

# 10.1

## Adding and Subtracting Polynomials

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## Guided Practice

### CRITICAL THINKING about the Lesson

- Describe a polynomial in one variable.
- Name the terms of  $-3x^3 - 2x^2 + 4x - 5$ .
- Name the coefficients in  $-7x^3 + 12x - 31$ .  
 $-7, 0, 12, -31$
- Write  $15y - 6 + 10y^3 - 3y^2$  in standard form.  $10y^3 - 3y^2 + 15y - 6$
- What is the degree of  $2x^2 - 4x^3 + 7$ ? **3**
- Subtract  $(2x^2 - 4x + 1)$  from  $(x^2 + 8)$ .  
 $-x^2 + 4x + 7$

1. See page 508.

## Independent Practice

In Exercises 7–10, classify the polynomial by degree and by number of terms.

- $-5x - 4$  **Linear, binomial**
- $-7$  **Constant, monomial**
- $16 - 4x + 3x^2 - x^4$  **Quadratic, polynomial**
- $3x^2 + 6x + 1$  **Quadratic, trinomial**

In Exercises 11–16, add the polynomials. (Use a horizontal format.)

- $x^2 - 3$ ;  $3x^2 + 5$   **$4x^2 + 2$**
- $-3y + 2$ ;  $y^2 + 3y + 2$   **$y^2 + 4$**
- $2x^2 + 3x + 1$ ;  $x^2 - 2x + 2$   **$3x^2 + x + 3$**
- $2x^2 - x + 3$ ;  $3x^2 - 4x + 7$   **$5x^2 - 5x + 10$**
- $12x^3 + 2x^2 - 4$ ;  $9x^2 + 3x - 8$   **$12x^3 + 11x^2 + 3x - 12$**
- $-4x^3 - 2x^2 + x - 5$ ;  $2x^3 + 3x + 4$   **$-2x^3 - 2x^2 + 4x - 1$**

In Exercises 17–20, add the polynomials. (Use a vertical format.)

- $2z - 8z^2 - 3$ ;  $z^2 + 5z$   **$-7z^2 + 7z - 3$**
- $6x^2 + 5$ ;  $3 - 2x^2$   **$4x^2 + 8$**
- $5x^4 - 2x + 7$ ;  $-3x^4 + 6x^2 - 5$   **$2x^4 + 6x^2 - 2x + 2$**
- $4x^2 - 7x + 2$ ;  $-x^2 + x - 2$   **$3x^2 - 6x$**

In Exercises 21–24, subtract the second polynomial from the first. (Use a horizontal format.)

- $z^3 + z^2 + 1$ ;  $z^2$   **$z^3 + 1$**
- $10$ ;  $u^2 + 5$   **$-u^2 + 5$**
- $2x^2 + 3x - 4$ ;  $x^2 + x - 1$   **$x^2 + 2x - 3$**
- $3x^3 - 4x^2 + 3$ ;  $x^3 + 3x^2 - x - 4$   **$2x^3 - 7x^2 + x + 7$**

In Exercises 25–28, subtract the second polynomial from the first. (Use a vertical format.)

25.  $10x^3 + 15$ ;  $17x^3 - 4x + 5$   $-7x^3 + 4x + 10$

26.  $y^2 + 3y^4$ ;  $y^5 - y^4$   $-y^6 + 4y^4 + y^2$

27.  $-2x^3 + 5x^2 - x + 8$ ;  $-2x^3 + 3x - 4$

28.  $3x^2 + 7x - 6$ ;  $3x^2 + 7x - 6$

$5x^2 - 4x + 12$

In Exercises 29–34, perform the indicated operations. Use either a horizontal or vertical format and explain why you chose the method you used. *Reasons will vary.*

29.  $(6x - 5) - (8x + 15) + (3x - 4)$   $x - 24$

30.  $(2x^2 + 1) + (x^2 - 2x + 1) - (2x^2 + 8)$   $x^2 - 2x - 6$

31.  $-(x^3 - 2) + (4x^3 - 2x) - (2x^2 + 3)$

32.  $-(5x^2 - 1) - (-3x^2 + 5) - (x^2 - x)$

33.  $2(t^2 + 5) - 3(t^2 + 5) + 5(t^2 + 5)$   $4t^2 + 20$

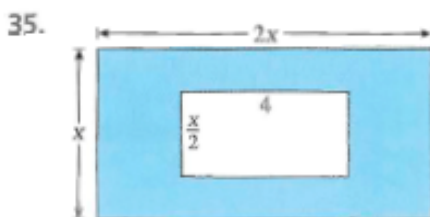
34.  $-10(u + 1) + 8(u - 1) - 3(u + 6)$

31.  $3x^3 - 2x^2 - 2x - 1$   $-5u - 36$

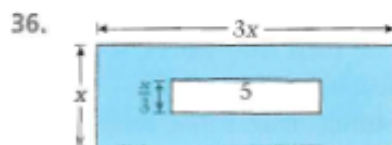
32.  $-3x^2 + x - 4$

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**Geometry** In Exercises 35 and 36, find the area of the shaded region.



$2x^2 - 2x$



$3x^2 - \frac{5}{4}x$

# 10.2

## Multiplying Polynomials



## EXERCISES

### Guided Practice

► **CRITICAL THINKING** about the Lesson See below.

1. Show how the Distributive Properties can be used to multiply  $(2x - 3)$  and  $(x + 4)$ .
2. Multiply:  $(x + 1)(x^2 - x + 1)$ . Explain your use of the Distributive Property.
3. Multiply:  $(x - 3)(2x + 5)$ .  $2x^2 - x - 15$
4. What does FOIL represent?

### Independent Practice

In Exercises 5–10, multiply.

5.  $(3x - 7)(-2x)$   $-6x^2 + 14x$
6.  $3x^2(5x - x^3 + 2)$   $15x^3 - 3x^5 + 6x^2$
7.  $(-x)(2x^2 - 3x)$   $-2x^3 + 3x^2$
8.  $2x(3x^2 - 4x + 1)$   $6x^3 - 8x^2 + 2x$
9.  $4x^2(5x^3 - 2x^2 + x)$   $20x^5 - 8x^4 + 4x^3$
10.  $-x^2(6x^3 - 14x + 9)$   $-6x^5 + 14x^3 - 9x^2$

In Exercises 11–16, use the FOIL pattern to multiply.

11.  $(3x - 2)(5x + 7)$   $15x^2 + 11x - 14$
12.  $(3x + 5)(2x + 1)$   $6x^2 + 13x + 5$
13.  $(x - 4)(x + 4)$   $x^2 - 16$
14.  $(2x - 3)(x + 3)$   $2x^2 + 3x - 9$
15.  $(x - 5)(2x + 10)$   $2x^2 - 50$
16.  $(3x - 5)(2x + 1)$   $6x^2 - 7x - 5$

In Exercises 17–22, use an area model (or algebra tiles) to multiply. See Additional Answers.

17.  $(x + 1)(x + 5)$   $x^2 + 6x + 5$
18.  $(x + 2)(x + 6)$   $x^2 + 8x + 12$
19.  $(x + 1)(x + 2)$   $x^2 + 3x + 2$
20.  $(3x + 1)(2x + 2)$   $6x^2 + 8x + 2$
21.  $(x + 2)(2x + 3)$   $2x^2 + 7x + 6$
22.  $(2x + 1)(x + 3)$   $2x^2 + 7x + 3$

In Exercises 23–28, use the Distributive Property to multiply.

23.  $(x - 3)(3x + 1)$   $3x^2 - 8x - 3$
24.  $(2x + 1)(3x + 1)$   $6x^2 + 5x + 1$
25.  $(3x^2 + x - 5)(2x - 1)$   $6x^3 - x^2 - 11x + 5$
26.  $(2x^2 - 7x + 1)(4x + 3)$   $8x^3 - 22x^2 - 17x + 3$
27.  $(x^2 + 9)(x^2 - x - 4)$   $x^4 - x^3 + 5x^2 - 9x - 36$
28.  $(x + 3)(x^2 - 6x + 2)$   $x^3 - 3x^2 - 16x + 6$

In Exercises 23–28, use the Distributive Property to multiply.

23.  $(x - 3)(3x + 1)$   $3x^2 - 8x - 3$  24.  $(2x + 1)(3x + 1)$   $6x^2 + 5x + 1$  25.  $(3x^2 + x - 5)(2x - 1)$   
 26.  $(2x^2 - 7x + 1)(4x + 3)$  27.  $(x^2 + 9)(x^2 - x - 4)$  28.  $(x + 3)(x^2 - 6x + 2)$   
 $8x^3 - 22x^2 - 17x + 3$   $x^4 - x^3 + 5x^2 - 9x - 36$   $x^3 - 3x^2 - 16x + 6$

In Exercises 29–37, multiply.

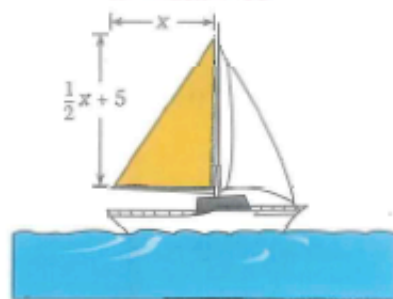
29.  $(x + 3)(x - 4)$   $x^2 - x - 12$  30.  $(2x - 1)(x + 9)$   $2x^2 + 17x - 9$  31.  $(2x - 5)(x + 6)$   
 32.  $(3x - 4)(\frac{1}{3}x + 1)$   $x^2 + \frac{5}{3}x - 4$  33.  $(x + \frac{6}{5})(4x - 5)$   $4x^2 - \frac{1}{5}x - 6$  34.  $(x + \frac{1}{4})(x - \frac{5}{4})$   
 35.  $(\frac{1}{2}x + 3)(\frac{1}{2}x - 2)$   $\frac{1}{4}x^2 + \frac{1}{2}x - 6$  36.  $(-3x^2 + x - 1)(x + 3)$  37.  $(x^2 + 4x - 9)(x - 4)$   
 $x^3 - 25x + 36$

38. **Area of a Sail** The base of a triangular sail is  $x$  feet and its height is  $\frac{1}{2}x + 5$  feet. Find an expression for the area,  $A$ , of the sail.  $A = \frac{1}{4}x^2 + \frac{5}{2}x$

39. Use the expression in Exercise 38 to complete the table.

Base, $x$	5	6	7	8	9	10
Area, $A$	?	?	?	?	?	?

$18\frac{3}{4}, 24, 29\frac{3}{4}, 36, 42\frac{3}{4}, 50$



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34.  $x^2 - x - \frac{5}{16}$

1.  $(2x - 3)(x + 4) = 2x(x + 4) - 3(x + 4)$   
 $= 2x^2 + 8x - 3x - 12 = 2x^2 + 5x - 12$  2.  $(x^3 + 1), x(x^2 - x + 1) + 1(x^2 - x + 1)$   
 $= x^3 - x^2 + x + x^2 - x + 1$

# 10.3

## Multiplying Polynomials: Two Special Cases

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I'VE DECIDED TO SIMPLIFY MY LIFE, ROY. YOU'RE OUT.  
MY iPHONE IS IN."



## Guided Practice

### CRITICAL THINKING about the Lesson

1. True or False? The product of  $(a - b)$  and  $(a - b)$  is  $a^2 - b^2$ . Explain. **False. See margin.**  $(a + b)^2 = a^2 + \boxed{?} + b^2$ ,  $2ab$
2. Find the missing term:  $(a + b)^2 = a^2 + \boxed{?} + b^2$ ,  $2ab$
3. Write two expressions for the area of a square whose sides are each  $x - 4$ .  
 $(x - 4)^2$ ,  $x^2 - 8x + 16$
4. Give an example of each of the types of special products in this lesson.  
 $(x - 3)(x + 3) = x^2 - 9$ ;  $(x + 3)^2 = x^2 + 6x + 9$ ;  
 $(x - 3)^2 = x^2 - 6x + 9$

#### Answer

$$\begin{aligned} 1. \text{ False } (a - b)(a - b) \\ &= a^2 - ab - ab + b^2 \\ &= a^2 - 2ab + b^2 \neq a^2 - b^2 \end{aligned}$$

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## Independent Practice

In Exercises 5–10, use an area model (or algebra tiles) to write the square as a trinomial. See Additional Answers.

5.  $(x + 2)^2$   $x^2 + 4x + 4$
6.  $(x + 3)^2$   $x^2 + 6x + 9$
7.  $(2n + 1)^2$   $4n^2 + 4n + 1$
8.  $(3a + 2)^2$   $9a^2 + 12a + 4$
9.  $(2x + 2)^2$   $4x^2 + 8x + 4$
10.  $(3x + 1)^2$   $9x^2 + 6x + 1$

In Exercises 11–16, write the square as a trinomial.

11.  $(n + 6)^2$   $n^2 + 12n + 36$
12.  $(x + 4)^2$   $x^2 + 8x + 16$
13.  $(2x + 1)^2$   $4x^2 + 4x + 1$
14.  $(2m - 3)^2$   $4m^2 - 12m + 9$
15.  $(3t - 2)^2$   $9t^2 - 12t + 4$
16.  $(x - 9)^2$   $x^2 - 18x + 81$

In Exercises 17–22, multiply.

17.  $(x + 5)(x - 5)$   $x^2 - 25$
18.  $(x - 2)(x + 2)$   $x^2 - 4$
19.  $(2x - 2)(2x + 2)$   $4x^2 - 4$
20.  $(5x - 6)(5x + 6)$   $25x^2 - 36$
21.  $(a + 2b)(a - 2b)$   $a^2 - 4b^2$
22.  $(4x - 7y)(4x + 7y)$   $16x^2 - 49y^2$

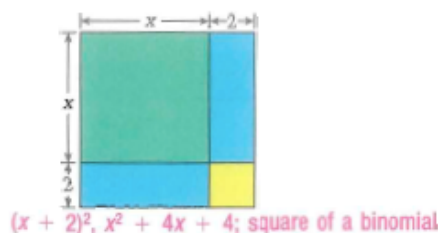
In Exercises 23–28, write the square as a trinomial.

23.  $(x + 6)^2$   $x^2 + 12x + 36$
24.  $(x + 10)^2$   $x^2 + 20x + 100$
25.  $(a - 2)^2$   $a^2 - 4a + 4$
26.  $(2x - 5)^2$   $4x^2 - 20x + 25$
27.  $(2x - 5y)^2$   $4x^2 - 20xy + 25y^2$
28.  $(4s + 3t)^2$   $16s^2 + 24st + 9t^2$



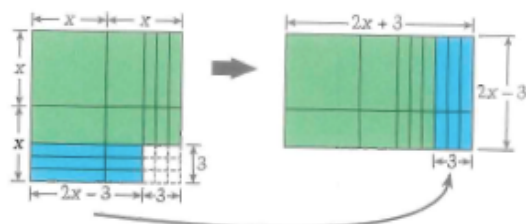
**Area Model (or Algebra Tiles)** In Exercises 29 and 30, write two different expressions for the area of the figure. Describe the special-product pattern that is represented.

29.



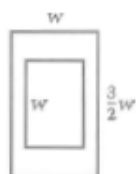
30.

$4x^2 - 9$ ,  $(2x + 3)(2x - 3)$ ;  
Product of a sum and difference of two terms



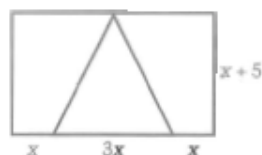
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33. **Geometry** The ratio of the height and width of the smaller rectangle is equal to the ratio of the height and width of the larger rectangle. Find expressions for the perimeters and areas of both.



Large:  $P = 5w$ ,  $A = \frac{3}{2}w^2$ . Small:  $P = \frac{10}{3}w$ ,  $A = \frac{2}{3}w^2$  Triangle:  $A = \frac{3}{2}x^2 + \frac{15}{2}x$ . Rectangle:  $A = 5x^2 + 25x$

34. **Geometry** Find the area of the rectangle and the area of the triangle.



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# 10.4

## Factoring: Special Products



## Guided Practice

### CRITICAL THINKING about the Lesson

- Describe the relationship between multiplying polynomials and factoring a polynomial. **Each is the reverse process of the other.**
- Show how the Distributive Property can be used to factor  $2x(2x - 3) + 3(2x - 3)$ .  
**3., 4. See margin.**
- Factor out the greatest common monomial factor:  $3x^3 - 6x^2 + 9$ .  
 **$3(x^3 - 2x^2 + 3)$**
- Give an example of each of the three special-product factoring patterns in the lesson.

## Independent Practice

In Exercises 5–10, find the greatest common factor of the given terms.

- $6x^5, 30x^4, 12x^3$   **$6x^3$**
- $7x^3, 28x, 14x^4$   **$7x$**
- $24x^3, 32x^2$   **$8x^2$**
- $99x^6, 45x^3$   **$9x^3$**
- $16x^2y, 84xy^2, 36x^2y^2$   **$4xy$**
- $10xy^2, 25x^3y^2, 80x^2y$   **$5xy$**

In Exercises 11–19, factor out the greatest common monomial factor. *(difficult!)*

- $2x^2 - 4$   **$2(x^2 - 2)$**
- $3x + 6$   **$3(x + 2)$**
- $4a - 12$   **$4(a - 3)$**
- $14z^3 + 21$   **$7(2z^3 + 3)$**
- $24x^2 - 18$   **$6(4x^2 - 3)$**
- $-a^3 - 4a$   **$-a(a^2 + 4)$**
- $21u^2 - 14u$   **$7u(3u - 2)$**
- $36y^4 + 24y^2$   **$12y^2(3y^2 + 2)$**
- $4x^2 - 8x + 8$   **$4(x^2 - 2x + 2)$**

In Exercises 20–28, factor the expression. **25.  $(u + \frac{1}{4})(u - \frac{1}{4})$**

- $x^2 - 64$   **$(x + 8)(x - 8)$**
- $y^2 - 144$   **$(y + 12)(y - 12)$**
- $2x^2 + 16x + 32$
- $9x^2 - 30xy + 25y^2$   **$(3x - 5y)^2$**
- $4y^2 + 20yz + 25z^2$   **$(2y + 5z)^2$**
- $u^2 - \frac{1}{16}$
- $v^2 - \frac{9}{25}$   **$(v + \frac{3}{5})(v - \frac{3}{5})$**
- $81 - (z + 5)^2$   **$(4 - z)(14 + z)$**
- $3(x - 3)^2 - 12$   **$3(x - 5)(x - 1)$**

10.5

## Factoring Quadratic Trinomials



## EXERCISES

### Guided Practice

#### CRITICAL THINKING about the Lesson

- Factor  $x^2 - 4x + 3$ . When testing possible factorizations, why is it unnecessary to test  $(x - 1)(x + 3)$  and  $(x + 1)(x - 3)$ ?
- Factor  $x^2 + 2x - 3$ . When testing possible factorizations, why is it unnecessary to test  $(x - 1)(x - 3)$  and  $(x + 1)(x + 3)$ ?
- What is the discriminant of  $ax^2 + bx + c$ ?  
 $b^2 - 4ac$
- If the discriminant of  $ax^2 + bx + c$  is 35, can the trinomial be factored with integer coefficients? Explain.  
No. The discriminant must be the square of an integer.

1., 2. See margin.

### Independent Practice

In Exercises 5–10, choose the correct factorization. (If neither is correct, find the correct factorization.)

- |                             |                             |                              |
|-----------------------------|-----------------------------|------------------------------|
| 5. $x^2 + x - 20$ <b>a</b>  | 6. $x^2 + 8x + 16$ <b>b</b> | 7. $x^2 - 10x + 24$ <b>a</b> |
| a. $(x - 4)(x + 5)$         | a. $(x + 2)(x + 8)$         | a. $(x - 6)(x - 4)$          |
| b. $(x + 4)(x - 5)$         | b. $(x + 4)(x + 4)$         | b. $(x - 12)(x + 2)$         |
| 8. $3x^2 - 7x - 6$ <b>a</b> | 9. $6x^2 - 7x - 5$ <b>b</b> | 10. $2x^2 - 7x - 9$          |
| a. $(x - 3)(3x + 2)$        | a. $(6x + 1)(x - 5)$        | a. $(x - 1)(2x + 9)$         |
| b. $(x + 3)(3x - 2)$        | b. $(2x + 1)(3x - 5)$       | b. $(2x - 1)(x + 9)$         |
|                             |                             | Neither, $(x + 1)(2x - 9)$   |

In Exercises 11–28, factor the trinomial.

- |                                        |                                          |                       |
|----------------------------------------|------------------------------------------|-----------------------|
| 11. $x^2 + 3x - 4$ $(x + 4)(x - 1)$    | 12. $x^2 - 5x + 6$ $(x - 2)(x - 3)$      | 13. $x^2 + 3x - 18$   |
| 14. $y^2 - 16y - 36$ $(y - 18)(y + 2)$ | 15. $x^2 - 10x + 24$ $(x - 6)(x - 4)$    | 16. $x^2 + 13x + 22$  |
| 17. $x^2 + 15x + 50$ $(x + 10)(x + 5)$ | 18. $y^2 + 30y + 216$ $(y + 12)(y + 18)$ | 19. $y^2 - 35y + 300$ |
| 20. $t^2 - 4t - 21$ $(t - 7)(t + 3)$   | 21. $3x^2 + 8x + 5$ $(3x + 5)(x + 1)$    | 22. $6x^2 + 5x - 4$   |
| 23. $2x^2 - x - 21$ $(2x - 7)(x + 3)$  | 24. $3x^2 + 11x + 10$ $(3x + 5)(x + 2)$  | 25. $48 - 16y + y^2$  |
| 26. $32 + 12x + x^2$ $(x + 4)(x + 8)$  | 27. $2x^2 - x - 6$ $(2x + 3)(x - 2)$     | 28. $5 + 34x - 7x^2$  |

In Exercises 29–34, use the discriminant to decide whether the polynomial can be factored with integer coefficients. If it can be factored, then find the factors:

29.  $12x^2 - 11x + 3$  **Cannot**

30.  $2x^2 - 5x - 12$   $(2x + 3)(x - 4)$

31.  $6x^2 - 10x + 4$   $2(3x - 2)(x - 1)$

32.  $10x^2 - 9x + 6$  **Cannot**

33.  $14x^2 - 19x - 40$   $(7x + 8)(2x - 5)$

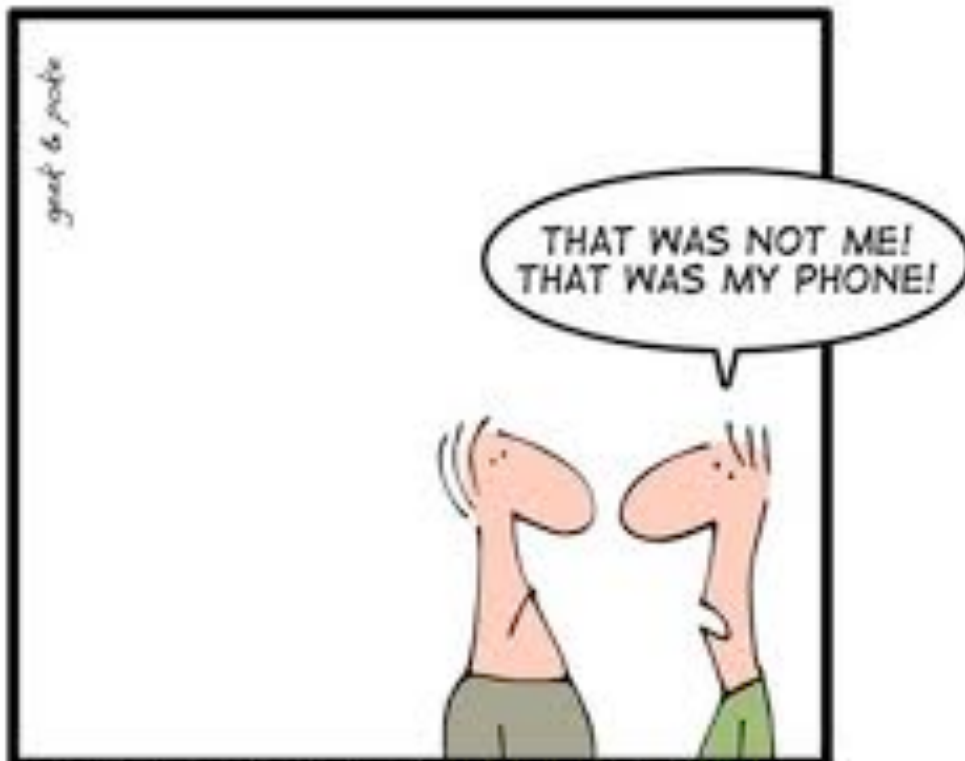
34.  $24x^2 + 3x - 11$  **Cannot**

35. **Geometry** The area of a rectangle is given by  $A = x^2 + 4x - 5$ .

Find expressions for possible lengths and widths of the rectangle.  $x + 5$ ,  $x - 1$

36. **Geometry** The area of a circle is given by  $A = \pi(4x^2 + 12x + 9)$ .

Find an expression for the radius of the circle.  $2x + 3$



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# 10.6

## Solving Quadratic Equations by Factoring



## Guided Practice

### CRITICAL THINKING about the Lesson

1. Use the Zero-Product Property to complete the statement. If  $ab = 0$ , then  $\boxed{?}$ .
2. Solve the equation:  $(x - 2)(x + 1) = 0$ .  
 $2, -1$
3. Solve the equation:  $3x^2 + 4x = 0$ .  $0, -\frac{4}{3}$
4. Which two numbers satisfy the statement, "The sum of a number and its square is zero."?  $0, -1$
5. True or False? If  $(5x - 1)(x + 3) = 1$ , then  $5x - 1 = 1$  or  $x + 3 = 1$ . Explain.  
False. The product of any number and its reciprocal is 1; so neither factor has to equal 1.
6. True or False? If  $(x + 3)(x - 3) = 0$ , then  $x + 3 = 0$  or  $x - 3 = 0$ . Explain  
True. A product cannot equal 0, unless one of the factors is 0.

## Independent Practice

In Exercises 7–10, solve the equation.

7.  $(x + 1)(x + 2) = 0$   $-1, -2$
8.  $(x - 3)(x + 7) = 0$   $3, -7$
9.  $(x + 3)(x + 4) = 0$   $-3, -4$
10.  $(x + 6)(x - 5) = 0$   $-6, 5$

In Exercises 11–16, solve the equation by factoring.

11.  $x^2 + 5x - 6 = 0$   $-6, 1$
12.  $3x^2 + 11x - 4 = 0$   $\frac{1}{3}, -4$
13.  $2x^2 + 5x + 3 = 0$   $-\frac{3}{2}, -1$
14.  $6x^2 + 13x + 5 = 0$   $-\frac{1}{2}, -\frac{5}{3}$
15.  $3x^2 + 7x + 2 = 0$   $-\frac{1}{3}, -2$
16.  $12x^2 - 5x - 3 = 0$   $\frac{3}{4}, -\frac{1}{3}$

In Exercises 17–24, match the equation with its solutions.

17.  $x^2 - 5x + 6 = 0$  **f**    18.  $x^2 + 5x + 6 = 0$  **b**    19.  $x^2 - 7x + 6 = 0$  **g**    20.  $x^2 + 7x + 6 = 0$  **a**  
 21.  $x^2 - 5x - 6 = 0$  **d**    22.  $x^2 + 5x - 6 = 0$  **c**    23.  $x^2 + x - 6 = 0$  **e**    24.  $x^2 - x - 6 = 0$  **h**  
 a.  $-1, -6$                       b.  $-2, -3$                       c.  $1, -6$                       d.  $-1, 6$   
 e.  $2, -3$                       f.  $2, 3$                       g.  $1, 6$                       h.  $-2, 3$

In Exercises 25–33, solve the equation by finding square roots, by the quadratic formula, or by factoring.

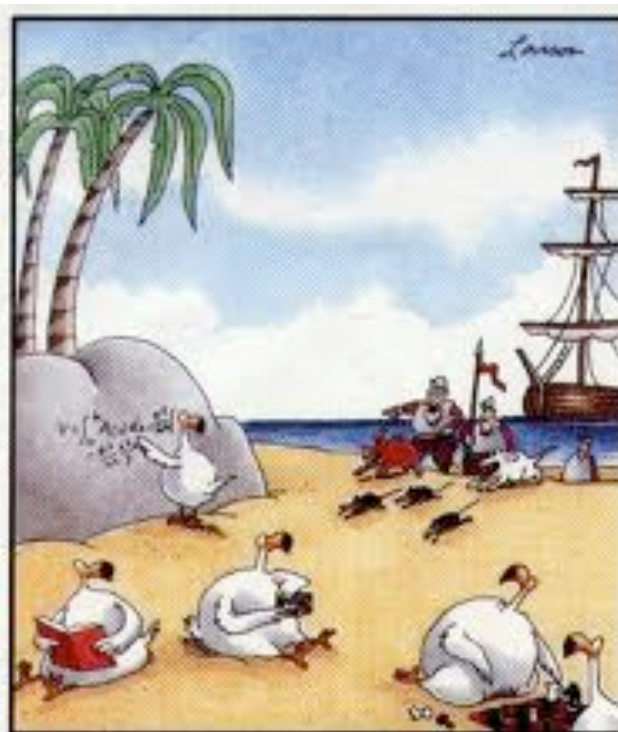
25.  $x(x - 9) = 0$  **0, 9**                      26.  $2y(y + 6) = 0$  **0, -6**                      27.  $y^2 - 7y + 6 = -6$  **3, 4**  
 28.  $x^2 - 12 = -3$   **$\pm 3$**                       29.  $x^2 - 8x = -16$  **4**                      30.  $x^2 + 4x + 7 = 3$  **-2**  
 31.  $4x^2 + 2x = 0$  **0,  $-\frac{1}{2}$**                       32.  $4y^2 - 18y = 0$  **0,  $\frac{9}{2}$**                       33.  $x^2 - 12x + 40 = 4$  **6**

In Exercises 34 and 35, multiply both sides of the equation by an appropriate power of ten to obtain integer coefficients. Then solve by factoring.

- ★ 34.  $0.8x^2 + 3.2x + 2.4 = 0$   **$-1, -3$**                       ★ 35.  $0.23x^2 - 0.54x + 0.16 = 0$   **$\frac{8}{23}, 2$**

# 9.2

## Solving Quadratic Equations by Finding Square Roots



Unbeknownst to most ornithologists, the dodo was actually a very advanced species, living alone quite peacefully until, in the 17th century, it was annihilated by men, rats, and dogs. As usual.

## Guided Practice

### CRITICAL THINKING about the Lesson

1. Which of the following are quadratic equations?

a.  $-3x + 5 = 0$

b.  $x^2 - 1 = 0$

c.  $x^2 - 3x^3 = 0$

d.  $-3 + 4x + x^2 = 0$

In Exercises 2–4, write in standard form and find the leading coefficient.

2.  $-3x^2 + 5 = 0$  As is,  $-3$

3.  $\frac{1}{2}x^2 + 9x - 3 = 0$  As is,  $\frac{1}{2}$

4.  $-8x - x^2 + 4 = 0$   
 $-x^2 - 8x + 4 = 0$ ,  $-1$

In Exercises 5–8, solve the equation. If there are no solutions, state the reason.

5.  $x^2 = 17 \pm \sqrt{17}$

6.  $x^2 = 0$   $0$

7.  $x^2 = -4$   
No real solution

8.  $x^2 = 6 \pm \sqrt{6}$

## Independent Practice

In Exercises 9–20, solve the equation.

9.  $x^2 = 9 \pm 3$

10.  $h^2 = 25 \pm 5$

11.  $6x^2 = 600 \pm 10$

16.  $\pm 12$

12.  $\frac{1}{5}x^2 = 5 \pm 5$

13.  $3x^2 = 363 \pm 11$

14.  $2b^2 = 98 \pm 7$

15.  $t^2 + 2 = 11 \pm 3$

16.  $t^2 - 57 = 87$

17.  $\frac{1}{2}x^2 - 1 = 7 \pm 4$

18.  $4y^2 + 7 = 8 \pm \frac{1}{2}$

19.  $2s^2 - 5 = 27 \pm 4$

20.  $81x^2 - 5 = 20 \pm \frac{5}{9}$

**Technology** In Exercises 21–28, use a calculator to solve the equation.

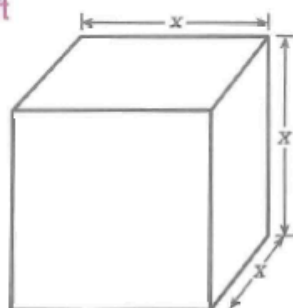
Round the results to two decimal places.

21.  $3x^2 + 2 = 56$   $\pm 4.24$  22.  $7y^2 - 12 = 23$   $\pm 2.24$  23.  $2x^2 - 5 = 7$   $\pm 2.45$  24.  $\frac{2}{3}n^2 - 6 = 2$   $\pm 3.46$   
 25.  $\frac{1}{2}x^2 + 3 = 8$   $\pm 3.16$  26.  $4x^2 + 9 = 41$   $\pm 2.83$  27.  $6s^2 - 2 = 0$   $\pm 0.58$  28.  $5a^2 + 10 = 20$   $\pm 1.41$

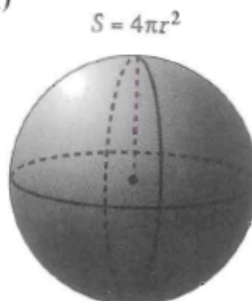
In Exercises 29–32, an object is dropped from a height  $h$ . How long does it take to reach the ground? (Assume there is no air resistance.)

29.  $h = 64$  feet 2 seconds 30.  $h = 144$  feet 3 seconds 31.  $h = 500$  feet 32.  $h = 600$  feet  
 31.  $\approx 5.59$  seconds  
 32.  $\approx 6.12$  seconds

- ★ 33. **Geometry** The surface area of a cube is 150 square feet. Find the length of each edge. 5 ft

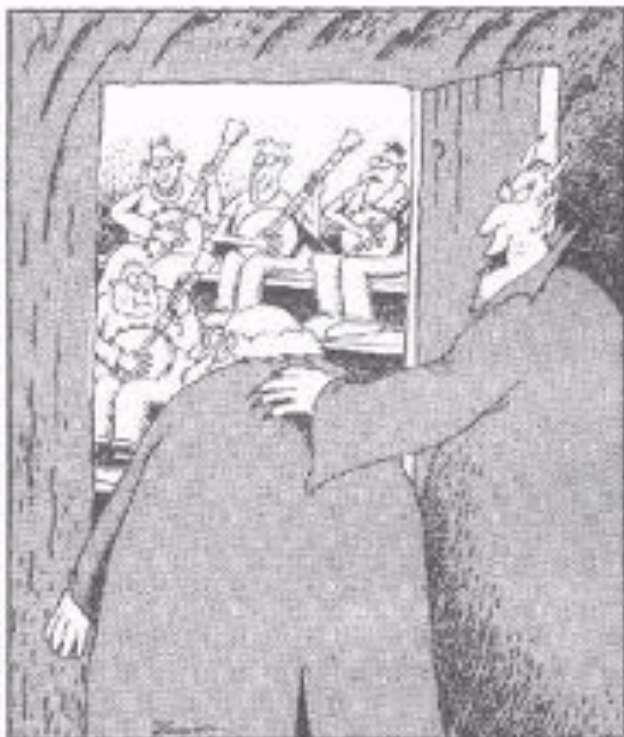


- ★ 34. **Geometry** The surface area of a sphere is 80 square meters. Find the radius. (Use  $\pi \approx 3.14$ .)  $\approx 2.52$  m



# 9.3

## Graphs of Quadratic Equations



"Your room is right in here, Maestro."



## Guided Practice

### CRITICAL THINKING about the Lesson

1. Write the equation  $y = -3 + 4x - x^2$  in standard form.  $y = -x^2 + 4x - 3$
2. The graph of a quadratic equation is called a ?. **parabola**
3. How can you use  $a$  to decide whether the graph of  $y = ax^2 + bx + c$  opens up or down? **If  $a > 0$ , graph opens up; if  $a < 0$ , graph opens down.**
4. **True or False?** The axis of symmetry of the graph of  $y = ax^2 + bx + c$  is parallel to the  $y$ -axis (or is the  $y$ -axis). **True**
5. Find the vertex of the graph of  $y = 2x^2 + 4x - 2$ .  **$(-1, -4)$**
6. Find the axis of symmetry of the graph of  $y = -3x^2 + 3x + 1$ .  **$x = \frac{1}{2}$**

## Independent Practice

In Exercises 7–12, decide whether the graph of the equation opens up or down. Then find the coordinates of the vertex.

- |                                                                          |                                                                                       |                                                         |
|--------------------------------------------------------------------------|---------------------------------------------------------------------------------------|---------------------------------------------------------|
| 7. $y = 2x^2 + 4$ <b>Up, <math>(0, 4)</math></b>                         | 8. $y = -5x^2$ <b>Down, <math>(0, 0)</math></b>                                       | 9. $y = -x^2 + 2x - 1$ <b>Down, <math>(0, 0)</math></b> |
| 10. $y - 3x^2 = -2x$ <b>Up, <math>(\frac{1}{3}, -\frac{1}{3})</math></b> | 11. $y + 5x^2 = -x + 10$<br><b>Down, <math>(-\frac{1}{10}, \frac{201}{20})</math></b> | 12. $y = 3x + 12$<br><b>Up, <math>(-8, -24)</math></b>  |

In Exercises 13–18, find the coordinates of the vertex and the equation of the axis of symmetry. See below.

13.  $y = 3x^2 + 2x + 4$

14.  $y = 2x^2 + 3x + 6$

15.  $y = -4x^2 - 4x + 8$

16.  $y = 3x^2 - 9x - 12$

17.  $y = 2x^2 + 7x - 21$

18.  $y = -x^2 + 4x + 16$

In Exercises 19–36, sketch the graph of the equation. Label the vertex. See Additional Answers.

19.  $y = x^2 + x + 2$

20.  $y = -x^2 + 2x - 1$

21.  $y = -2x^2 + 6x - 9$

22.  $y = 2x^2 - 3x + 4$

23.  $y = 6x^2 - 3x + 4$

24.  $y = 5x^2 + 4x - 5$

25.  $y = 4x^2 - x + 6$

26.  $y = -3x^2 - x + 7$

27.  $y = -5x^2 + 2x - 2$

28.  $y = 6x^2 - 4x - 1$

29.  $y = -3x^2 - 5x + 3$

30.  $y = -2x^2 - 3x + 2$

31.  $y = x^2 + 6x + 5$

32.  $y = -4x^2 - 3x + 6$

★ 33.  $y = -\frac{1}{2}x^2 - 3x + 4$

34.  $y = \frac{1}{3}x^2 + 3x - 2$

★ 35.  $y = -2x^2 + \frac{1}{3}x - 1$

★ 36.  $y = 3x^2 - \frac{1}{2}x + 4$

**You've Got to Have the Right Angle** In Exercise 37, use the information given in Example 3.

37. Natalya Lisovskaya's winning throw in the shot put was at a  $45^\circ$  angle. If the shot had been thrown at a  $40^\circ$  angle or  $50^\circ$  angle, would it have gone farther? Explain. See margin.

Throw at  $40^\circ$  angle:  $y = -0.0125x^2 + 0.84x + 5$

Throw at  $50^\circ$  angle:  $y = -0.0177x^2 + 1.19x + 5$

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13.  $x = -\frac{1}{3}; (-\frac{1}{3}, \frac{11}{3})$

14.  $x = -\frac{3}{4}; (-\frac{3}{4}, \frac{39}{8})$

15.  $x = -\frac{1}{2}; (-\frac{1}{2}, 9)$

16.  $x = \frac{3}{2}; (\frac{3}{2}, -\frac{75}{4})$

17.  $x = -\frac{7}{4}; (-\frac{7}{4}, -\frac{217}{8})$

18.  $x = 2; (2, 20)$

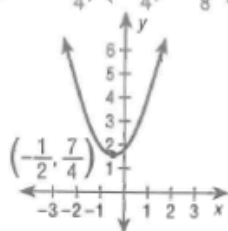
■ 9.3 Independent Practice pp. 468–469

7. Up  $(0, 4)$  9. Down,  $(0, 0)$  11. Down,  $(-\frac{1}{10}, \frac{201}{20})$

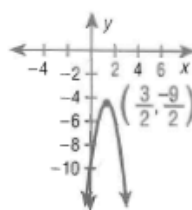
13.  $x = -\frac{1}{3}, (-\frac{1}{3}, \frac{11}{3})$  15.  $x = -\frac{1}{2}, (-\frac{1}{2}, 9)$

17.  $x = -\frac{7}{4}, (-\frac{7}{4}, -\frac{217}{8})$

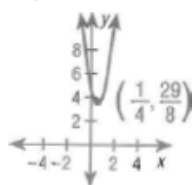
19.



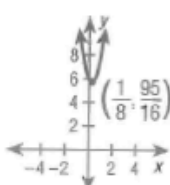
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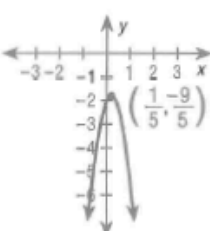
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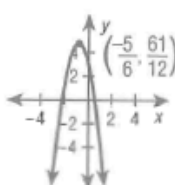
25.



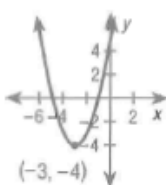
27.



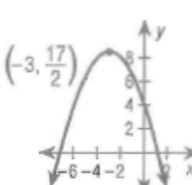
29.



31.



33.



# 9.4

## The Quadratic Formula



Knowing how it could change the lives of canines everywhere, the dog scientists struggled diligently to understand the Doorknob Principle.

## Guided Practice

### CRITICAL THINKING about the Lesson

1. **True or False?** The quadratic formula states that the solutions of the equation  $ax^2 + bx + c = 0$  are "the opposite of  $b$ , plus or minus the square root of  $b$  minus  $4ac$ , all divided by  $2a$ ." **False**
2. Describe the two models for vertical motion. **See top of page 474.**
3. State the values of  $a$ ,  $b$ , and  $c$  from the standard form of the equation  $5 = 6 + 9x - x^2$ .  **$a = -1$ ,  $b = 9$ ,  $c = 1$**
4. Solve  $x^2 + x - 2 = 0$ .  **$1, -2$**
5. Sketch the graph of  $y = x^2 + x - 2$  and label the  $x$ -intercepts. **See Additional Answers.**
6. Describe the relationship between the  $x$ -intercepts found in Exercise 5 and the solutions found in Exercise 4. **They are the same.**

## Independent Practice

In Exercises 7–10, write the quadratic equation in standard form.

7.  $-3x^2 + 5x = 9$   **$-3x^2 + 5x - 9 = 0$**

8.  $5 - 2x + x^2 = 0$   **$x^2 - 2x + 5 = 0$**

9.  $-4 + 3x + x^2 = 5$   **$x^2 + 3x - 9 = 0$**

10.  $9x - 7x^2 = 16$   **$-7x^2 + 9x - 16 = 0$**

In Exercises 11–14, find the value of  $b^2 - 4ac$  for the equation.

11.  $2x^2 - 3x - 1 = 0$  **17**

12.  $4x^2 + 4x + 1 = 0$  **0**

13.  $3x^2 - 2x - 5 = 0$  **64**

14.  $x^2 - 11x + 30 = 0$  **1**

18.  **$3 + \sqrt{2} \approx 4.41$ ,  $3 - \sqrt{2} \approx 1.59$**

$$18. 3 + \sqrt{2} \approx 4.41, 3 - \sqrt{2} \approx 1.59$$

In Exercises 15–20, use the quadratic formula to solve the equation.

$$15. 4x^2 - 13x + 3 = 0 \quad 3, \frac{1}{4}$$

$$16. 3y^2 + 11y + 10 = 0 \quad -\frac{5}{3}, -2$$

$$17. 2x^2 + 7x + 3 = 0 \quad -\frac{1}{2}, -3$$

$$18. x^2 - 6x + 7 = 0$$

$$19. 5y^2 + 2y - 2 = 0$$

$$20. 2x^2 + 4x - 3 = 0$$

19.–20. See below.

In Exercises 21–26, solve the quadratic equation by the most convenient method (finding square roots or the quadratic formula). Explain why you chose your method.

$$\sqrt{27} \approx 5.20, -\sqrt{27} \approx -5.20$$

$$21. \frac{-10 + \sqrt{70}}{6} \approx -0.27$$

$$\frac{-10 - \sqrt{70}}{6} \approx -3.06$$

$$21. 6x^2 + 20x + 5 = 0$$

$$22. t^2 = 27$$

$$23. x^2 - 625 = 0 \quad 25, -25$$

$$24. 4u^2 - 49 = 0 \quad \frac{7}{2}, -\frac{7}{2}$$

$$25. -2x^2 + 6x + 1 = 0$$

$$26. x^2 + 14x + 49 = 0 \quad -7$$

In Exercises 27–32, find the x-intercepts of the graph of the equation.

$$27. y = x^2 + 2x + 15 \quad \text{None}$$

$$28. y = x^2 - 6x - 7 \quad 7, -1$$

$$29. y = x^2 + x - 20 \quad 4, -5$$

$$30. y = x^2 + 8x + 12 \quad -2, -6$$

$$31. y = x^2 + x - \frac{3}{4} \quad \frac{1}{2}, -\frac{3}{2}$$

$$32. y = x^2 + \frac{7}{3}x - 2 \quad \frac{2}{3}, -3$$

$$25. \frac{3 + \sqrt{11}}{2} \approx 3.16, \frac{3 - \sqrt{11}}{2} \approx -0.16$$

*Isol what  
ask what  
mean*

9.4 • The Quadratic Formula 475

$$19. \frac{-1 + \sqrt{11}}{5} \approx 0.46; \frac{-1 - \sqrt{11}}{5} \approx -0.86 \quad 20. \frac{-2 + \sqrt{10}}{2} \approx 0.58; \frac{-2 - \sqrt{10}}{2} \approx -2.58$$



Math phobic's nightmare



## What is a Metaphor?

The **definition of a metaphor** is "a figure of speech containing an implied comparison, in which a word or phrase ordinarily and primarily used of one thing is applied to another (Ex.: the curtain of night, "all the world's a stage")."

1.)  $-1$  or  $-3$

2.)  $5$  or  $2$

3.)  $-2$  or  $-3$

4.)  $4$  or  $-1$

5.)  $2$  or  $-4$

6.)  $\frac{5 \pm \sqrt{17}}{2} \approx 4.56$  or  $0.44$

7.)  $\frac{-3 \pm \sqrt{37}}{2} \approx 1.54$  or  $-4.54$

8.)  $2$  or  $\frac{1}{2}$

9.)  $\frac{5}{2}$  or  $-1$

10.)  $\frac{-5 \pm \sqrt{13}}{6} \approx -0.23$  or  $-1.43$

11.)  $2$  or  $-\frac{4}{3}$

# 10.7

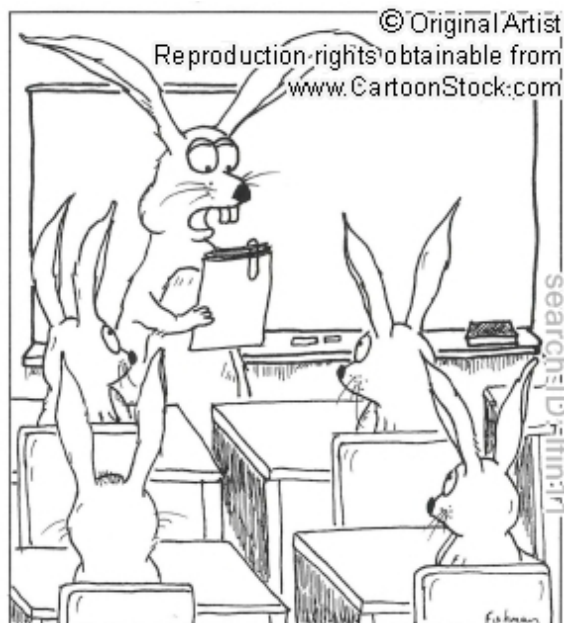
## Solving Quadratic Equations by Completing the Square

off the mark .com by Mark Parisi



# 10.7

## Solving Quadratic Equations by Completing the Square



"Well, I've finished grading your exams,  
and there was a lot of improvement in  
addition and subtraction. Of course,  
you're all still multiplying like rabbits."

## Guided Practice

### CRITICAL THINKING about the Lesson

$$2 + \sqrt{12}, 2 - \sqrt{12}; \text{no difference}$$

- Which is a perfect square trinomial?  
a.  $x^2 - 8x + 8$     b.  $x^2 - 8x + 16$   
c.  $x^2 - 8x + 64$
- Solve  $x^2 - 4x = 8$  by completing the square. Solve the same equation by the quadratic formula. Explain the difference in the results.
- What term must be added to  $x^2 + 6x$  to create a perfect square trinomial? **9**
- Name the five methods for solving a quadratic equation. **See chart on page 550.**

## Independent Practice

In Exercises 5–10, find the term that must be added to the expression to create a perfect square trinomial.

5.  $x^2 - 18x$  **81**

6.  $x^2 + 6x$  **9**

7.  $x^2 + 12x$  **36**

8.  $x^2 - 10x$  **25**

9.  $x^2 - 7x$   **$\frac{49}{4}$**

10.  $x^2 - 5x$   **$\frac{25}{4}$**

In Exercises 11–28, solve the equation by completing the square. **16.**  $-\frac{1}{2}, -\frac{17}{2}$     **25.**  $-\frac{1}{3}, -1$     **21, 3**

11.  $x^2 + 10x - 11 = 0$  **1, -11**

12.  $x^2 + 14x - 15 = 0$  **1, -15**

13.  $y^2 - 24y + 63 = 0$

14.  $y^2 - 8y + 12 = 0$  **6, 2**

15.  $t^2 + 3t - \frac{7}{4} = 0$   **$\frac{1}{2}, -\frac{7}{2}$**

16.  $y^2 + 9y + \frac{17}{4} = 0$

17.  $x^2 - \frac{2}{3}x - 3 = 0$

18.  $x^2 + \frac{4}{5}x - 1 = 0$

19.  $x^2 + x - 1 = 0$

20.  $1 + x - x^2 = 0$

21.  $4y^2 + 4y - 9 = 0$

22.  $3x^2 - 24x - 5 = 0$

23.  $2x^2 - 6x - 15 = 5$  **5, -2**

24.  $5x^2 - 20x - 20 = 5$  **5, -1**

25.  $3x^2 + 4x + 4 = 3$

26.  $4x^2 + 6x - 6 = 2$

27.  $x^2 + 2x = 2$

28.  $x^2 - 2x = 2$

**17–22., 26., 27. See margin.**

**$1 + \sqrt{3}, 1 - \sqrt{3}$**

### Answers

$$17. \frac{1}{3} + \frac{\sqrt{28}}{3}, \frac{1}{3} - \frac{\sqrt{28}}{3}$$

$$18. -\frac{2}{5} + \frac{\sqrt{29}}{5}, -\frac{2}{5} - \frac{\sqrt{29}}{5}$$

$$19. -\frac{1}{2} + \frac{\sqrt{5}}{2}, -\frac{1}{2} - \frac{\sqrt{5}}{2}$$

$$20. \frac{1}{2} + \frac{\sqrt{5}}{2}, \frac{1}{2} - \frac{\sqrt{5}}{2}$$

$$21. -\frac{1}{2} + \frac{\sqrt{10}}{2}, -\frac{1}{2} - \frac{\sqrt{10}}{2}$$

$$22. 4 + \frac{\sqrt{159}}{3}, 4 - \frac{\sqrt{159}}{3}$$

$$26. -\frac{3}{4} + \frac{\sqrt{41}}{4}, -\frac{3}{4} - \frac{\sqrt{41}}{4}$$

$$27. -1 + \sqrt{3}, -1 - \sqrt{3}$$

$$29. \frac{3}{2} + \frac{\sqrt{13}}{2}, \frac{3}{2} - \frac{\sqrt{13}}{2}$$

$$34. -\frac{5}{2} + \frac{\sqrt{17}}{2}, -\frac{5}{2} - \frac{\sqrt{17}}{2}$$

$$36. -1 + \sqrt{27}, -1 - \sqrt{27}$$

$$37. -\frac{5}{9} + \frac{\sqrt{244}}{18}, -\frac{5}{9} - \frac{\sqrt{244}}{18}$$

In Exercises 29–43, use the most convenient method to solve the equation. Explain why you made your choice. 29., 34., 36., 37. See margin.

29.  $x^2 - 3x - 1 = 0$

32.  $4x^2 - 25 = 0$   $\frac{5}{2}, -\frac{5}{2}$

35.  $3x^2 - 5x = 0$   $0, \frac{5}{3}$

38.  $4x^2 + 4x + 1 = 0$   $-\frac{1}{2}$

41.  $8x^2 - 10x + 3 = 0$   $\frac{3}{4}, \frac{1}{2}$

30.  $4x^2 - 12 = 0$   $\sqrt{3}, -\sqrt{3}$

33.  $x^2 + 7x + 10 = 0$   $-5, -2$

36.  $y^2 + 2y - 26 = 0$

39.  $7x^2 - 14x = 0$   $0, 2$

42.  $7x^2 - 14 = 0$   $\sqrt{2}, -\sqrt{2}$

$-3 + \frac{\sqrt{132}}{2}, -3 - \frac{\sqrt{132}}{2}$

31.  $y^2 + 6y - 24 = 0$

34.  $u^2 + 5u + 2 = 0$

37.  $9z^2 + 10z - 4 = 0$

40.  $4x^2 - 13x + 3 = 0$

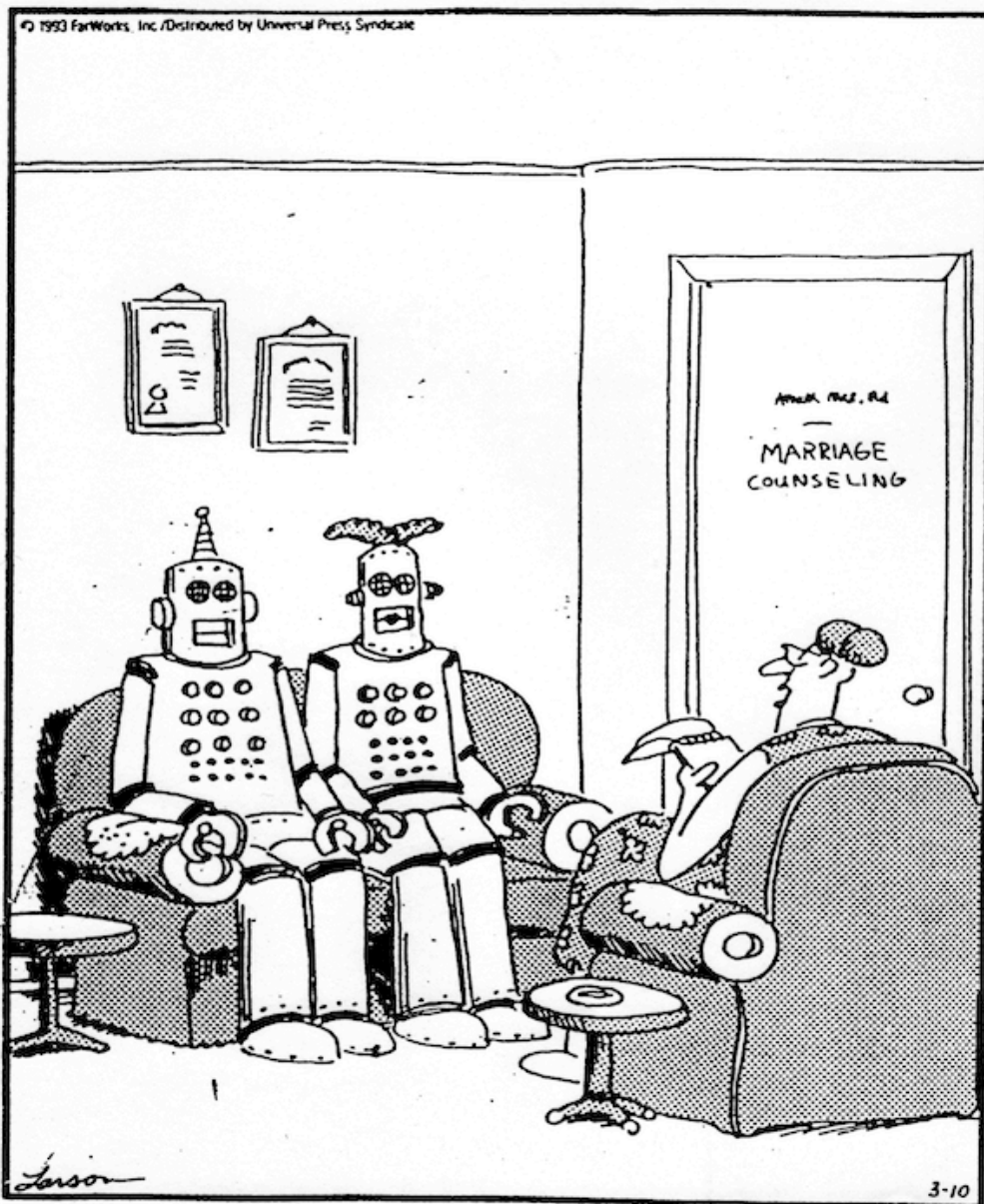
43.  $y^2 + 20y + 10 = 0$   
 $-10 + \frac{\sqrt{360}}{2}, -10 - \frac{\sqrt{360}}{2}$

40.  $3, \frac{1}{4}$

44. **Money in the Bank** At your seventh grade graduation, you and your twin sister each received \$200. You each deposited the money in savings accounts that compound interest annually. Two years later your sister's deposit has grown by \$28.98. Your account is in a different bank that pays an interest rate that is 1% more than your sister receives. What is your balance after two years? **\$233.28**

# THE FAR SIDE

By Gary Larson



"The problem, as I see it, is that you both are extremely adept at pushing each other's buttons."

