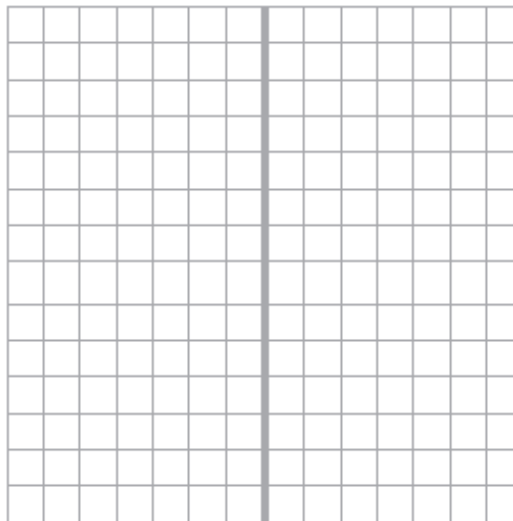


Tell the truth - All the time!

## Investigations 1 & 2 Extra Practice

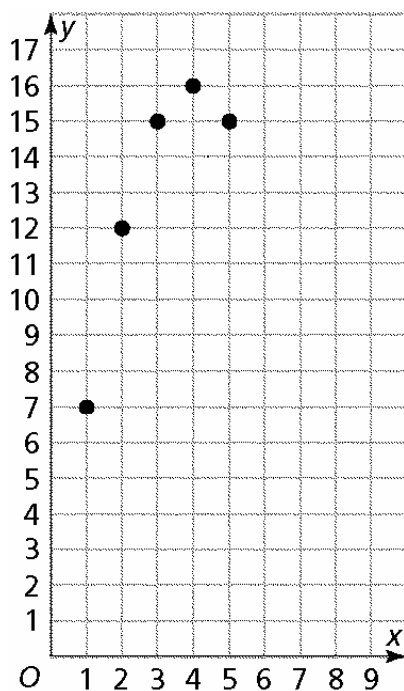
**Short Answer SHOW ALL THINKING FOR MAXIMUM CREDIT and circle final answers.**

1. A rectangle has a perimeter of 100 centimeters and a side of length  $l$  centimeters.
  - a. Draw a rectangle to represent this situation. Label each side of the rectangle in terms of  $l$ .
  - b. Write an equation for the area  $A$  of the rectangle in terms of  $l$ .
  - c. Make a table showing how the area changes as the length of a side increases, for as many rectangles as you can.
  - d. Sketch a graph of the relationship between the length of a side and the area. Be sure to include important features.



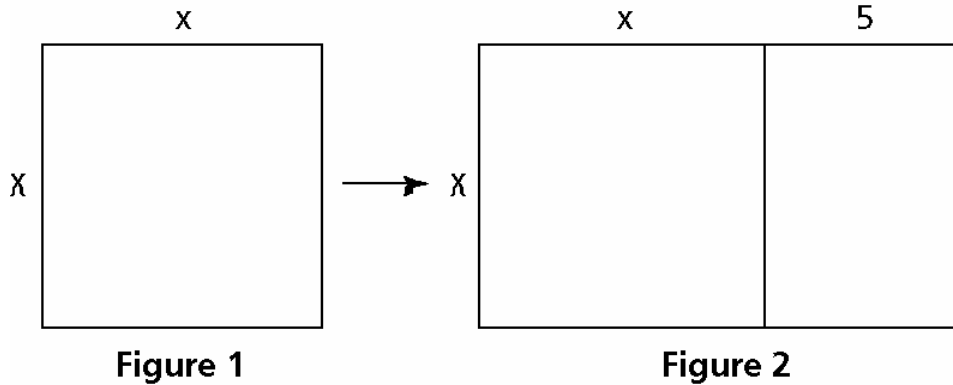
- e. Estimate the greatest area possible for a rectangle with this perimeter. What are the dimensions of the rectangle with this maximum area?
- f. Study your graph, table, and equation for the area of rectangles with a fixed perimeter of 100 centimeters. Which representation is the most useful for predicting the maximum area? Explain your choice.

2. Below is an incomplete graph for an unknown quadratic relationship.



- Make a table of the data shown in the graph.
- What appears to be the maximum  $y$ -value? Explain why you believe that this is the maximum value.
- Continue the graph and the table, adding points that fit this relationship. Explain why you chose the points that you chose.
- What is the value of  $y$  when  $x = 6$ ?
- Give the  $x$ -intercept(s) for this quadratic relationship. Explain how you found your answer.

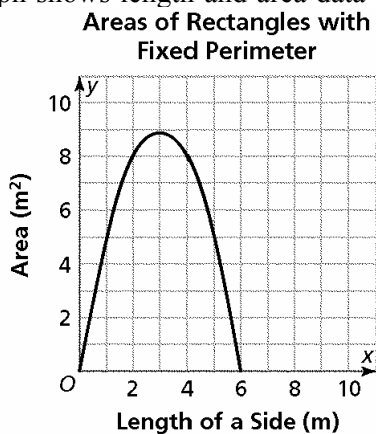
3. Figure 2 was created from Figure 1.



- What is the area of Figure 1?
  - Two of the expressions below are equivalent, each representing the area of Figure 2. Circle the two equivalent expressions.
 

$x^2 + 5x$        $x^2 + 5$        $x(x + 5)$        $x + x + 5$
  - For each expression you chose in part (b), explain how it represents the area of Figure 2.
  - How much greater is the area of Figure 2 than the area of Figure 1 (in terms of  $x$ )?
4. Which of these four expressions represent(s) a quadratic relationship? Circle your choice(s). Explain how you know.
- $x^2 + 5x$        $x^2 + 5$        $x(x + 5)$        $x + x + 5$
- Draw a rectangle divided to show that its area is represented by the expression  $(x + 1)(x + 3)$ . Label the lengths and areas on your drawing.
  - Write an equivalent expression in expanded form.
  - Find the  $x$ - and  $y$ -intercepts, maximum or minimum, and line of symmetry of the graph of  $A = (x + 1)(x + 3)$ . Explain how you found these.
6. Find an equivalent expression in factored form for  $x^2 + 8x + 15$ .

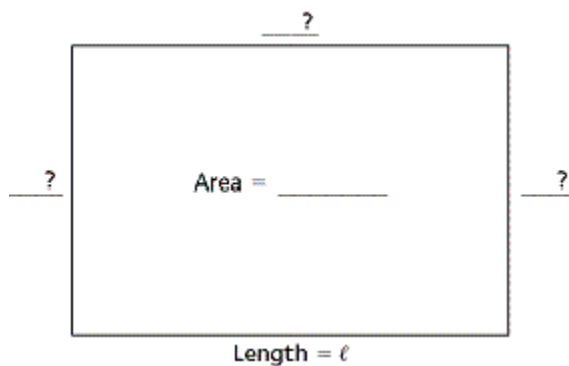
7. The area  $A$  of a rectangle with a side of length  $l$  meters (m) and a fixed perimeter is given by the equation  $A = l(240 - l)$ .
- Suppose one dimension of the rectangle is 180 meters. What is the other dimension? What is the area of the rectangle?
  - Suppose one dimension of the rectangle is 110 meters. What is the other dimension? What is the area of the rectangle?
  - What are the dimensions of the rectangle with the greatest area possible for this perimeter? Explain how you found your answer.
  - What are the dimensions of the rectangle with this perimeter and an area of  $8,000 \text{ m}^2$ ? Explain your answer.
  - What is the fixed perimeter for the rectangles represented by this equation? Explain how you found the perimeter.
8. The graph shows length and area data for rectangles with a fixed perimeter.



- What are the dimensions of the rectangle with this perimeter and an area of  $8 \text{ m}^2$ ?
- What are the dimensions of the rectangle with this perimeter and an area of  $5 \text{ m}^2$ ?
- What is the greatest possible area for a rectangle with this perimeter? What are the dimensions of this rectangle?

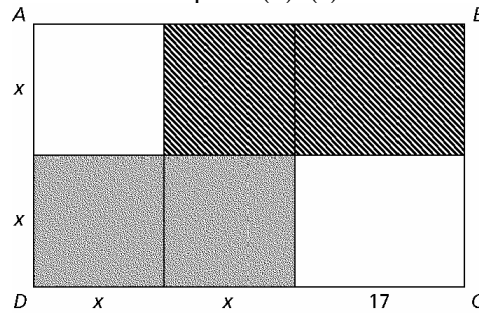
9. Find the maximum area for a rectangle with a perimeter of 10 meters. Include the following in your answer and explain how each piece of evidence supports your answer:
- Sketch rectangles with a perimeter of 10 meters that do not have the maximum area and a rectangle you think does have the maximum area.
  - Make a table of the length of a side and the area for rectangles with a perimeter of 10 meters. Use increments of 1 meter for the lengths.
  - Make a graph of the relationship between length and area of rectangles with a perimeter of 10 meters.
10. Find the maximum area for a rectangle with a perimeter of 200 meters. Include the following in your answer and explain how each piece of evidence supports your answer:
- Sketch rectangles with a perimeter of 200 meters that do not have the maximum area and a rectangle you think does have the maximum area.
  - Make a table of the length of a side and the area for rectangles with a perimeter of 200 meters. Use increments of 10 meters for the lengths.
  - Make a graph of the relationship between length and area of rectangles with a perimeter of 200 meters.

11. The rectangle below has a perimeter of 60 meters and a side length  $l$  meters.



- a. Express the lengths of the other sides in terms of  $l$ .
- b. Write an equation for the area  $A$  in terms of  $l$ .
- c. Make a graph of your equation.
- d. Use your equation to find the area of the rectangle if the length of one side is 10 meters.
- e. Describe how you could use your graph to find the area of the rectangle if the length of one side is 10 meters.
- f. Describe how you could use a table to find the area of the rectangle if the length of one side is 10 meters.
- g. What is the maximum area possible for a rectangle with a perimeter of 60 meters? What are the dimensions of the rectangle with maximum area?

12. Refer to the diagram below to answer parts (a)–(f).



- Write an expression for the area of the diagonally shaded region.
- Write an expression for the area of the gray region.
- Write an expression for the total area of the white regions.
- Write an expression for the difference in area between the diagonally shaded region and the gray region.
- Write an expression for the perimeter of rectangle  $ABCD$ .
- Write an expression for the area of rectangle  $ABCD$ .

**Draw and label a rectangle whose area is represented by the expression. Then write an equivalent expression in expanded form.**

13.  $(x + 1)(x + 5)$

14.  $3x(x - 4)$

15.  $(x + 6)(x + 2)$

**Write the expression in factored form. You may want to draw a rectangle to illustrate the area represented by the expression.**

16.  $x^2 + 2x + 9x + 18$

17.  $x^2 + 4x$

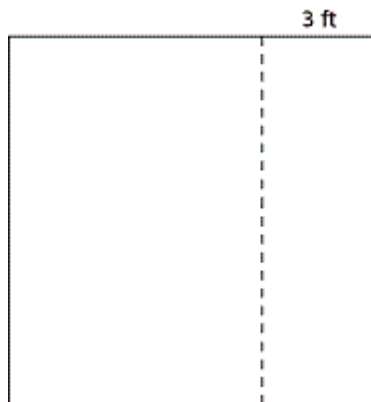
18.  $x^2 + 12x + 36$

19.  $x^2 + 2x + 7x + 14$

20.  $x^2 + 7x + 12$

21.  $x^2 + 12x + 27$

22. Serena and Chuck had a large square piece of cardboard for designing a poster advertising the upcoming drama club fund-raiser. They decided to trim 3 feet from the length of the cardboard.



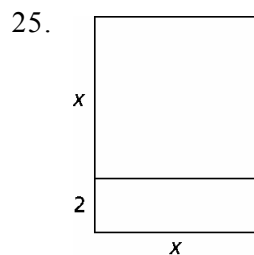
Suppose each side of the original square of cardboard had a length of  $x$  feet.

- Write an expression for the area of the strip that Serena and Chuck trimmed from the large piece.
- Write an expression for the area of the remaining piece of cardboard.
- Write an expression for the perimeter of the strip that Serena and Chuck trimmed from the large piece.
- Write an expression for the perimeter of the remaining piece of cardboard.
- The perimeter of the original piece of cardboard was 36 feet.
  - What is the area of the strip that Serena and Chuck trimmed from the large piece?
  - What is the area of the remaining piece of cardboard?
  - What is the perimeter of the remaining piece of cardboard?

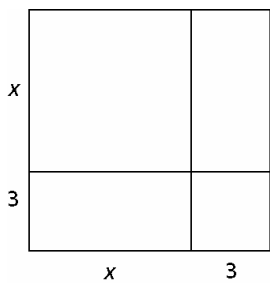


23. A square has sides of length  $x$  centimeters. A new rectangle is made by increasing one dimension by 2 centimeters and decreasing the other dimension by 2 centimeters. A square has sides of length  $x$  centimeters. A new rectangle is made by increasing one dimension by 2 centimeters and decreasing the other dimension by 2 centimeters.
- Make a table showing the area of the square and the area of the new rectangle for whole number  $x$ -values from 0 to 10.
  - Which values for area are not reasonable? Explain.
  - On the same set of axes, graph the  $(x, \text{area})$  data for both the square and the rectangle. Graph only those values for which the area is positive.
  - Write an equation for the area of the original square and an equation for the area of the new rectangle. Use these equations to label the graphs you made in part (c).
24. A square has sides of length  $x$  centimeters. A new rectangle is created by increasing one dimension by 2 centimeters.
- Make a sketch to show how the square is transformed into the new rectangle.
  - Make a table showing the area of the square and the area of the new rectangle for whole number  $x$ -values from 0 to 10.
  - On the same set of axes, graph the  $(x, \text{area})$  data for both the square and the rectangle.
  - Write an equation for the area of the original square and an equation for the area of the new rectangle.

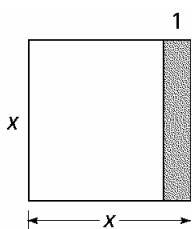
**Write two expressions, one in factored form and one in expanded form, for the area of the unshaded region.**



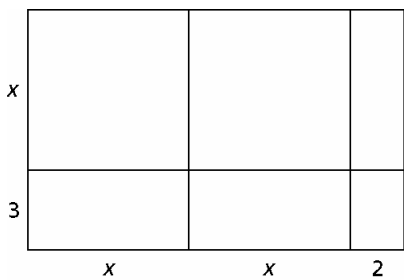
26.



27.



28.



**Draw and label a rectangle whose area is represented by the expression.  
Write an equivalent expression in factored form.**

29.  $x^2 + 4x$

30.  $x^2 + x + x + 1$

31.  $x^2 + 3x + 2$

32.  $x^2 + 2x + 1$

**Multiple Choice:** *Identify the choice that best completes the statement or answers the question.*

\_\_\_\_\_ 1. Combine like terms:  $-21a + 16a$ .  
 A.  $-5a$                       B.  $5a$                       C.  $37a$                       D.  $-37a$

\_\_\_\_\_ 2. Combine like terms:  $q + 12q$ .  
 A.  $-13q$                       B.  $-11q$                       C.  $11q$                       D.  $13q$

**Simplify the expression.**

\_\_\_\_\_ 3.  $9a - b - 2a - 10b$   
 A.  $-7a + 11b$                       B.  $11a + 9b$                       C.  $-11a - 9b$                       D.  $7a - 11b$

\_\_\_\_\_ 4.  $5(x + 10) + x$   
 A.  $6x + 15$                       B.  $5x + 15$                       C.  $6x + 50$                       D.  $4x - 50$

\_\_\_\_\_ 5.  $-6 - 7(c + 10)$   
 A.  $64 - 7c$                       B.  $-76 - 7c$                       C.  $4 - 13c$                       D.  $-16 - 13c$

\_\_\_\_\_ 6.  $-x(7x - 8)$   
 A.  $6x^2 - 9x$                       B.  $-7x - 8x$                       C.  $-7x^2 + 8x$                       D.  $7x^2 + 8x$

\_\_\_\_\_ 7.  $5k^2(-6k^2 - 2k + 6)$   
 A.  $-30k^3 + 3k^2 + 30k$                       C.  $-k^4 + 3k^3 + 11k^2$   
 B.  $30k^4 - 10k^3 + 11k^2$                       D.  $-30k^4 - 10k^3 + 30k^2$

\_\_\_\_\_ 8.  $(-8x) \cdot 3x^2$   
 A.  $-5x^3$                       B.  $-24x^2$                       C.  $-24x^3$                       D.  $-5x^2$

\_\_\_\_\_ 9. Lena buys 4 cans of soft drinks and 12 sandwiches for a picnic. Patrick buys 9 cans of soft drinks and 7 sandwiches for the same picnic. Let  $c$  be the cost of a soft drink. Let  $s$  be the cost of a sandwich. Write an expression that represents the total cost.  
 A.  $12c + 4s + 7c + 9s$                       C.  $4c + 12s + 9c + 7s$   
 B.  $4c + 12s + 7c + 9s$                       D.  $12c + 4s + 9c + 7s$

**Simplify the product.**

- \_\_\_\_\_ 10.  $(x - 4)(x + 3)$   
 A.  $x^2 - 7x - 12$  C.  $x^2 - x - 12$   
 B.  $x^2 + x - 12$  D.  $x^2 - 12x - 1$
- \_\_\_\_\_ 11. Simplify  $(x - 2)(x - 2)$  using the Distributive Property.  
 A.  $2x^2 - 4x + 4$  B.  $x^2 - 4x + 4$  C.  $2x^2 - 4$  D.  $2x - 4$
- \_\_\_\_\_ 12. Suppose  $3s$  represents an even integer. What polynomial represents the product of  $3s$ , the *even* integer that comes just before  $3s$ , and the *even* integer that comes just after  $3s$ ?  
 A.  $27s^3 + 12s$  B.  $27s^3 - 3s$  C.  $27s^3 - 12s$  D.  $3s^3 - 12s$
- \_\_\_\_\_ 13. The base of a triangle is  $(6h + 16)$  centimeters. The height of the triangle is  $(3h - 8)$  centimeters. Find the area of the triangle.  
 A.  $(18h^2 - 96h - 64) \text{ cm}^2$  C.  $(18h^2 + 64) \text{ cm}^2$   
 B.  $(9h^2 - 16h - 64) \text{ cm}^2$  D.  $(9h^2 - 64) \text{ cm}^2$

**Factor the expression.**

- \_\_\_\_\_ 14.  $d^2 + 10d + 9$   
 A.  $(d + 9)(d - 1)$  C.  $(d - 9)(d - 1)$   
 B.  $(d - 9)(d + 1)$  D.  $(d + 9)(d + 1)$
- \_\_\_\_\_ 15.  $k^2 + kf - 2f^2$   
 A.  $(k - 2f)(k + f)$  C.  $(k + 2f)(k + f)$   
 B.  $(k + 2f)(k - f)$  D.  $(k - 2f)(k - f)$
- \_\_\_\_\_ 16.  $50k^3 - 40k^2 + 75k - 60$   
 A.  $5(2k^2 - 3)(5k + 4)$  C.  $(2k^2 + 15)(5k - 20)$   
 B.  $(10k^2 - 3)(25k + 4)$  D.  $5(2k^2 + 3)(5k - 4)$

**Complete.**

\_\_\_\_\_ 17.  $z^2 + 9z - 90 = (z - 6)(z + \blacksquare)$

A.  $-9$

B.  $15$

C.  $90$

D.  $-15$

**Factor by grouping.**

\_\_\_\_\_ 18.  $a^2 + ab - 56b^2$

A.  $(a + 8b)(a + 7b)$

B.  $(a - 8)(a + 7b)$

C.  $(a + 8b)(a - 7b)$

D.  $(a - 8b)(a - 7b)$

\_\_\_\_\_ 19.  $40p^2 - 13p - 36$

A.  $(8p + 9)(5p + 4)$

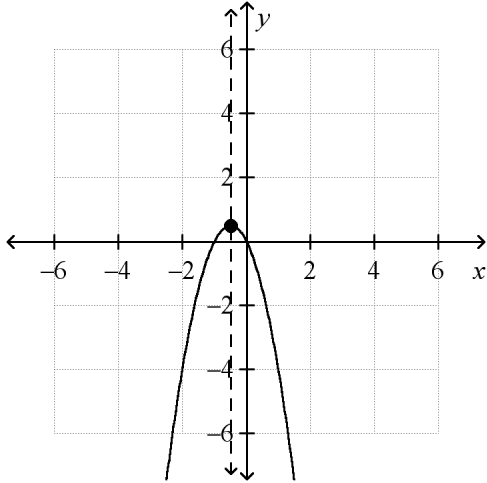
B.  $(8p - 9)(5p - 4)$

C.  $(8p - 9)(5p + 4)$

D.  $(8p + 9)(5p - 4)$

\_\_\_\_\_ 20. Graph  $f(x) = -2x^2 - 2x - 1$ . Label the axis of symmetry and the vertex.

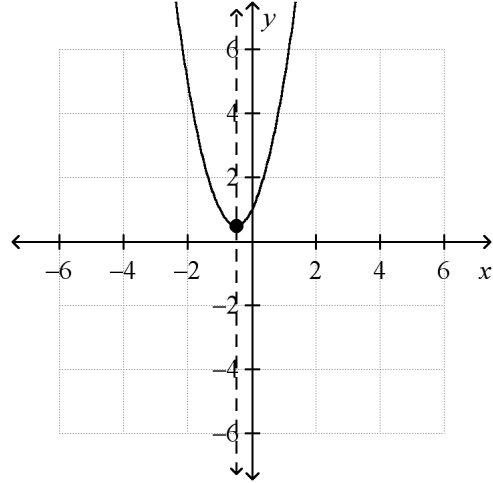
A.



Axis of symmetry:  $x = -0.5$

Vertex:  $(-0.5, 0.5)$

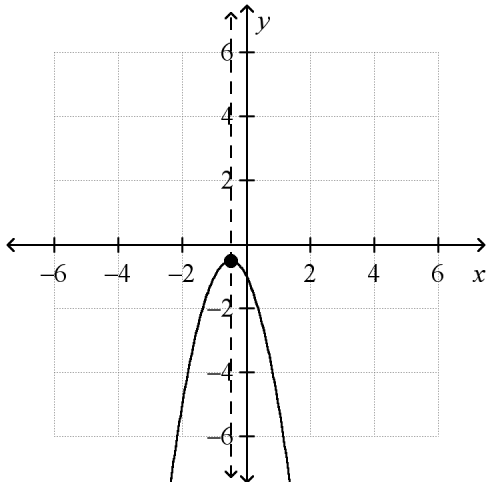
C.



Axis of symmetry:  $x = -0.5$

Vertex:  $(-0.5, 0.5)$

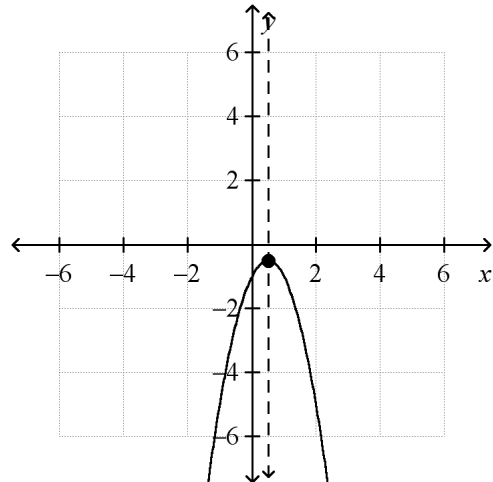
B.



Axis of symmetry:  $x = -0.5$

Vertex:  $(-0.5, -0.5)$

D.



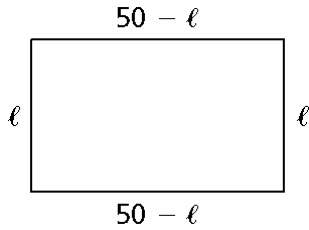
Axis of symmetry:  $x = 0.5$

Vertex:  $(0.5, -0.5)$

# Investigations 1 & 2 Extra Practice Answer Section

## SHORT ANSWER

1. a.



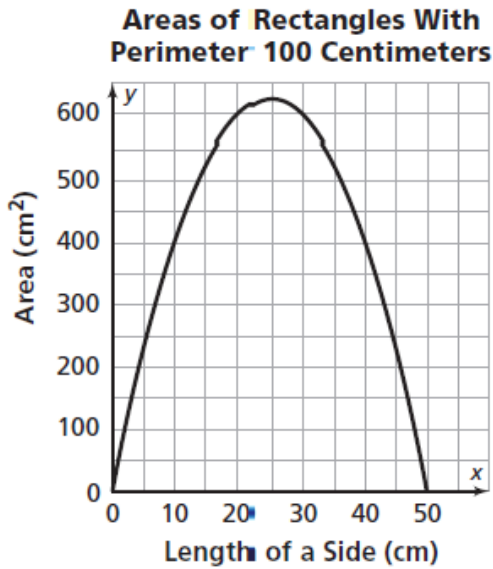
b.  $\ell(50 - \ell)$

c.  $\ell$  must be greater than 0, but less than 50.

d. Possible table:

Length of Side (cm)	Area (m <sup>2</sup> )
1	49
2	96
3	141
4	184
5	225
6	264
7	301

e.



f. The greatest area is 625 cm<sup>2</sup> for a square with a side length of 25 cm.

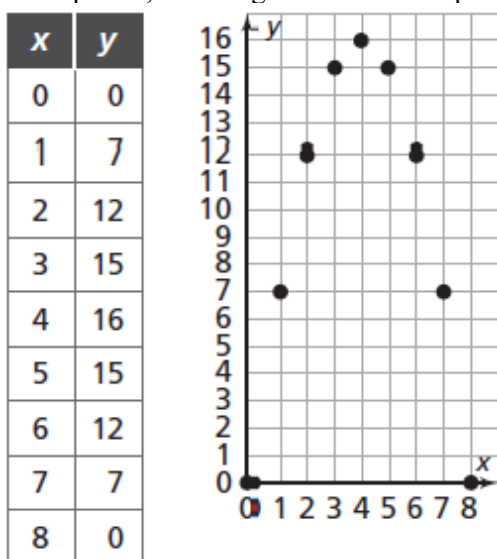
- g. Answers will vary. Some students may claim the table is the most useful because it is easy to find where the data “turns around.” Others may say the graph is the most useful because it is easy to locate the maximum point, which is where the maximum area is represented. Less likely is the claim that the equation is the most helpful, unless a student reasons that half of half the perimeter is equal to the side length of the rectangle with the greatest area and that value of half the perimeter, 50, can be found in the equation.

2. a.

$x$	$y$
0	0
1	7
2	12
3	15
4	16
5	15

- b. The maximum  $y$ -value appears to be 16. It is the highest of the plotted points, and the two points on either side of it have the same  $y$ -value, 15. This implies that the graph is symmetric halfway between those points, meaning the maximum point is where  $y = 16$ .

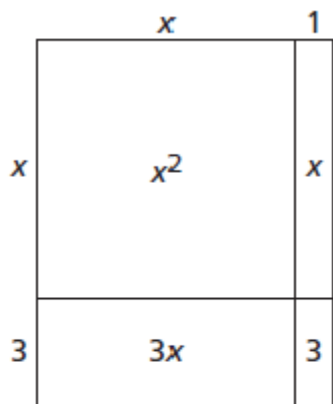
c.



- d. When  $x = 6$ ,  $y = 12$ .
- e. When  $y = 0$ ,  $x = 0$  or  $x = 8$ . Students may find this answer from their table or graph.
3. a.  $x^2$
- b.  $x^2 + 5x$  and  $x(x + 5)$
- c. Viewed as two separate rectangles with dimensions  $x$  by  $x$  and 5 by  $x$ , Figure 2 has an area of  $x^2 + 5x$ .  
Viewed as one large rectangle with dimensions  $x$  by  $x + 5$ , Figure 2 has an area of  $x(x + 5)$ .
- d. Figure 2's area is greater than Figure 1's area by a difference of  $5x$ .
4. Possible answer: The expressions  $x^2 + 5x$ ,  $x^2 + 5$ , and  $x(x + 5)$  all represent quadratic relationships. Each, when simplified, has an  $x^2$  term. (**Note:** Students might show tables or graphs of the expressions to explain their answers.)



5. a.

b.  $x^2 + 3x + x + 3$  or  $x^2 + 4x + 3$ 

c. The  $x$ -intercepts are  $(-3, 0)$  and  $(-1, 0)$  which can be found by the value of  $x$  that will make the factors  $(x + 3)$  and  $(x + 1)$  both equal to zero. The  $y$ -intercept  $(0, 3)$  is found by substituting 0 in for  $y$  in the equation  $y = x^2 + 4x + 3$ . The line of symmetry,  $x = -2$ , is located halfway between the  $x$ -intercepts. The minimum point  $(-2, -1)$  is found by substituting  $-2$  into the equation  $y = x^2 + 4x + 3$ .

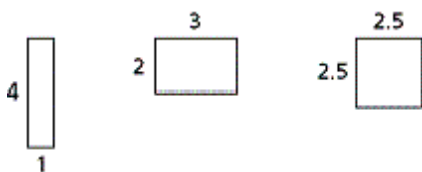
6.  $(x + 5)(x + 3)$ 7. a. 60 m; 10,800 m<sup>2</sup>; Let  $l = 180$ . Then,  $240 - l = 60$  m; thus,  $A = l(240 - l) = 180 \times 60 = 10,800$  m<sup>2</sup>.b. 130 m; 14,300 m<sup>2</sup>; Let  $l = 110$ . Then,  $240 - 110 = 130$  m; thus, area =  $180(130) = 14,300$  m<sup>2</sup>.c. 120 m by 120 m; The greatest possible area is 14,400 m<sup>2</sup>, which corresponds to a square with side lengths of 120 meters.d. 40 m and 200 m; The dimensions of a rectangle with an area of 8,000 m<sup>2</sup> are 40 meters and 200 meters since  $(40 + 200) = 240$  and  $40(200) = 8,000$ .e. 480 m; Possible explanation: For part (a), one rectangle with this fixed perimeter and area defined by the equation  $A = l(240 - l)$  had dimensions of 60 meters and 180 meters. Substituting these dimensions into the equation  $P = 2l + 2w$  gives a perimeter of  $(2 \times 180 \text{ m}) + (2 \times 60 \text{ m}) = 480$  m.

8. a. 2 m and 4 m

b. 1 m and 5 m

c. The greatest area possible is 9 m<sup>2</sup>, which corresponds to a square with side lengths of 3 meters.

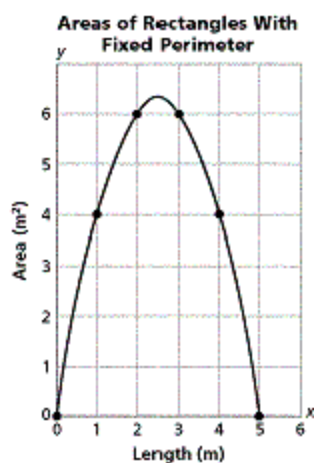
9. The maximum area for a rectangle with a perimeter of 10 meters is  $2.5 \times 2.5 = 6.25 \text{ m}^2$ .  
Here are some examples of rectangles students may sketch:



The rectangle with maximum area is the rectangle that is 2.5 meters by 2.5 meters.

This is an example of a table of lengths from 0 to 5 and areas of rectangles with areas determined by taking length,  $l$ , multiplying by the other dimension, and having the sum of  $l + w = 5$ .

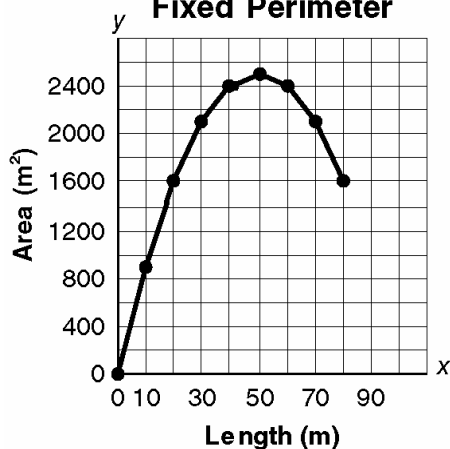
$l$	$A$
0	0
1	4
2	6
3	6
4	4
5	0



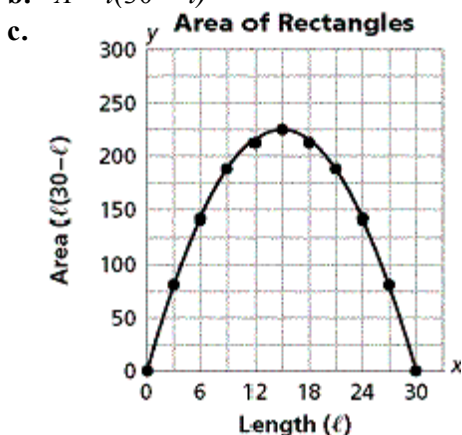
10. The maximum area for a rectangle with a perimeter of 200 meters is  $50 \times 50 = 2,500 \text{ m}^2$ .  
 The rectangle with maximum area is the rectangle that is  $50 \times 50 = 2,500 \text{ m}^2$ .  
 This is an example of a table of lengths from 0 to 5 and areas of rectangles with areas determined by taking length,  $l$ , multiplying by the other dimension, and having the sum of  $l + w = 100$ .

$l$	$A$
0	5
10	900
20	1,600
30	2,100
40	2,400
50	2,500
60	2,400
70	2,100
80	1,600

**Areas of Rectangles With  
Fixed Perimeter**



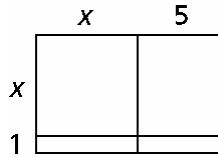
11. a.  $30 - l$ ; That is half the perimeter minus the length of the side. The sides are  $l$ ,  $30 - l$ ,  $l$ , and  $30 - l$ .  
 b.  $A = l(30 - l)$



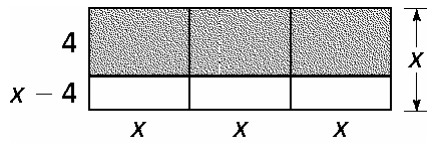
- d.  $A = 10(20) = 200 \text{ m}^2$   
 e. Find the  $y$ -value on the graph of the parabola corresponding to the value of 10 on the  $x$ -axis.  
 f. Find the  $A$ -value in the table corresponding to the  $l$ -value of 10.  
 g. The maximum area is for a square with side 15 meters ( $60 \div 4 = 15$ ). Area is  $225 \text{ m}^2$ .

12. **a.**  $x(x + 17) = x^2 + 17x$   
**b.**  $x(2x) = 2x^2$  or  $x(x + x)$   
**c.**  $x(x) + x(17) = x^2 + 17x$  or  $x(x + 17)$   
**d.**  $x(x + 17) - 2x^2 = 17x - x^2$   
**e.**  $8x + 34$  or an equivalent expression  
**f.**  $2x(2x + 17)$  or an equivalent expression

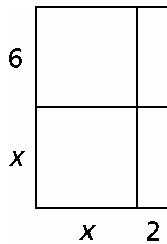
13.  $x^2 + 6x + 5$



14.  $3x^2 - 12x$



15.  $x^2 + 8x + 12$



16.  $(x + 9)(x + 2)$

17.  $x(x + 4)$

18.  $(x + 6)(x + 6) = (x + 6)^2$

19.  $(x + 2)(x + 7)$

20.  $(x + 3)(x + 4)$

21.  $(x + 3)(x + 9)$

22. **a.**  $3x$

**b.**  $x(x - 3)$

**c.**  $2x + 6 = 2(x + 3)$

**d.**  $2x + 2(x - 3) = 4x - 6$

**e. i.** Since perimeter is  $4x = 36$ ,  $x = 9$ . Thus,  $3 \times 9 = 27 \text{ ft}^2$ .

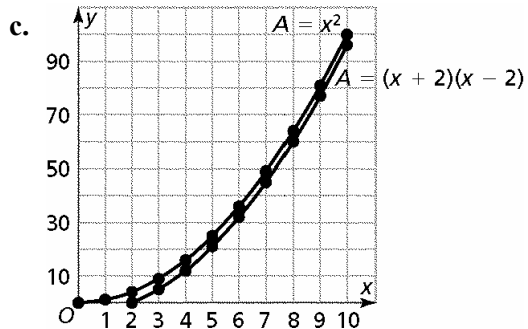
**ii.**  $9 \times (9 - 3) = 54 \text{ ft}^2$

**iii.**  $2(9) + 2(9 - 3) = 30 \text{ ft}$

23. a.

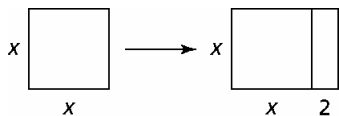
$x$	area of square	area of rectangle
0	$0 \times 0 = 0$	$2 \times -2 = -4$ : not possible
1	$1 \times 1 = 1$	$3 \times -1 = -3$ ; not possible
2	$2 \times 2 = 4$	$4 \times 0 = 0$
3	$3 \times 3 = 9$	$5 \times 1 = 5$
4	$4 \times 4 = 16$	$6 \times 2 = 12$
5	$5 \times 5 = 25$	$7 \times 3 = 21$
6	$6 \times 6 = 36$	$8 \times 4 = 32$
7	$7 \times 7 = 49$	$9 \times 5 = 45$
8	$8 \times 8 = 64$	$10 \times 6 = 60$
9	$9 \times 9 = 81$	$11 \times 7 = 77$
10	$10 \times 10 = 100$	$12 \times 8 = 96$

b. The values of  $x$  for which the areas are negative or zero; for example,  $x = 0$ ,  $x = 1$ , and  $x = 2$



d.  $A = x^2$  and  $A = (x + 2)(x - 2)$ , or  $A = x^2 - 4$ .

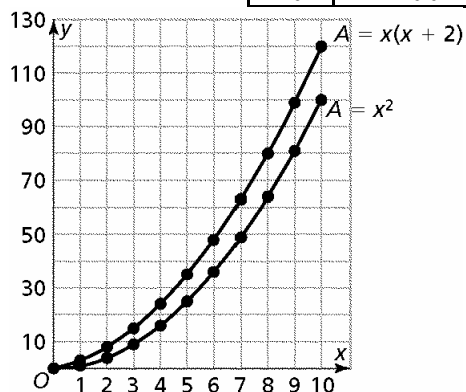
24. a.



b.

$x$	Area of square	Area of rectangle
0	0	0
1	1	3
2	4	8
3	9	15
4	16	24
5	25	35
6	36	48
7	49	63
8	64	80
9	81	99
10	100	120

c.

d.  $A = x^2$  and  $A = (x+2)(x) = x^2 + 2x$ 

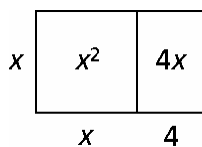
25.  $x(x+2) = x^2 + 2x$

26.  $(x+3)(x+3) = x^2 + 3x + 3x + 9$  or  $x^2 + 6x + 9$

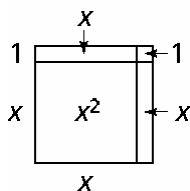
27.  $x(x-1) = x^2 - x$

28. Factored form:  $(x+3)(x+x+2)$  or  $(x+3)(2x+2)$ ; expanded form:  $2x^2 + 6x + 2x + 6 = 2x^2 + 8x + 6$

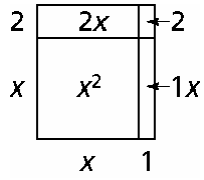
29.  $x(x+4)$



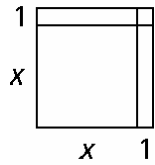
30.  $(x+1)(x+1) = (x+1)^2$



31.  $(x + 1)(x + 2)$



32.  $(x + 1)(x + 1)$



### MULTIPLE CHOICE

1. A
2. D
3. D
4. C
5. B
6. C
7. D
8. C
9. C
10. C
11. B
12. C
13. D
14. D
15. B
16. D
17. B
18. C
19. C
20. B