

HW Solutions:

$$21.) -\frac{1}{2} \pm \frac{\sqrt{10}}{2}$$

$$23.) 5, -2$$

$$25.) -\frac{1}{3}, -1$$

$$27.) -1 \pm \sqrt{3}$$

$$29.) \frac{3}{2} \pm \frac{\sqrt{13}}{2}$$

$$31.) -3 \pm \frac{\sqrt{132}}{2}$$

$$33.) -5, -2$$

$$35.) 0, \frac{5}{3}$$

$$37.) -\frac{5}{9} \pm \frac{\sqrt{244}}{18}$$

$$39.) 0, 2$$

$$41.) \frac{3}{4}, \frac{1}{2}$$

$$43.) -10 \pm \frac{\sqrt{360}}{2}$$

$$\sqrt{\frac{10}{4}} = \frac{\sqrt{10}}{\sqrt{4}} = \frac{\sqrt{10}}{2}$$

$$\sqrt{\frac{49}{4}} = \frac{\sqrt{49}}{\sqrt{4}} = \frac{7}{2}$$

(from 31)

$$\sqrt{\frac{132}{4}} = \frac{\sqrt{132}}{\sqrt{4}} = \frac{\sqrt{132}}{2}$$

$$\sqrt{\frac{132}{4}} = \sqrt{33}$$

$$\text{So } \sqrt{\frac{132}{4}} \leq \frac{\sqrt{132}}{2} \leq \sqrt{33}$$

$$(x+3)^2 = x^2 + 6x + 9$$

Perfect trinomial

Square root method

$$ax^2 + c = 0$$

$$\begin{array}{r} x^2 - 7 = 9 \\ + 7 \quad + 7 \\ \hline \end{array}$$

$$\sqrt{x^2} = \sqrt{16}$$

$$x = \pm 4$$

You can derive the quadratic formula by completing the square on the standard form of a quadratic equation?

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a} \quad (1st)$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\begin{array}{r} x^2 + \frac{b}{a}x \\ \hline \end{array} \quad \begin{array}{l} -\frac{c}{a} \quad -\frac{c}{a} \\ = -\frac{c}{a} \end{array} \quad (2nd)$$

$$\begin{array}{r} x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 \\ \hline \end{array} \quad \begin{array}{l} + \left(\frac{b}{2a}\right)^2 \\ = -\frac{c}{a} + \frac{b^2}{4a^2} \end{array} \quad (3rd)$$

Can you continue the proof from here?

Quadratic

$$x = \frac{-(b) \pm \sqrt{(b)^2 - 4(a)(c)}}{2(a)}$$

Formula

$$3x^2 = 2x + 1$$

$$\Rightarrow 3x^2 - 2x = 1$$

$$\Rightarrow 3x^2 - 2x - 1 = 0$$

$$ax^2 + bx + c = 0$$

$$a=3 \quad b=-2 \quad c=-1$$

$$x = \frac{(2) \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{4 + 12}}{6}$$

$$x = \frac{2 \pm \sqrt{16}}{6}$$

$$x = \frac{2 \pm 4}{6}$$

$$\frac{2+4}{6} = 1 \quad \frac{2-4}{6} = -\frac{2}{3}$$

Euclid and Pythagoras

The first purely mathematical try to come up with a quadratic formula was done by Pythagoras in 500 BC. Euclid did the same thing in Alexandria, Egypt. Euclid used a purely geometric method. And he came up with a general formula to solve the equation. For his part Pythagoras observed that the ratios between the area of a square and the length of the sides did not add up. To him there was no other proportion except the rational. Euclid thought otherwise: If there are rational numbers, there must be irrational numbers. He then wrote a book called Elements in which he lay down the mathematics of solving quadratic equation.



$$37.) 9z^2 + 10z - 4 = 0$$

$$a=9 \quad b=10 \quad c=-4$$

$$x = \frac{-10 \pm \sqrt{(10)^2 - 4(9)(-4)}}{2(9)}$$

$$x = \frac{-10 \pm \sqrt{100 + 144}}{18}$$

$$x = \frac{-10 \pm \sqrt{244}}{18}$$

$$x = -\frac{5}{9} \pm \frac{\sqrt{244}}{18}$$

$$2x^2 - 3x = 8$$

- 8 - 8

$$2x^2 - 3x - 8 = 0$$

$$a=2 \quad b=-3 \quad c=-8$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-8)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{9 + 64}}{4}$$

$$x = \frac{3 \pm \sqrt{73}}{4}$$

↙ 4 (d) ↘

$$x = \frac{3 + \sqrt{73}}{4} \approx 2.89 \quad x = \frac{3 - \sqrt{73}}{4} \approx -1.39$$

$$x^2 = 3x + 1$$

$$\begin{array}{r} -3x \quad -3x \\ \hline x^2 - 3x = 1 \\ -1 \quad -1 \\ \hline \end{array}$$

$$x^2 - 3x - 1 = 0$$

$$ax^2 + bx + c = 0$$

$$a=1 \quad b=-3 \quad c=-1$$

$$x = \frac{(3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9+4}}{2}$$

$$\rightarrow x = \frac{3 \pm \sqrt{13}}{2}$$

$$\begin{array}{c} \swarrow \quad \searrow \\ x = \frac{3 + \sqrt{13}}{2} \quad \text{or} \quad x = \frac{3 - \sqrt{13}}{2} \\ \approx 3.30 \quad \approx -0.30 \end{array}$$

Use the Quadratic formula to solve

$$7x^2 - 12x - 2 = 2$$

$\begin{matrix} -2 & -2 \end{matrix}$

$$7x^2 - 12x - 4 = 0$$

$$ax^2 + bx + c = 0$$

$$a=7 \quad b=-12 \quad c=-4$$

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(7)(-4)}}{2(7)}$$

$$x = \frac{12 \pm \sqrt{144 + 112}}{14}$$

$$\rightarrow x = \frac{12 \pm \sqrt{256}}{14}$$

$$x = \frac{12 \pm 16}{14}$$

$$x = \frac{12+16}{14} \quad \text{or} \quad x = \frac{12-16}{14}$$

$$x = \frac{28}{14} \quad x = \frac{-4}{14}$$

$$x = 2 \quad x = -\frac{2}{7}$$



Say what?

