

"If you tell the truth, you don't have to remember anything." - Mark Twain

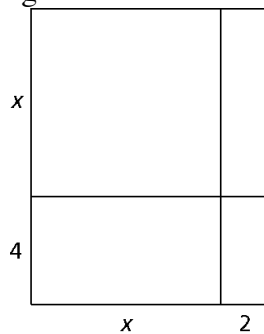
## Algebra Final Review D

**Short Answer Show all thinking for maximum credit. Each question is worth 10 points.**

1. One of the equations below represents the height,  $h$ , in feet of a thrown baseball as it changes over time,  $t$ , in seconds. The ball starts about shoulder height, rises to a maximum, and then falls to the ground.

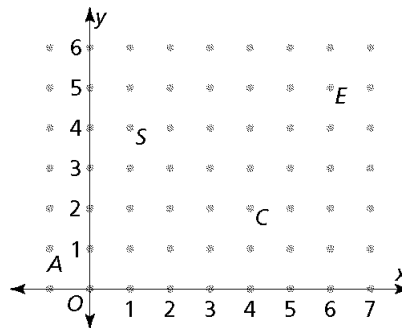
$$h = 16t^2 + 40t + 4 \quad h = -16t^2 + 40t + 4 \quad h = 16t \quad h = 16^2 + 4$$

- Which of these equations is a reasonable model for the given situation? Explain your choice.
  - How is "shoulder height" represented in the correct equation?
  - What maximum height will the ball reach? Explain how you found your solution.
2. Write an equation for the area of this rectangle in factored form and expanded form.



3. The number sequence 2, 6, 12, 20, 30, ..., follows a pattern.
- Describe the pattern of change between the numbers in the sequence.
  - Use the pattern you have described to predict the next three numbers in the sequence.
  - Write an equation for calculating the  $n$ th number in the sequence.
  - Is 117 in the sequence? If it is in the sequence, explain which number in the sequence it is. If it is not in the sequence, explain why not.
  - Is 10,100 in the sequence? If it is in the sequence, explain which number in the sequence it is. If it is not in the sequence, explain why not.
4.
  - Suppose you had 40 meters of fencing. What is the greatest rectangular area that you can enclose?
  - Suppose you enclose the area from part a and then increase each side by 1 meter. What will the new dimensions of the rectangle be? What will the new area be?
  - In part b, you added 4 meters to the perimeter of the rectangle from part a. How much did this increase the area?
  - Suppose you did parts a–c for several other lengths of fencing. Would you always get the same increase in area? Show work to support your answer.

5. Draw line segments to connect points  $A$ ,  $C$ ,  $E$ , and  $S$ , in that order.



a. What shape is figure  $ACES$ ?

b. Without using a ruler, find the lengths of line segments  $AS$ ,  $SE$ ,  $CE$ , and  $AC$ .

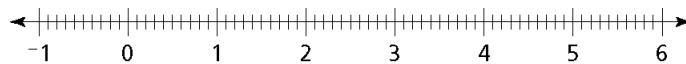
$AS$  \_\_\_\_\_  $SE$  \_\_\_\_\_  $CE$  \_\_\_\_\_  $AC$  \_\_\_\_\_

c. How do the lengths of the sides compare?

d. What is the area of the figure?

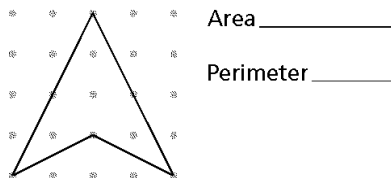
6. Arrange the following numbers on a number line.

$$\sqrt{3}, \frac{15}{7}, \sqrt{17}, \sqrt{36}, \sqrt{5}, 1.5, \sqrt{11}$$

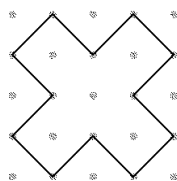


**Find the perimeter and area of the figure.**

7.



8.



Area \_\_\_\_\_

Perimeter \_\_\_\_\_

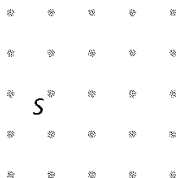
9. **a.** On the dot grid below, draw and label a line segment with length  $\sqrt{2}$ .



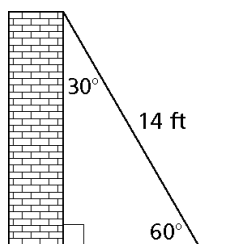
- b.** Draw and label a line segment with length  $\sqrt{4}$ .

- c.** Which is greater,  $\sqrt{2} + \sqrt{2}$  or  $\sqrt{4}$ ? Explain how you know.

10. Label the grid dot below with the letter  $T$  so that the length of  $ST$  is  $\sqrt{10}$ .

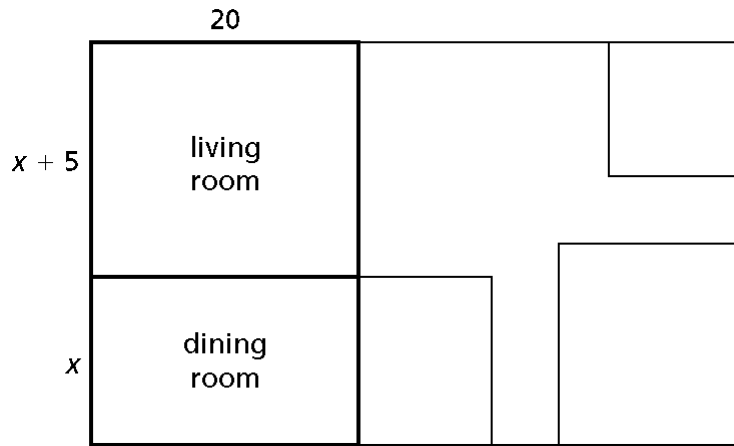


11. A 14-foot piece of wire is strung between a building and the ground, making a 30-60-90 triangle as shown.



- a.** How far straight out from the base of the building is the wire attached to the ground?
- b.** How far up the side of the building is the wire attached?

12. The Morales family is remodeling their home. The wall between the living room and dining room is going to be removed to make one big living space. Write two equivalent expressions for the area of the new living space.



13. Three of the following expressions are equivalent. Choose the expression that is not equivalent to the others and explain how you can tell, without using a calculator, that it is not equivalent.

A.  $8x - 12x + 4$

B.  $12x - 16x + 4$

C.  $4 - 4x$

D.  $4(1 - 4x)$

14. Write an expression equivalent to each of these expressions.

**a.**  $(15 - 4x) - (10 - x)$

**b.**  $3(10 + x) - (30 + 3x)$

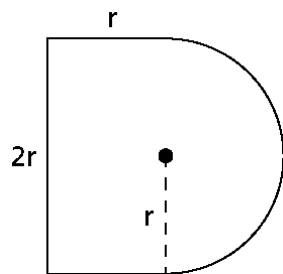
15. Explain how you can tell, without using a calculator, that these expressions are not equivalent.

$5 - 4x^2$

$4x^2 - 5$

$4x(x - 5)$

16. **a.** Choose the expression(s) that represent the perimeter of the shape below.



A.  $2r + r + 2\pi r + r$

B.  $4r + \pi r$

C.  $2r + \pi r$

D.  $r(2 + \pi)$

E.  $r(4 + \pi)$

**b.** Explain why the expression(s) you chose are correct.

17. Solve the following equations. Show your work.

**a.**  $(x + 4)(x - 6) = 0$

**b.**  $3(x + 10) + 5(x + 2) = 0$

**c.**  $x^2 - x - 20 = 0$

**d.**  $-2(7x + 15) = 18 + 2x$

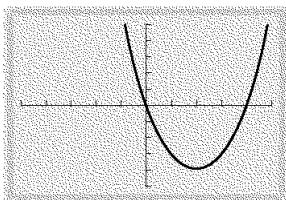
Match each equation with two other equivalent representations.

A.  $y = x(3 - x)$

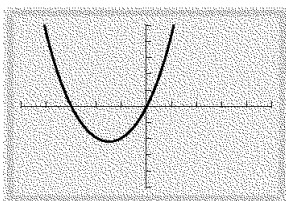
B.  $y = x^2 + 2x - 8$

C.  $y = x^2 + 3x$

D.



E.



F.

$x$	$y$
-1	-4
0	0
1	2
2	2
3	0
4	-4
5	-10

G.

$x$	$y$
-4	0
-3	-5
-2	-8
-1	-9
0	-8
1	-5
2	0

H.

$x$	$y$
-2	12
-1	5
0	0
1	-3
2	-4
3	-3
4	0

18.  $y = x(x + 3)$
19.  $y = 3x - x^2$
20.  $y = x^2 - 4x$
21.  $y = (x - 2)(x + 4)$
22. Write an equation for a line that is parallel to side  $AD$  and passes through the point  $(0, 1)$ .

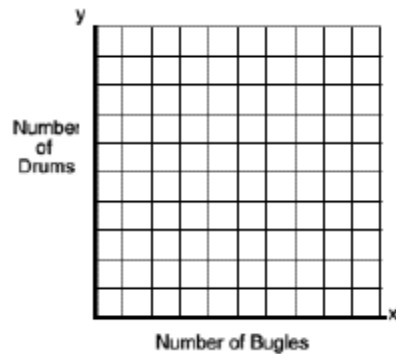
José is batboy for the *Lansing Lugnuts* baseball team during the summer school vacation. His pay includes two season tickets worth a total of \$80 and he also gets paid \$5 per hour of practice or game time that he works. Charlie works at the concession stand for most *Lugnut* games, earning \$6 per hour plus a \$50 bonus if he works at least 20 games.

23. What equation shows how José's summer pay depends on the number of hours he works?
24. Write and solve an inequality that answers the question, "For how many hours of work will José's total summer pay be less than \$260?"
25. Suppose that Charlie does plan to work at least 20 games. What equation shows how his total summer earnings are related to the number of hours he works?
26. Write and solve an inequality to find the number of hours worked for which Charlie will earn more in total summer pay than José.

The *Plano Texans* are a youth drum and bugle corps that competes with music and precision marching against other groups all over the country. The corps rents instruments to members. Each bugle rents for \$10 per month and each drum rents for \$5 per month.

27. What is the corps monthly income from instrument rentals if members rent:
- 7 bugles and 9 drums?
  - 9 bugles and 7 drums?

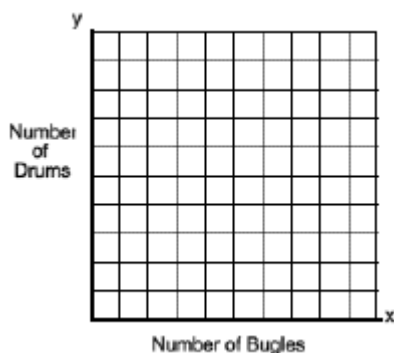
28. **a.** What equation relates the number of bugle rentals  $x$  and the number of drum rentals  $y$  to the business manager's goal of \$100 in monthly rental income?
- b.** Draw a graph showing solutions of the rental income equation you found in part (a) on the following grid and give coordinates of 3 points that represent solutions.



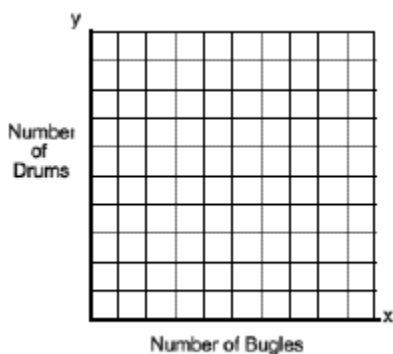
29. Suppose that there are 12 members of the drum and bugle corps who rent an instrument.
- a.** What equation relating  $x$  and  $y$  expresses this fact?
- b.** Give two solutions to the equation in (a).
- c.** Write and solve a system of equations that finds numbers of bugle rentals  $x$  and drum rentals  $y$  that supply 12 members and rental income of \$100.



30. Suppose there are at most 18 members of the drum and bugle corps who rent an instrument.
- Write an inequality relating  $x$  and  $y$  that express this fact, keeping in mind that in this context,  $x \geq 0$  and  $y \geq 0$ .
  - Draw a graph illustrating the solutions to the inequality you wrote in part a.



- Suppose that the group would like a rental income of at least \$100, but they will rent at most 18 instrument rentals. Write a system of inequalities to express these relationships.
- Draw a graph exhibiting the possible numbers of drum and bugle rentals in part (c).



- Use your graph from part d to list two possibilities for numbers of drum and bugle rentals which satisfy the conditions of part c.
31. Solve the following system of linear equations. Name the strategy that you use and show all of your work.
- $$\begin{cases} 2x + y = 7 \\ x + 3y = 11 \end{cases}$$
32. Draw a graph illustrating solutions to the following system of linear inequalities. Use the graph to find one possible solution to the system.
- $$\begin{cases} 2x + y < 7 \\ x + 3y < 11 \end{cases}$$

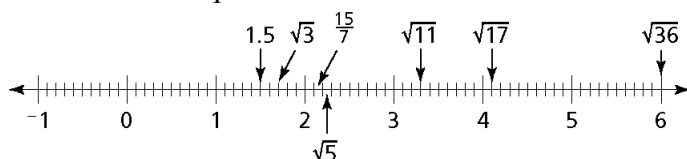
## Algebra Final Review D

### Answer Section

#### SHORT ANSWER

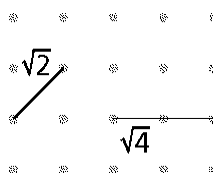
1.
  - a. The first two equations represent quadratic relationships, but only  $h = -16t^2 + 40t + 4$  has a maximum so it is the correct equation. The equation  $h = 16t^2 + 40t + 4$  is quadratic but has a minimum (which occurs when  $t$  is a negative number). The third equation is linear, and the fourth equation is exponential.
  - b. The shoulder height is represented by the “+ 4.” This constant term shows how high off the ground the baseball is when it is released. This height can be found by substituting the value of 0 into the equation, which represents time = 0.
  - c. The maximum height is 29 ft. Students may find this in several ways. They could graph the function on a calculator and trace the graph. They could look at a table of values to find the greatest value for  $h$ . They could look at the table or the graph to find where  $h = 4$  again (at  $t = 2.5$ ), then use what they know about symmetry of parabolas to reason that the maximum would occur when  $t = 1.25$ , and substitute this value into the equation.
2. Factored form:  $A = (x + 2)(x + 4)$ ; expanded form:  $A = x^2 + 2x + 4x + 8$ , or  
 $A = x^2 + 6x + 8$
3.
  - a. The difference between numbers is 4, 6, 8, 10, which is an increase of 2 from one number to the next.
  - b. 42, 56, 72
  - c.  $c = n(n + 1)$ , where  $c$  is the  $n$ th number in the sequence; The first four numbers predicted by this equation are  $1(2) = 2$ ,  $2(3) = 6$ ,  $3(4) = 12$ , and  $4(5) = 20$ .
  - d. No, 117 is not in the sequence because it is not a product of consecutive integers.
  - e. Yes, 10,100 is in the sequence because  $100(100 + 1) = 100 \times 101 + 10,100$ . It is the 100<sup>th</sup> number in the sequence.
4.
  - a. The greatest rectangular area that can be enclosed is a square 10 m by 10 m with an area of  $100 \text{ m}^2$ .
  - b. By increasing each side by 1 m, the new square would measure 11 m by 11 m and have an area of  $121 \text{ m}^2$ .
  - c. This 4-meter increase in the perimeter increased the area by  $21 \text{ m}^2$ .
  - d. Students should demonstrate that a constant increase in the perimeter, achieved by adding 1 meter to each side, does not yield a constant increase in the area.

5. **a.** Figure *ACES* is a parallelogram.  
**b.** The lengths are as follows:  $AS$ ,  $\sqrt{13}$ ;  $SE$ ,  $\sqrt{26}$ ;  $CE$ ,  $\sqrt{13}$ ;  $AC$ ,  $\sqrt{26}$ .  
**c.** Opposite sides are the same length.  
**d.** 13 square units



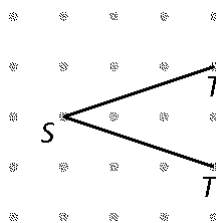
6.  
 7. area = 6 square units, perimeter =  $2\sqrt{20} + 2\sqrt{5} \approx 13.4$   
 8. area = 10 square units, perimeter =  $12\sqrt{2} \approx 17$

9. **a** and **b**.



- c.**  $\sqrt{2}$  must be greater than 1 but less than 2, as  $1^2 = 1$  and  $2^2 = 4$ . So,  $\sqrt{2} + \sqrt{2}$  must be greater than 2 and therefore greater than  $\sqrt{4}$ .

10. There are two possible positions (labeled  $T$  and  $T'$ ):



11. **a.** Since this is a 30-60-90 triangle, the length of the shorter leg is half the length of the hypotenuse. The wire will be attached to the ground 7 ft from the building.  
**b.** Using the Pythagorean Theorem, since  $14^2 - 7^2 = 147$ , the wire will be attached  $\sqrt{147} \approx 12.12$  ft up the side of the building.  
 12. The area, if you see this as two parts, is  $20x + 20(x + 5)$  or, as one room,  $20(x + x + 5) = 20(2x + 5)$  or  $40x + 100$   
 13. D. The 4 was not distributed to the negative  $4x$  inside the parentheses. A, B, C are all equivalent to the expression  $-4x + 4$  whereas D is equivalent to the expression  $-16x + 4$ .

From tables of values, it can be seen that D is not equivalent to the others. Since these are all linear expressions students may plug in a point (or possibly two points) in order to find the expression, which is not equivalent to the others.

14. Answers may vary. Some possibilities are:

**a.**  $(15 - 4x) - (10 - x) = 15 - 4x - 10 + x = 5 - 3x$

**b.**  $3(10 + x) - (30 + 3x) = 30 + 3x - 30 - 3x = 3(10 + x - 10 - x) = 0$

15.  $y = 5 - 4x^2$

$$y = 4x^2 - 5$$

$$y = 4x(x - 5)$$

The first two expressions are not equivalent; the “5” is positive in the first and negative in the second. In fact all the signs of the terms in the second are opposite from those in the first. If the third expression is expanded, using the distributive property, it would be  $4x^2 - 20x$ , which is not equivalent to the other two expressions.

16. **a.** B.  $4r + \pi r$  or E.  $r(4 + \pi)$

**b.** Explanation: The perimeter of a circle is  $2\pi r$ ; so the perimeter of the semicircular part is  $\pi r$ . The three sides of the rectangle form the rest of the perimeter. These are  $2r$ ,  $r$  and  $r$ . The total is, therefore,  $2r + r + r + \pi r$  or  $4r + \pi r$  (B) and using the distributive property we get E:  $r(4 + \pi)$ .

17. **a.**  $(x + 4)(x - 6) = 0$

$$x + 4 = 0 \quad \text{and} \quad x - 6 = 0$$

$$x = -4 \quad \text{and} \quad x = 6$$

**b.**  $3(x + 10) + 5(x + 2) = 0$

$$3x + 30 + 5x + 10 = 0$$

$$8x + 40 = 0$$

$$8x = -40$$

$$x = -5$$

**c.**  $x^2 - x - 20 = 0$

$$(x + 4)(x - 5) = 0$$

$$x + 4 = 0 \quad \text{and} \quad x - 5 = 0$$

$$x = -4 \quad \text{and} \quad x = 5$$

**d.**  $-2(7x + 15) = 18 + 2x$

$$-14x - 30 = 18 + 2x$$

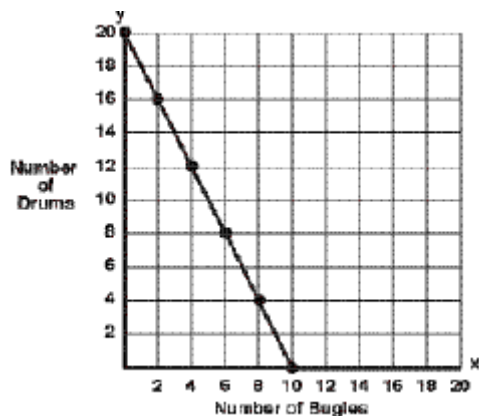
$$-16x - 30 = 18$$

$$-16x = 48$$

$$x = -3$$

18. C and E

19. A and F
20. D and H
21. B and G
22.  $y = \frac{1}{2}x + 1$
23.  $\text{Pay} = 5h + 80$  where  $h$  is the number of hours.
24.  $260 > 5h + 80$  and  $h < 36$  (less than 36 hours of work); To solve  $260 > 5h + 80$ , subtract 80 from both sides to obtain  $180 > 5h$  and then divide both sides of this inequality by 5 to obtain  $36 > h$ .
25.  $\text{Pay} = 6h + 50$
26.  $6h + 50 > 5h + 80$  has a solution of  $h > 30$  so Charlie needs to work more than 30 hours; To solve  $6h + 50 > 5h + 80$ :
- $$6h + 50 > 5h + 80$$
- $$6h + 50 - 5h > 5h + 80 - 5h$$
- $$h + 50 > 80$$
- $$h + 50 - 50 > 80 - 50$$
- $$h > 30$$
27. a. \$115 per month;  $(7 \times 10) + (9 \times 5) = 115$ .
- b. \$125 per month;  $(9 \times 10) + (7 \times 5) = 125$ .
28. a.  $10x + 5y = 100$
- b. Possible answers: (10, 0), (0, 20), (6, 8), (2, 16), (4, 12), (8, 4).



29. a.  $x + y = 12$

b. Possible answers: (0, 12), (12, 0), (5, 7), (8, 4)

c. (8, 4); Students may use a variety of methods to solve the system:  $\begin{cases} x + y = 12 \\ 10x + 5y = 100 \end{cases}$ ;

One way to solve this system is to solve both equations for  $y$  to get the equivalent system:

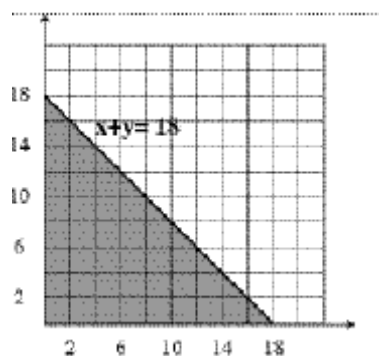
$$\begin{cases} y = -x + 12 \\ y = -2x + 20 \end{cases} \text{ . Then solve the equation } -x + 12 = -2x + 20 \text{ for } x \text{ to obtain}$$

$x = 8$ . To find  $y$  substitute this value for  $x$  into either equation to get that

$$y = -8 + 12 = 4.$$

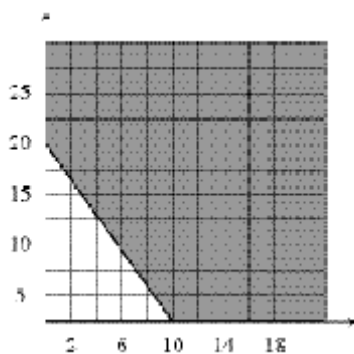
30. a.  $x + y \leq 18$

b.



c.  $\begin{cases} x + y \leq 18 \\ 10x + 5y \geq 100 \end{cases}$

d.



e. Answers will vary. Possible answers include (10, 8) and (9, 7).

31. Students may choose their solution methods. Possible solutions by substitution and by linear combination are shown below. In either case, the solution is (2,3).

**Solution by Substitution**

$$\begin{cases} 2x + y = 7 \\ x + 3y = 11 \end{cases} \text{ is equivalent to:}$$

$$\begin{cases} y = 7 - 2x \\ x + 3y = 11 \end{cases} \text{ . Substituting, we get:}$$

$$x + 3(7 - 2x) = 11. \text{ Solving for } x, \text{ we get:}$$

$$x + 21 - 6x = 11$$

$$-5x = -10$$

$$x = -2$$

Then we can find  $y = 3$

**Solution by Linear Combination**

$$\begin{cases} 2x + y = 7 \\ x + 3y = 11 \end{cases} \text{ is equivalent to:}$$

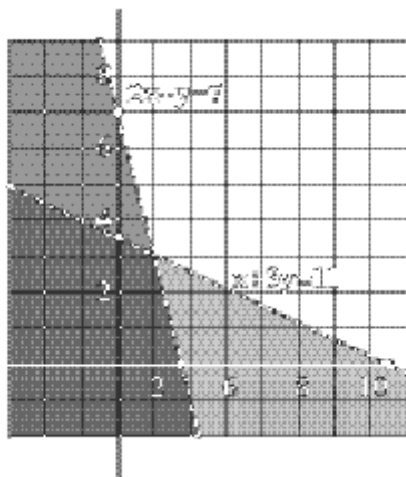
$$\begin{cases} 2x + y = 7 \\ 2x + 6y = 22 \end{cases} \text{ . Subtracting the top equation from the bottom gives us:}$$

$$5y = 15$$

$$y = 3$$

Then we can find  $x = -2$ .

32. 
$$\begin{cases} 2x + y < 7 \\ x + 3y < 11 \end{cases}$$



One possible solution is  $(0, 0)$ ; answers will vary.