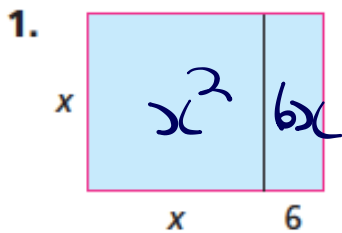


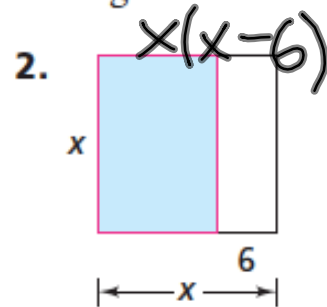
Problem 2.2 Quadratic Expressions

- A. Each diagram shows a large rectangle divided into two smaller rectangles. Write two expressions, one in factored form and one in expanded form, for the area of the rectangle outlined in red.



$$x(x+6) \text{ F.F.}$$

$$x^2 + 6x \text{ E.F.}$$

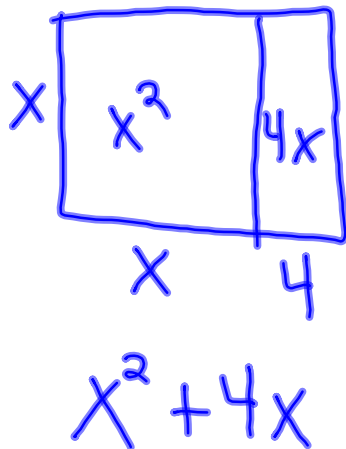


$$x^2 - 6x$$

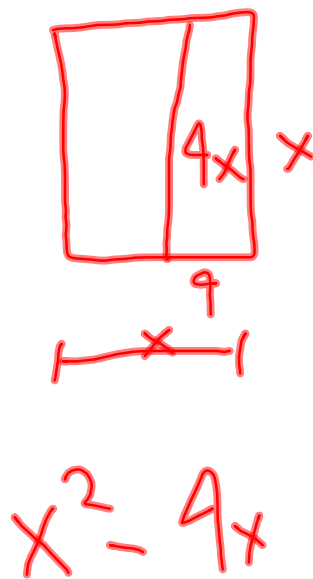
B. Complete the steps in bullets for each of the factored expressions in parts (1)–(3).

- Draw a divided rectangle whose area is represented by the expression. Label the lengths and area of each section.
- Write an equivalent expression in expanded form.

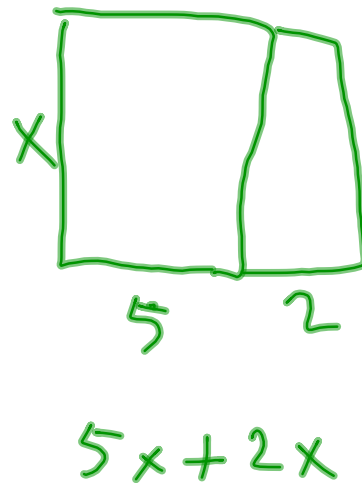
1. $x(x + 4)$



2. $x(x - 4)$



3. $x(5 + 2)$



C. Complete the steps in bullets for each of the factored expressions in parts (1)–(3).

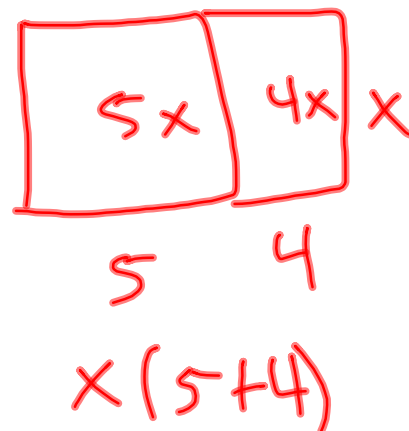
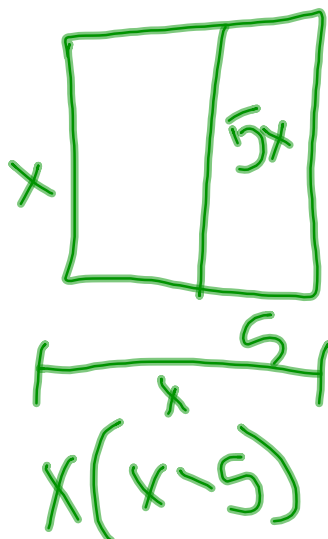
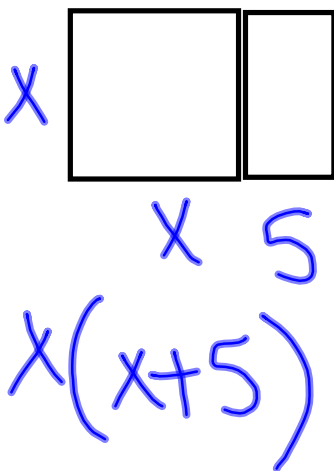
- Draw a divided rectangle whose area is represented by the expression. Label the lengths and area of each section.
- Tell what clues in the expanded expression helped you draw the divided rectangle.
- Write an equivalent expression in factored form.

1. $x^2 + 5x$

2. $x^2 - 5x$

3. $5x + 4x$

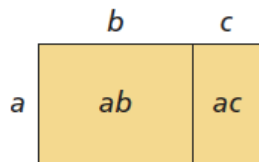
ACE Homework starts on page 30.



The equation $x(x + 3) = x^2 + 3x$ is an example of the **Distributive Property**, which you studied in earlier units.

The Distributive Property says that, for any three numbers a , b , and c ,

$$a(b + c) = ab + bc.$$



Area:
 $a(b + c)$ or $ab + ac$

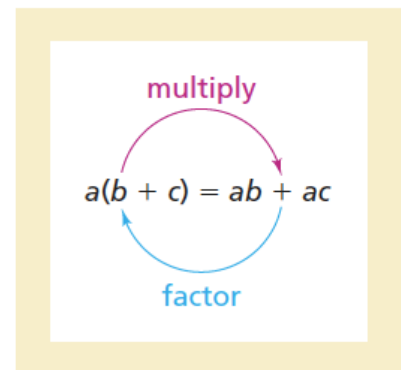
$$a(b+c) = ab+ac$$

$$a(b-c) = ab-ac$$

When you write $a(b + c)$ as $ab + ac$, you are *multiplying*, or writing the expression in expanded form. When you write $ab + ac$ as $a(b + c)$, you are *factoring*, or writing the expression in factored form.

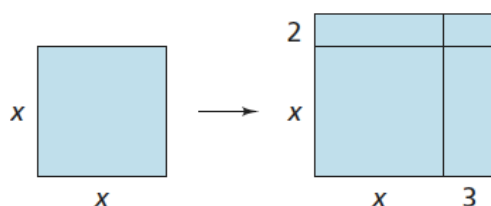
The terms $2x$ and $3x$ are *like terms*. The Distributive Property can be used to add like terms. For example, $2x + 3x = (2 + 3)x = 5x$.

In Problem 2.3, you will explore what happens to the area of a square when both dimensions are changed. You will see how the Distributive Property can be used to change the expression for area from factored form to expanded form.



Getting Ready for Problem 2.3

A new rectangle is made by increasing one side of a square with sides of length x by 2 centimeters and increasing the other side by 3 centimeters



$$(x+2)(x+3)$$

$$x^2 + 3x + 2x + 6$$

$$x^2 + 5x + 6$$

- How do the areas of the square and the new rectangle compare?
- How can you represent the area of the new rectangle?

First
Outer
Inner
Last

$$(x+5)(x-2)$$

$$x^2 - 2x + 5x - 10$$



On March 22 of this year, the first women's collegiate basketball game was played at Smith College in Northampton, Massachusetts. Sandra Berenson, "the Mother of Women's Basketball," supervised the game.

- My hundreds and tens digits are consecutive integers with a sum of 17 and a product of 72.
- My units digit is equal to $|\sqrt{9}|$.
- The sum of all of my digits is equal to the value of x in this equation: $\sqrt{36} - x = -15$.

What year am I?

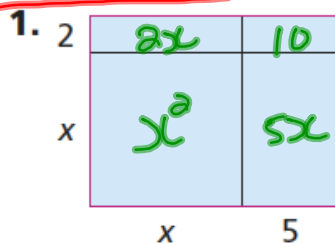
<u>1</u>	<u>8</u>	<u>9</u>	<u>3</u>
Thousands	Hundreds	Tens	Units



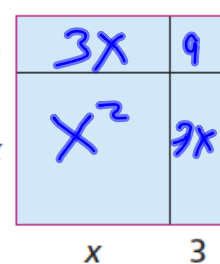
Napoleon Dynamite gave Chuck Norris some of his tots.

Problem 2.3 The Distributive Property

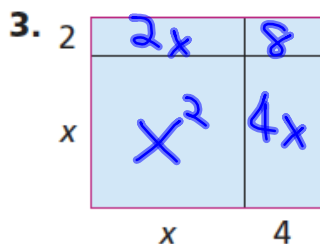
A. Each rectangle has been subdivided into four smaller rectangles. Write two expressions for the area of the rectangle outlined in red, one in factored form and one in expanded form.



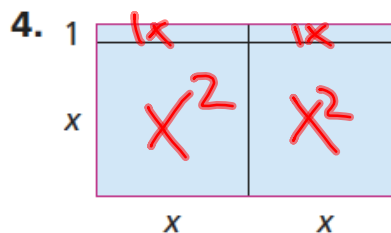
$(x+2)(x+5)$
 $x^2 + 5x + 2x + 10$
 $= x^2 + 7x + 10$



$(x+3)^2 =$
 $(x+3)(x+3)$
 $x^2 + 6x + 9$

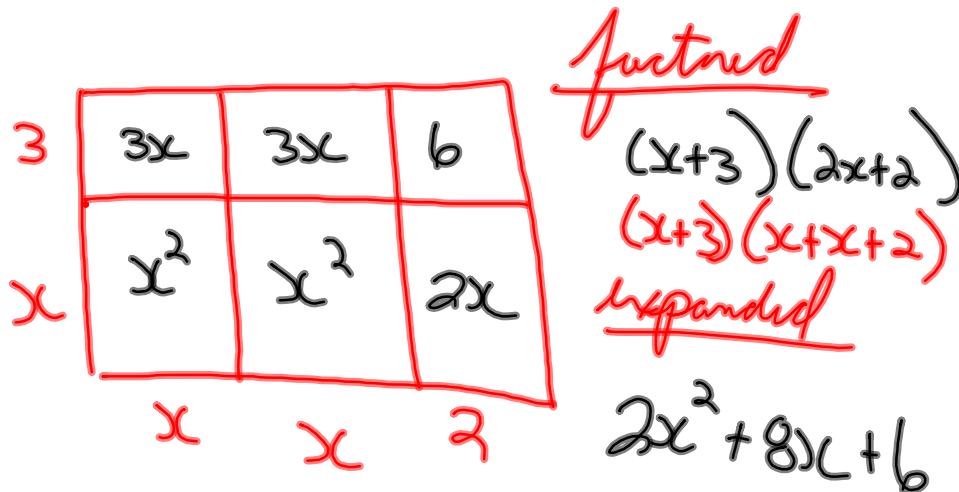


$(x+4)(x+2)$
 $x^2 + 6x + 8$



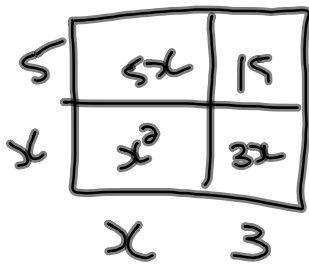
$(x+1)(2x)$
 $F = [(x+1)(x+1)]$
 $E = x^2 + 1 + x^2 + 1$
 $2x^2 + 2x$

- B.** A square has sides of length x centimeters. One dimension is doubled and then increased by 2 centimeters. The other dimension is increased by 3 centimeters.
1. Make a sketch of the new rectangle. Label the area of each section.
 2. Write two expressions, one in factored form and one in expanded form, for the area of the new rectangle.



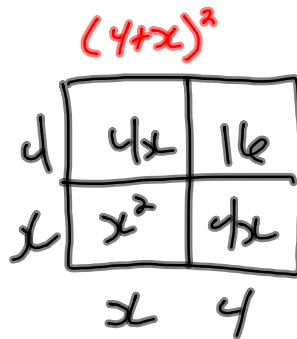
C. Use a rectangle model to help write each expression in expanded form.

1. $(x + 3)(x + 5)$



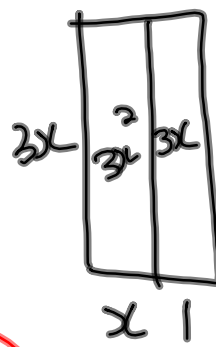
$x^2 + 8x + 15$

2. $(4 + x)(4 + x) =$



$x^2 + 8x + 16$

3. $3x(x + 1)$



$3x^2 + 3x$

$3x(x)$
 $3xx = 3x^2$

*

$$a(b+c) = ab+ac$$

- D. Carminda says she doesn't need a rectangle model to multiply $(x + 3)$ by $(x + 2)$. She uses the Distributive Property.

$$\begin{aligned} (x+3)(x+2) &= (x+3)x + (x+3)2 & (1) \\ &= x^2 + 3x + 2x + 6 & (2) \\ &= x^2 + 5x + 6 & (3) \end{aligned}$$

1. Is Carminda correct? Explain what she did at each step.
2. Show how using the Distributive Property to multiply $(x + 3)$ and $(x + 5)$ is the same as using a rectangle model.

$$\begin{array}{c|cc} & 3 & 5 \\ \hline x & 3x & 5x \\ & x^2 & 5x \end{array}$$

24 Frogs, Fleas, and Painted Cubes

$$(x+3)x + (x+3)5 = x^2 + 8x + 15$$



E. Use the Distributive Property to write each expression in expanded form.

1. $(x+5)(x+5)$
 $x^2 + 10x + 25$

2. $(x+3)(x-4)$
 $x^2 - x - 12$

3. $2x(5-x)$
 $-2x^2 + 10x$

4. $(x-3)(x-4)$
 $x^2 - 7x + 12$

5. $(x+2)(x-2)$
 $x^2 - 4$

ACE Homework starts on page 30.

Distributive Property

$(x-3)(x+5)$
 $x(x+5) - 3(x+5)$
 $x^2 + 5x - 3x - 15$
 $x^2 + 2x - 15$

Area Model

$(x-3)(x+3)$

-3	-3x	-9
x	x ²	3x

$x + 3$
 $x^2 - 3x + 3x - 9$
 $x^2 - 9$

Vertical Method

$x-3$
 $\times x+3$
 $\hline 3x-9$
 $+ x^2-3x$
 $\hline x^2+0x-9$
 x^2-9

F.O.I.L (CLAW)

$(x-2)(x-6)$
 $x^2 - 6x - 2x + 12$
 $x^2 - 8x + 12$

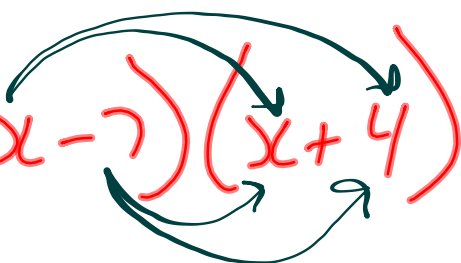
(27.) $(x-3)(x+3)$

$x(x+3) - 3(x+3)$

$x^2 + 3x - 3x - 9$

$x^2 + 0x - 9$

$x^2 - 9$

$$(x-7)(x+4)$$


Vertical

$$\begin{array}{r} x-7 \\ \times x+4 \\ \hline \end{array}$$

$$\begin{array}{r} 4x-28 \\ + x^2-7x \\ \hline x^2-3x-28 \end{array}$$

$+4$	$4x$	-28
x	x^2	$-7x$

$x \quad -7$

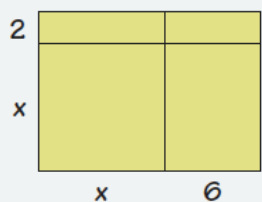
2.4

Factoring Quadratic Expressions

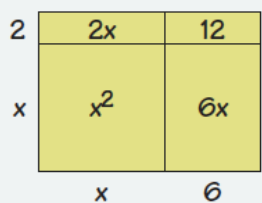
You know two ways to change a factored expression, such as $(x + 2)(x + 6)$, to expanded form.

Rectangle Model

Subdivide.



Label areas.



Distributive Property

$$(x + 2)(x + 6) = (x + 2)x + (x + 2)6$$

$$= x^2 + 2x + 6x + 12$$

x		
	x	6

Label areas.

2	$2x$	12
x	x^2	$6x$
	x	6

Add the areas of the sections:

$$(x + 2)(x + 6) = x^2 + 2x + 6x + 12$$

$$= x^2 + 8x + 12$$

$$(x + 2)(x + 6) = (x + 2)x + (x + 2)6$$

$$= x^2 + 2x + 6x + 12$$

$$= x^2 + 8x + 12$$

How can you write an expanded expression, such as $x^2 + 8x + 12$, in factored form?

In the next problem, we will use the distributive property to write expressions in factored form.

On March 2 of this year, the first school for the blind in the United States was established in Massachusetts. Solve this puzzle to learn the year.

- My hundreds digit is equal to 2^3 , while my units digit is equal to 3^2 .
- The digit in my tens place is equal to the slope m in the equation $y = mx + b$ for the ordered pair $(4, 6)$ and a y -intercept b of -2 .
- The sum of all of my digits is equal to the number of equilateral triangles in an icosahedron.

What year am I?



<u>1</u>	<u>8</u>	<u>2</u>	<u>9</u>
Thousands	Hundreds	Tens	Units

Only Chuck Norris knows a bigger number than infinity, and it's not infinite plus one.

Problem 2.4 Factoring Quadratic Expressions

$y = mx + b$
 $xy = k$
 $y = a(b)x$

A. 1. Copy the diagram. Replace each question mark with the correct length or area.

2. Write two expressions for the area of the rectangle outlined in red.

B. Consider this expression.

$$x^2 + 6x + 8$$

- Choose a value for b that gives an expression you can factor. Then, write the expression in factored form. $(x+2)(x+4)$
- Compare your work with your classmates. Did everyone write the same expressions? Explain.

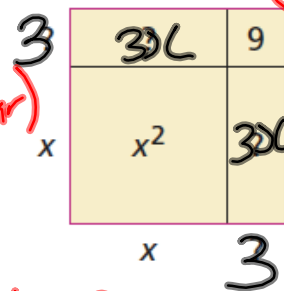
C. In parts (1)–(4), find values of r and s that make the equations true.

1. $x^2 + 10x + 24 = (x + 6)(x + 4)$

2. $x^2 + 11x + 24 = (x + 8)(x + 3)$

3. $x^2 + 25x + 24 = (x + 24)(x + 1)$

4. Describe the strategies you used to factor the expressions in parts (1)–(3).



E. Use the Distributive Property to factor each expression.

1. $x^2 + 5x + 2x + 10$

2. $x^2 + 11x + 10$

3. $x^2 + 3x - 10$

4. $x^2 + 16x + 15$

5. $x^2 - 8x + 15$

6. $x^2 - 12x + 36$

ACE

Homework starts on page 30.

$y = ax^2 + bx + c$

$(x+2)(x+5)$

$(x+10)(x+1)$

$(x+5)(x-2)$

$(x-3)(x-5)$

$(x+15)(x+1)$

$(x-6)(x-6)$
 $(x-6)^2$

Is Alyse correct? Explain what she did at each step.

E. Use the Distributive Property to factor each expression.

1. $x^2 + 5x + 2x + 10$

2. $x^2 + 11x + 10$

3. $x^2 + 3x - 10$

4. $x^2 + 16x + 15$

5. $x^2 - 8x + 15$

6. $x^2 - 12x + 36$

ACE Homework starts on page 30.

$(x+5)(x+2)$

$(x+1)(x+10)$

$(x+5)(x-2)$

$(x+1)(x+15)$

$(x-3)(x-5)$

$(x-6)^2$

$(x-6)(x-6)$

$$x^2 + 6x + 8$$

$$(x+4)(x+2)$$

$$x^2 - 5x + 6$$

$$(x-3)(x-2)$$

$$x^2 + 3x + 2.$$

$$(x+2)(x+1)$$

$$x^2 + 3x - 4$$

$$(x+4)(x-1)$$

$$x^2 + 2x - 3.$$

$$(x-1)(x+3)$$

$$x^2 - 4x + 3.$$

$$(x-3)(x-1)$$

$$x^2 + 8x + 16$$

$$(x+4)(x+4)$$

$$x^2 - 10x + 24$$

$$(x-6)(x-4)$$

$$x^2 - 2x - 8.$$

$$(x+2)(x-4)$$

$$x^2 + x - 20$$

28. $(x - 3)(x + 5)$

First outer
inner last

$$x^2 + 5x - 3x - 15$$

$$x^2 + 2x - 15$$

F.O.I.L.

+ 5	$5x$	-15
x	x^2	$-3x$

$5x \quad -3$

$$x^2 + 5x - 3x - 15$$

$$x^2 + 2x - 15$$

Review of distributive property $a(b+c)$
 $= ab + ac$

ex. 1 $2x(x+3) = 2x^2 + 6x$

ex. 2 $(x+3)(x+2) = x(x+2) + 3(x+2)$
 $= x^2 + 2x + 3x + 6$
 $= x^2 + 5x + 6$

Box method $(x+2)(x-3)$

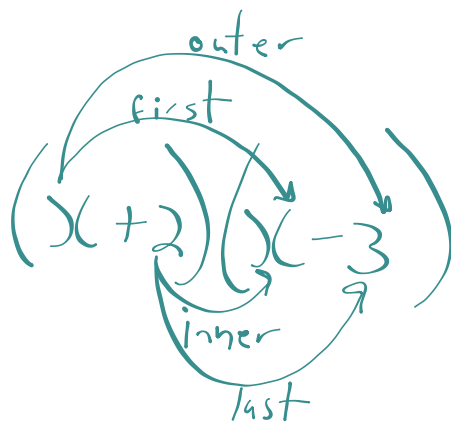
$+2$	$2x$	-6
x	x^2	$-3x$
	x	-3

$$\begin{aligned} &= x^2 - 3x + 2x - 6 \\ &= x^2 - 1x - 6 \end{aligned}$$

Vertical Method $(x+2)(x-3)$

$$\begin{array}{r} x+2 \\ \times \quad x-3 \\ \hline -3x-6 \\ + \quad x^2+2x \\ \hline x^2-x-6 \end{array}$$

F.O.I.L. method $(x+2)(x-3)$



$$\begin{aligned} x^2 - 3x + 2x - 6 &= \\ x^2 - 1x - 6 \end{aligned}$$

For Exercises 31–39, determine whether the equation represents a quadratic relationship *without* making a table or a graph. Explain.

31. $y = 5x + x^2$

32. $y = 2x + 8$

33. $y = (9 - x)x$

34. $y = 4x(3 + x)$

35. $y = 3^x$

36. $y = x^2 + 10x$

37. $y = x(x + 4)$

38. $y = 2(x + 4)$

39. $y = 7x + 10 + x^2$

😊 (1.) $y = mx + b$ $y = 2x + 3$ linear

★ (2.) $y = \frac{k}{x}$ $y = \frac{200}{x}$ inverse

🌸 (3.) $y = a(b)^x$ $y = 16(1.8)^x$ exponential

❄️ (4.) $y = ax^2 + bx + c$ $y = x^2 + 5x + 6$ quadratic

Factor each trinomial and check the evens

1. $x^2 + 4x + 3$

$(x+3)(x+1)$

4. $x^2 - x - 6$

7. $c^2 - 4c - 12$

10. $x^2 + 6x + 5$

13. $x^2 - 2x - 3$

16. $x^2 + 12x + 20$



3. $r^2 - 3r + 2$

6. $x^2 - 22x + 121$

9. $9 - 10x + x^2$

12. $y^2 - 7y - 8$

15. $m^2 + 9m + 20$

18. $18 + 11y + y^2$

Exercises

Factor each trinomial.

1. $x^2 + 4x + 3$
 $(x + 3)(x + 1)$

4. $x^2 - x - 6$
 $(x - 3)(x + 2)$

7. $c^2 - 4c - 12$
 $(c + 2)(c - 6)$

10. $x^2 + 6x + 5$
 $(x + 5)(x + 1)$

13. $x^2 - 2x - 3$
 $(x - 3)(x + 1)$

16. $x^2 + 12x + 20$
 $(x + 10)(x + 2)$

19. $x^2 + 2xy + y^2$
 $(x + y)(x + y)$



3. $r^2 - 3r + 2$
 $(r - 2)(r - 1)$

6. $x^2 - 22x + 121$
 $(x - 11)(x - 11)$

9. $9 - 10x + x^2$
 $(9 - x)(1 - x)$

12. $y^2 - 7y - 8$
 $(y - 8)(y + 1)$

15. $m^2 + 9m + 20$
 $(m + 4)(m + 5)$

18. $18 + 11y + y^2$
 $(9 + y)(2 + y)$

21. $x^2 + 6xy - 7y^2$
 $(x + 7y)(x - y)$