences	Second Differences
-5	49			
-4	36		-13	
-3	25		-11	2
-2	16		-9	2
-1	9		-7	2
0	4		-5	2
1	1		-3	2
2	0		-1	2
3	1		1	2
4	4		3	2
5	9		5	2

d.

$$y = x^2 + 5x + 6$$

x	$x^2 + 5x + 6$		First Differences	Second Differences
-5	6			
-4	2		-4	
-3	0		-2	2
-2	0		0	2
-1	2		2	2
0	6		4	2
1	12		6	2
2	20		8	2
3	30		10	2
4	42		12	2
5	56		14	2

2. For $y = 2x(x + 3)$, $y = (x - 2)^2$, and $y = x^2 + 5x + 6$, the y -value first decreases and then increases. For the equation $y = 3x - x^2$ the y -value first increases and then decreases. In all four equations, the first differences are not constant: for $y = 2x(x + 3)$, they increase by 4; for $y = (x - 2)^2$ and $y = x^2 + 5x + 6$, they increase by 2; and for $y = 3x - x^2$ they decrease by 2.

3. In all four equations, the second differences are constant.

B. 1. a. $y = x + 2$

x	y		First Differences		Second Differences
0	2				
1	3	↖	1	↖	0
2	4	↖	1	↖	0
3	5	↖	1	↖	0
4	6	↖	1	↖	0
5	7	↖	1	↖	0

b. $y = 2x$

x	y		First Differences		Second Differences
0	0				
1	2	↖	2	↖	0
2	4	↖	2	↖	0
3	6	↖	2	↖	0
4	8	↖	2	↖	0
5	10	↖	2	↖	0

c. $y = 2^x$

x	y		First Differences		Second Differences
0	1				
1	2	↖	1	↖	1
2	4	↖	2	↖	2
3	8	↖	4	↖	4
4	16	↖	8	↖	8
5	32	↖	16	↖	

d. $y = x^2$

x	y		First Differences		Second Differences
0	1				
1	2	↖	1	↖	2
2	4	↖	3	↖	2
3	9	↖	5	↖	2
4	16	↖	7	↖	2
5	25	↖	9	↖	2

2. In all the tables, for $x > 0$, the y -value increases as the x value increases. For $y = x + 2$ and $y = 2x$, the change in the y -value is constant, which means that the y -value increases at a constant rate. For $y = 2^x$ and $y = x^2$, the y -value increases at an increasing rate. The second differences for $y = x^2$ are constant, while the second differences for $y = 2^x$ increase exponentially.
3. The equations $y = x + 2$ and $y = 2x$ fit the general form of linear equations, $y = mx + b$. In the table, the constant first differences tell that the equation is linear. The third equation, $y = 2^x$, fits the form of an exponential equation, $y = b^x$. Since the variable is in the exponent, the base 2 tells the factor by which the y -value grows. In the table, the growth factor of 2 shows up in the ratio of consecutive y -values: each difference is twice the previous difference. In $y = x^2$, the exponent is 2 and the base is the variable, so the y -values are the square numbers. In the table we note that first differences are not constant, but second differences are all 2.