

10.1

Adding and Subtracting Polynomials

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Guided Practice

CRITICAL THINKING about the Lesson

- Describe a polynomial in one variable.
- Name the terms of $-3x^3 - 2x^2 + 4x - 5$.
- Name the coefficients in $-7x^3 + 12x - 31$.
 $-7, 0, 12, -31$
- Write $15y - 6 + 10y^3 - 3y^2$ in standard form. $10y^3 - 3y^2 + 15y - 6$
- What is the degree of $2x^2 - 4x^3 + 7$? **3**
- Subtract $(2x^2 - 4x + 1)$ from $(x^2 + 8)$.
 $-x^2 + 4x + 7$

1. See page 508.

Independent Practice

In Exercises 7–10, classify the polynomial by degree and by number of terms.

- $-5x - 4$ Linear, binomial
- -7 Constant, monomial
- $16 - 4x + 3x^2 - x^4$ Quadratic, polynomial
- $3x^2 + 6x + 1$ Quadratic, trinomial

In Exercises 11–16, add the polynomials. (Use a horizontal format.)

- $x^2 - 3$; $3x^2 + 5$ $4x^2 + 2$
- $-3y + 2$; $y^2 + 3y + 2$ $y^2 + 4$
- $2x^2 + 3x + 1$; $x^2 - 2x + 2$ $3x^2 + x + 3$
- $2x^2 - x + 3$; $3x^2 - 4x + 7$ $5x^2 - 5x + 10$
- $12x^3 + 2x^2 - 4$; $9x^2 + 3x - 8$ $12x^3 + 11x^2 + 3x - 12$
- $-4x^3 - 2x^2 + x - 5$; $2x^3 + 3x + 4$ $-2x^3 - 2x^2 + 4x - 1$

In Exercises 17–20, add the polynomials. (Use a vertical format.)

- $2z - 8z^2 - 3$; $z^2 + 5z$ $-7z^2 + 7z - 3$
- $6x^2 + 5$; $3 - 2x^2$ $4x^2 + 8$
- $5x^4 - 2x + 7$; $-3x^4 + 6x^2 - 5$ $2x^4 + 6x^2 - 2x + 2$
- $4x^2 - 7x + 2$; $-x^2 + x - 2$ $3x^2 - 6x$

In Exercises 21–24, subtract the second polynomial from the first. (Use a horizontal format.)

- $z^3 + z^2 + 1$; z^2 $z^3 + 1$
- 10 ; $u^2 + 5$ $-u^2 + 5$
- $2x^2 + 3x - 4$; $x^2 + x - 1$ $x^2 + 2x - 3$
- $3x^3 - 4x^2 + 3$; $x^3 + 3x^2 - x - 4$ $2x^3 - 7x^2 + x + 7$

In Exercises 25–28, subtract the second polynomial from the first. (Use a vertical format.)

25. $10x^3 + 15$; $17x^3 - 4x + 5$ $-7x^3 + 4x + 10$

26. $y^2 + 3y^4$; $y^5 - y^4$ $-y^6 + 4y^4 + y^2$

27. $-2x^3 + 5x^2 - x + 8$; $-2x^3 + 3x - 4$

28. $3x^2 + 7x - 6$; $3x^2 + 7x - 6$

$5x^2 - 4x + 12$

In Exercises 29–34, perform the indicated operations. Use either a horizontal or vertical format and explain why you chose the method you used. *Reasons will vary.*

29. $(6x - 5) - (8x + 15) + (3x - 4)$ $x - 24$

30. $(2x^2 + 1) + (x^2 - 2x + 1) - (2x^2 + 8)$ $x^2 - 2x - 6$

31. $-(x^3 - 2) + (4x^3 - 2x) - (2x^2 + 3)$

32. $-(5x^2 - 1) - (-3x^2 + 5) - (x^2 - x)$

33. $2(t^2 + 5) - 3(t^2 + 5) + 5(t^2 + 5)$ $4t^2 + 20$

34. $-10(u + 1) + 8(u - 1) - 3(u + 6)$

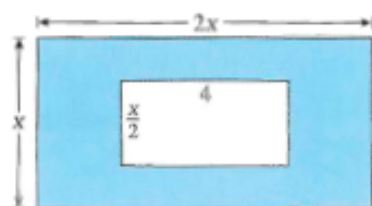
31. $3x^3 - 2x^2 - 2x - 1$ $-5u - 36$

32. $-3x^2 + x - 4$

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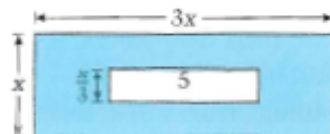
Geometry In Exercises 35 and 36, find the area of the shaded region.

35.



$2x^2 - 2x$

36.



$3x^2 - \frac{5}{3}x$

10.2

Multiplying Polynomials



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EXERCISES

Guided Practice

► **CRITICAL THINKING** about the Lesson See below.

1. Show how the Distributive Properties can be used to multiply $(2x - 3)$ and $(x + 4)$.
2. Multiply: $(x + 1)(x^2 - x + 1)$. Explain your use of the Distributive Property.
3. Multiply: $(x - 3)(2x + 5)$. $2x^2 - x - 15$
4. What does FOIL represent?

Independent Practice

In Exercises 5–10, multiply.

5. $(3x - 7)(-2x)$ $-6x^2 + 14x$
6. $3x^2(5x - x^3 + 2)$ $15x^3 - 3x^5 + 6x^2$
7. $(-x)(2x^2 - 3x)$ $-2x^3 + 3x^2$
8. $2x(3x^2 - 4x + 1)$ $6x^3 - 8x^2 + 2x$
9. $4x^2(5x^3 - 2x^2 + x)$ $20x^5 - 8x^4 + 4x^3$
10. $-x^2(6x^3 - 14x + 9)$ $-6x^5 + 14x^3 - 9x^2$

In Exercises 11–16, use the FOIL pattern to multiply.

11. $(3x - 2)(5x + 7)$ $15x^2 + 11x - 14$
12. $(3x + 5)(2x + 1)$ $6x^2 + 13x + 5$
13. $(x - 4)(x + 4)$ $x^2 - 16$
14. $(2x - 3)(x + 3)$ $2x^2 + 3x - 9$
15. $(x - 5)(2x + 10)$ $2x^2 - 50$
16. $(3x - 5)(2x + 1)$ $6x^2 - 7x - 5$

In Exercises 17–22, use an area model (or algebra tiles) to multiply. See Additional Answers.

17. $(x + 1)(x + 5)$ $x^2 + 6x + 5$
18. $(x + 2)(x + 6)$ $x^2 + 8x + 12$
19. $(x + 1)(x + 2)$ $x^2 + 3x + 2$
20. $(3x + 1)(2x + 2)$ $6x^2 + 8x + 2$
21. $(x + 2)(2x + 3)$ $2x^2 + 7x + 6$
22. $(2x + 1)(x + 3)$ $2x^2 + 7x + 3$

In Exercises 23–28, use the Distributive Property to multiply.

23. $(x - 3)(3x + 1)$ $3x^2 - 8x - 3$
24. $(2x + 1)(3x + 1)$ $6x^2 + 5x + 1$
25. $(3x^2 + x - 5)(2x - 1)$ $6x^3 - x^2 - 11x + 5$
26. $(2x^2 - 7x + 1)(4x + 3)$ $8x^3 - 22x^2 - 17x + 3$
27. $(x^2 + 9)(x^2 - x - 4)$ $x^4 - x^3 + 5x^2 - 9x - 36$
28. $(x + 3)(x^2 - 6x + 2)$ $x^3 - 3x^2 - 16x + 6$

In Exercises 23–28, use the Distributive Property to multiply.

23. $(x - 3)(3x + 1)$ $3x^2 - 8x - 3$ 24. $(2x + 1)(3x + 1)$ $6x^2 + 5x + 1$ 25. $(3x^2 + x - 5)(2x - 1)$

26. $(2x^2 - 7x + 1)(4x + 3)$ 27. $(x^2 + 9)(x^2 - x - 4)$ 28. $(x + 3)(x^2 - 6x + 2)$
 $8x^3 - 22x^2 - 17x + 3$ $x^4 - x^3 + 5x^2 - 9x - 36$ $x^3 - 3x^2 - 16x + 6$

In Exercises 29–37, multiply.

29. $(x + 3)(x - 4)$ $x^2 - x - 12$ 30. $(2x - 1)(x + 9)$ $2x^2 + 17x - 9$ 31. $(2x - 5)(x + 6)$

32. $(3x - 4)(\frac{1}{3}x + 1)$ $x^2 + \frac{5}{3}x - 4$ 33. $(x + \frac{6}{5})(4x - 5)$ $4x^2 - \frac{1}{5}x - 6$ 34. $(x + \frac{1}{4})(x - \frac{5}{4})$

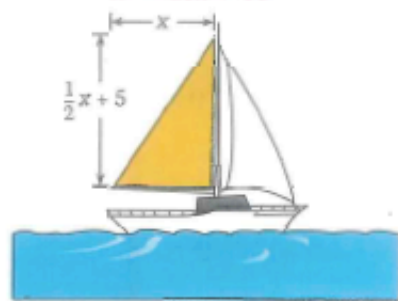
35. $(\frac{1}{2}x + 3)(\frac{1}{2}x - 2)$ $\frac{1}{4}x^2 + \frac{1}{2}x - 6$ 36. $(-3x^2 + x - 1)(x + 3)$ 37. $(x^2 + 4x - 9)(x - 4)$
 $x^3 - 25x + 36$

38. **Area of a Sail** The base of a triangular sail is x feet and its height is $\frac{1}{2}x + 5$ feet. Find an expression for the area, A , of the sail. $A = \frac{1}{4}x^2 + \frac{5}{2}x$

39. Use the expression in Exercise 38 to complete the table.

Base, x	5	6	7	8	9	10
Area, A	?	?	?	?	?	?

$18\frac{3}{4}, 24, 29\frac{3}{4}, 36, 42\frac{3}{4}, 50$



34. $x^2 - x - \frac{5}{16}$

1. $(2x - 3)(x + 4) = 2x(x + 4) - 3(x + 4)$
 $= 2x^2 + 8x - 3x - 12 = 2x^2 + 5x - 12$

2. $(x^3 + 1), x(x^2 - x + 1) + 1(x^2 - x + 1)$
 $= x^3 - x^2 + x + x^2 - x + 1$

10.3

Multiplying Polynomials: Two Special Cases

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I'VE DECIDED TO SIMPLIFY MY LIFE, ROY. YOU'RE OUT.
MY iPhone IS IN."

Guided Practice

CRITICAL THINKING about the Lesson

1. True or False? The product of $(a - b)$ and $(a - b)$ is $a^2 - b^2$. Explain. **False. See margin.** $(a + b)^2 = a^2 + \boxed{?} + b^2$, $2ab$
2. Find the missing term:
 $(x - 3)(x + 3) = x^2 - 9$; $(x + 3)^2 = x^2 + 6x + 9$;
 $(x - 3)^2 = x^2 - 6x + 9$
3. Write two expressions for the area of a square whose sides are each $x - 4$.
 $(x - 4)^2$, $x^2 - 8x + 16$
4. Give an example of each of the types of special products in this lesson.

Answer

$$\begin{aligned} 1. \text{ False } (a - b)(a - b) \\ &= a^2 - ab - ab + b^2 \\ &= a^2 - 2ab + b^2 \neq a^2 - b^2 \end{aligned}$$

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Independent Practice

In Exercises 5–10, use an area model (or algebra tiles) to write the square as a trinomial. **See Additional Answers.**

5. $(x + 2)^2$ $x^2 + 4x + 4$
6. $(x + 3)^2$ $x^2 + 6x + 9$
7. $(2n + 1)^2$ $4n^2 + 4n + 1$
8. $(3a + 2)^2$ $9a^2 + 12a + 4$
9. $(2x + 2)^2$ $4x^2 + 8x + 4$
10. $(3x + 1)^2$ $9x^2 + 6x + 1$

In Exercises 11–16, write the square as a trinomial.

11. $(n + 6)^2$ $n^2 + 12n + 36$
12. $(x + 4)^2$ $x^2 + 8x + 16$
13. $(2x + 1)^2$ $4x^2 + 4x + 1$
14. $(2m - 3)^2$ $4m^2 - 12m + 9$
15. $(3t - 2)^2$ $9t^2 - 12t + 4$
16. $(x - 9)^2$ $x^2 - 18x + 81$

In Exercises 17–22, multiply.

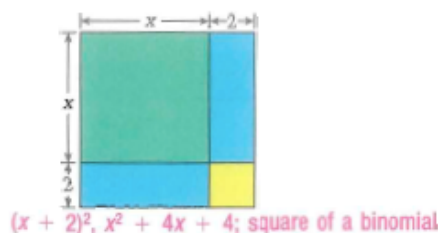
17. $(x + 5)(x - 5)$ $x^2 - 25$
18. $(x - 2)(x + 2)$ $x^2 - 4$
19. $(2x - 2)(2x + 2)$ $4x^2 - 4$
20. $(5x - 6)(5x + 6)$ $25x^2 - 36$
21. $(a + 2b)(a - 2b)$ $a^2 - 4b^2$
22. $(4x - 7y)(4x + 7y)$ $16x^2 - 49y^2$

In Exercises 23–28, write the square as a trinomial.

23. $(x + 6)^2$ $x^2 + 12x + 36$
24. $(x + 10)^2$ $x^2 + 20x + 100$
25. $(a - 2)^2$ $a^2 - 4a + 4$
26. $(2x - 5)^2$ $4x^2 - 20x + 25$
27. $(2x - 5y)^2$ $4x^2 - 20xy + 25y^2$
28. $(4s + 3t)^2$ $16s^2 + 24st + 9t^2$

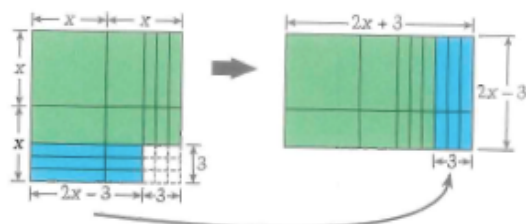
Area Model (or Algebra Tiles) In Exercises 29 and 30, write two different expressions for the area of the figure. Describe the special-product pattern that is represented.

29.



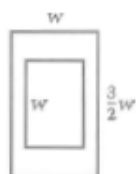
30.

$4x^2 - 9$, $(2x + 3)(2x - 3)$;
Product of a sum and difference of two terms



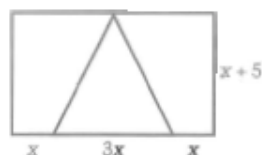
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33. **Geometry** The ratio of the height and width of the smaller rectangle is equal to the ratio of the height and width of the larger rectangle. Find expressions for the perimeters and areas of both.



Large: $P = 5w$, $A = \frac{3}{2}w^2$. Small: $P = \frac{10}{3}w$, $A = \frac{2}{3}w^2$ Triangle: $A = \frac{3}{2}x^2 + \frac{15}{2}x$. Rectangle: $A = 5x^2 + 25x$

34. **Geometry** Find the area of the rectangle and the area of the triangle.



10.4

Factoring: Special Products



Guided Practice

CRITICAL THINKING about the Lesson

- Describe the relationship between multiplying polynomials and factoring a polynomial. **Each is the reverse process of the other.**
- Factor out the greatest common monomial factor: $3x^3 - 6x^2 + 9$.
 $3(x^3 - 2x^2 + 3)$
- Show how the Distributive Property can be used to factor $2x(2x - 3) + 3(2x - 3)$.
3., 4. See margin.
- Give an example of each of the three special-product factoring patterns in the lesson.

Independent Practice

In Exercises 5–10, find the greatest common factor of the given terms.

- $6x^5, 30x^4, 12x^3$ **$6x^3$**
- $7x^3, 28x, 14x^4$ **$7x$**
- $24x^3, 32x^2$ **$8x^2$**
- $99x^6, 45x^3$ **$9x^3$**
- $16x^2y, 84xy^2, 36x^2y^2$ **$4xy$**
- $10xy^2, 25x^3y^2, 80x^2y$ **$5xy$**

In Exercises 11–19, factor out the greatest common monomial factor. *(difficult!)*

- $2x^2 - 4$ **$2(x^2 - 2)$**
- $3x + 6$ **$3(x + 2)$**
- $4a - 12$ **$4(a - 3)$**
- $14z^3 + 21$ **$7(2z^3 + 3)$**
- $24x^2 - 18$ **$6(4x^2 - 3)$**
- $-a^3 - 4a$ **$-a(a^2 + 4)$**
- $21u^2 - 14u$ **$7u(3u - 2)$**
- $36y^4 + 24y^2$ **$12y^2(3y^2 + 2)$**
- $4x^2 - 8x + 8$ **$4(x^2 - 2x + 2)$**
- $2x^2 + 16x + 32$ **$2(x + 4)^2$**

In Exercises 20–28, factor the expression. **25. $(u + \frac{1}{4})(u - \frac{1}{4})$**

- $x^2 - 64$ **$(x + 8)(x - 8)$**
- $y^2 - 144$ **$(y + 12)(y - 12)$**
- $2x^2 + 16x + 32$
- $9x^2 - 30xy + 25y^2$ **$(3x - 5y)^2$**
- $4y^2 + 20yz + 25z^2$ **$(2y + 5z)^2$**
- $u^2 - \frac{1}{16}$
- $v^2 - \frac{9}{25}$ **$(v + \frac{3}{5})(v - \frac{3}{5})$**
- $81 - (z + 5)^2$ **$(4 - z)(14 + z)$**
- $3(x - 3)^2 - 12$ **$3(x - 5)(x - 1)$**

10.5

Factoring Quadratic Trinomials

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REVOLUTIONIZE THE WAY
WE ANNOY EACH OTHER.



EXERCISES

16. $(x + 2)(x + 11)$
 22. $(2x - 1)(3x + 4)$
 28. $(1 + 7x)(5 - x)$

19. $(y - 15)(y - 20)$
 25. $(y - 12)(y - 4)$

Guided Practice

CRITICAL THINKING about the Lesson

- Factor $x^2 - 4x + 3$. When testing possible factorizations, why is it unnecessary to test $(x - 1)(x + 3)$ and $(x + 1)(x - 3)$?
- Factor $x^2 + 2x - 3$. When testing possible factorizations, why is it unnecessary to test $(x - 1)(x - 3)$ and $(x + 1)(x + 3)$?
- What is the discriminant of $ax^2 + bx + c$?
 $b^2 - 4ac$
 1., 2. See margin.
- If the discriminant of $ax^2 + bx + c$ is 35, can the trinomial be factored with integer coefficients? Explain.
 No. The discriminant must be the square of an integer.

Independent Practice

In Exercises 5–10, choose the correct factorization. (If neither is correct, find the correct factorization.)

- | | | |
|---|--|---|
| 5. $x^2 + x - 20$ a
a. $(x - 4)(x + 5)$
b. $(x + 4)(x - 5)$ | 6. $x^2 + 8x + 16$ b
a. $(x + 2)(x + 8)$
b. $(x + 4)(x + 4)$ | 7. $x^2 - 10x + 24$ a
a. $(x - 6)(x - 4)$
b. $(x - 12)(x + 2)$ |
| 8. $3x^2 - 7x - 6$ a
a. $(x - 3)(3x + 2)$
b. $(x + 3)(3x - 2)$ | 9. $6x^2 - 7x - 5$ b
a. $(6x + 1)(x - 5)$
b. $(2x + 1)(3x - 5)$ | 10. $2x^2 - 7x - 9$
a. $(x - 1)(2x + 9)$
b. $(2x - 1)(x + 9)$
Neither, $(x + 1)(2x - 9)$ |

In Exercises 11–28, factor the trinomial.

- | | | |
|--|--|-----------------------|
| 11. $x^2 + 3x - 4$ $(x + 4)(x - 1)$ | 12. $x^2 - 5x + 6$ $(x - 2)(x - 3)$ | 13. $x^2 + 3x - 18$ |
| 14. $y^2 - 16y - 36$ $(y - 18)(y + 2)$ | 15. $x^2 - 10x + 24$ $(x - 6)(x - 4)$ | 16. $x^2 + 13x + 22$ |
| 17. $x^2 + 15x + 50$ $(x + 10)(x + 5)$ | 18. $y^2 + 30y + 216$ $(y + 12)(y + 18)$ | 19. $y^2 - 35y + 300$ |
| 20. $t^2 - 4t - 21$ $(t - 7)(t + 3)$ | 21. $3x^2 + 8x + 5$ $(3x + 5)(x + 1)$ | 22. $6x^2 + 5x - 4$ |
| 23. $2x^2 - x - 21$ $(2x - 7)(x + 3)$ | 24. $3x^2 + 11x + 10$ $(3x + 5)(x + 2)$ | 25. $48 - 16y + y^2$ |
| 26. $32 + 12x + x^2$ $(x + 4)(x + 8)$ | 27. $2x^2 - x - 6$ $(2x + 3)(x - 2)$ | 28. $5 + 34x - 7x^2$ |

In Exercises 29–34, use the discriminant to decide whether the polynomial can be factored with integer coefficients. If it can be factored, then find the factors:

29. $12x^2 - 11x + 3$ **Cannot**

30. $2x^2 - 5x - 12$ $(2x + 3)(x - 4)$

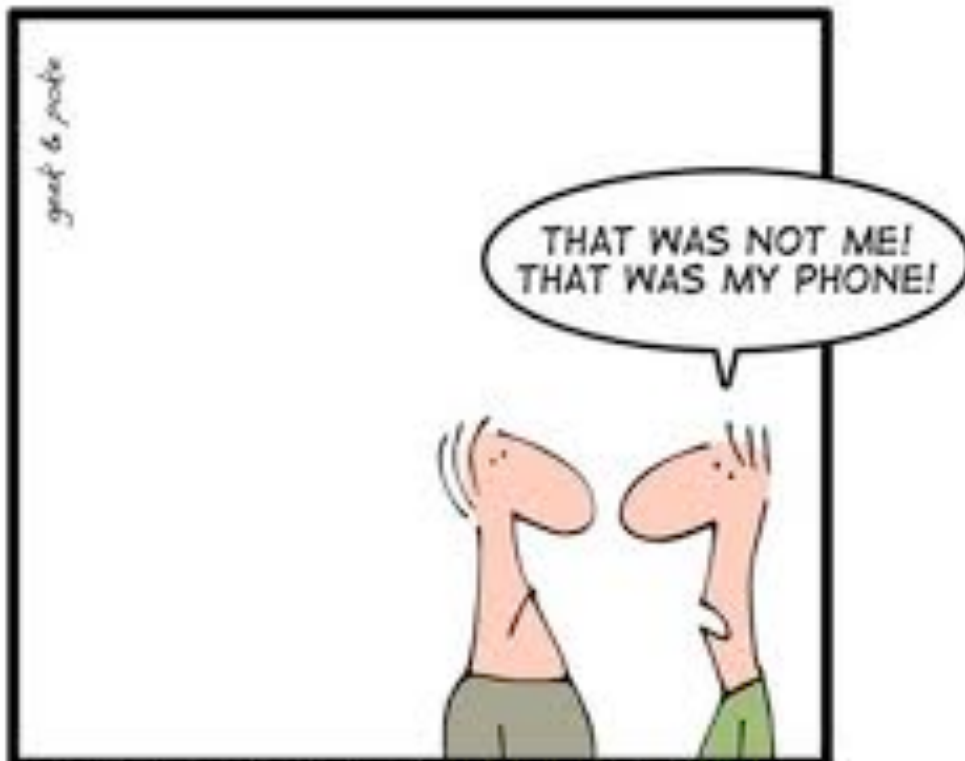
31. $6x^2 - 10x + 4$ $2(3x - 2)(x - 1)$

32. $10x^2 - 9x + 6$ **Cannot**

33. $14x^2 - 19x - 40$ $(7x + 8)(2x - 5)$

34. $24x^2 + 3x - 11$ **Cannot**

35. **Geometry** The area of a rectangle is given by $A = x^2 + 4x - 5$. Find expressions for possible lengths and widths of the rectangle. $x + 5$, $x - 1$
36. **Geometry** The area of a circle is given by $A = \pi(4x^2 + 12x + 9)$. Find an expression for the radius of the circle. $2x + 3$



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