

1. Which statements are true? Give an example of each true statement.

- A number can have no real square root. **True, -1**
- A number can have only one square root. **True, 0**
- A number can have two different square roots. **True, 25 has 5 and -5 .**
- A number can have more than two square roots. **False**

2. Evaluate $\sqrt{25}$, $-\sqrt{25}$, and $\pm\sqrt{25}$.
 5 , -5 , ± 5

3. Give three examples of perfect squares, one an integer, one a fraction, and one a decimal number. **9 , $\frac{1}{4}$, 0.01**

4. Find the numbers represented by
 $3 \pm \sqrt{(-3)^2 - 4(\frac{1}{2})(-8)}$. **8 and -2**

5. Is $\sqrt{2}$ irrational? Is $\frac{144}{99}$ a square root of 2 ? Is $\frac{7064}{4995}$ a square root of 2 ? Explain.
Yes; no; no. An irrational number cannot be represented by a fraction

Independent Practice

Exercises 7–14, find all square roots of the number. Check your results.

- 64 **8 , -8**
- 144 **12 , -12**
- -9 **None**
- 0 **0**
- $\frac{4}{9}$ **$\frac{2}{3}$, $-\frac{2}{3}$**
- $\frac{25}{16}$ **$\frac{5}{4}$, $-\frac{5}{4}$**
- 0.16 **0.4 , -0.4**
- 0.25 **0.5 , -0.5**

Exercises 15–30, evaluate the expression. Give the exact value if possible. Otherwise, give an approximation to two decimal places.

- $-\sqrt{256}$ **-16**
- $\sqrt{49}$ **7**
- $\sqrt{11}$ **3.32**
- $\sqrt{121}$ **11**
- $\sqrt{100}$ **10**
- $-\sqrt{169}$ **-13**
- $\sqrt{36}$ **6**
- $\sqrt{23}$
- $\sqrt{41}$ **6.40**
- $\sqrt{0.75}$ **0.87**
- $\sqrt{0.04}$ **0.2**
- $-\sqrt{\frac{25}{100}}$
- $\sqrt{\frac{81}{324}}$ **$\frac{9}{18}$ or $\frac{1}{2}$**
- $\sqrt{6.25}$ **2.5**
- $-\sqrt{\frac{1}{64}}$ **$-\frac{1}{8}$**
- $\sqrt{26}$ **5.10**

Exercises 31–34, evaluate $\sqrt{b^2 - 4ac}$ for the given values of a , b , and c .

- $a = 4$, $b = 5$, $c = 1$ **3**
- $a = -2$, $b = 8$, $c = -8$ **0**
- $a = 3$, $b = -7$, $c = 6$ **$\sqrt{-23}$ is undefined.**
- $a = 12$, $b = 13$, $c = 3$ **5**

Technology In Exercises 35–38, use a calculator to evaluate the expression. Round results to two decimal places. Use estimation to check results.

- $\frac{1 \pm 5\sqrt{6}}{2}$ **7.12 , -5.12**
- $\frac{3 \pm 4\sqrt{5}}{4}$ **2.99 , -1.49**
- $\frac{7 \pm 3\sqrt{2}}{-1}$ **-11.24 , -2.76**
- $\frac{5 \pm 6\sqrt{3}}{3}$ **5.13 , -1.80**

Exercises 1–4 are quadratic equations.

a. $-3x + 5 = 0$ b. $x^2 - 1 = 0$ c. $x^2 - 3x^3 = 0$ d. $-3 + 4x + x^2 = 0$

In Exercises 2–4, write in standard form and find the leading coefficient.

2. $-3x^2 + 5 = 0$ As is, -3 3. $\frac{1}{2}x^2 + 9x - 3 = 0$ As is, $\frac{1}{2}$ 4. $-8x - x^2 + 4 = 0$
 $-x^2 - 8x + 4 = 0$, -1

In Exercises 5–8, solve the equation. If there are no solutions, state the reason.

5. $x^2 = 17 \pm \sqrt{17}$ 6. $x^2 = 0$ 0 7. $x^2 = -4$
 No real solution 8. $x^2 = 6 \pm \sqrt{6}$

Independent Practice

In Exercises 9–20, solve the equation.

9. $x^2 = 9 \pm 3$ 10. $h^2 = 25 \pm 5$ 11. $6x^2 = 600 \pm 10$ 12. $\frac{1}{5}x^2 = 5 \pm 5$
 13. $3x^2 = 363 \pm 11$ 14. $2b^2 = 98 \pm 7$ 15. $t^2 + 2 = 11 \pm 3$ 16. $t^2 - 57 = 87$
 17. $\frac{1}{4}x^2 - 1 = 7 \pm 4$ 18. $4y^2 + 7 = 8 \pm \frac{1}{2}$ 19. $2s^2 - 5 = 27 \pm 4$ 20. $81x^2 - 5 = 20$
 $\pm \frac{5}{9}$

Technology In Exercises 21–28, use a calculator to solve the equation. Round the results to two decimal places.

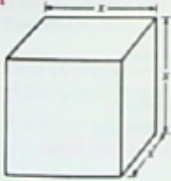

21. $3x^2 + 2 = 56 \pm 4.24$ 22. $7y^2 - 12 = 23 \pm 2.24$ 23. $2x^2 - 5 = 7 \pm 2.45$ 24. $\frac{2}{3}n^2 - 6 = 2$
 ± 3.46
 25. $\frac{1}{4}x^2 + 3 = 8 \pm 3.16$ 26. $4x^2 + 9 = 41 \pm 2.83$ 27. $6s^2 - 2 = 0 \pm 0.58$ 28. $5a^2 + 10 = 20$
 ± 1.41

In Exercises 29–32, an object is dropped from a height h . How long does it take to reach the ground? (Assume there is no air resistance.)

29. $h = 64$ feet 2 seconds 30. $h = 144$ feet 3 seconds 31. $h = 500$ feet 32. $h = 600$ feet
 ± 5.59 seconds
 ± 6.12 seconds

Geometry The surface area of a cube is 150 square feet. Find the length of each edge. 5 ft

Geometry The surface area of a sphere is 80 square meters. Find the radius. (Use $\pi \approx 3.14$)
 ≈ 2.52 m

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CRITICAL THINKING about the Lesson 4. True; $x = -\frac{c}{2a}$ is a vertical line

- Write the equation $y = -3 + 4x - x^2$ in standard form. $y = -x^2 + 4x - 3$
- The graph of a quadratic equation is called a **parabola**.
- How can you use a to decide whether the graph of $y = ax^2 + bx + c$ opens up or down? If $a > 0$, graph opens up; if $a < 0$, graph opens down.
- True or False? The axis of symmetry of the graph of $y = ax^2 + bx + c$ is parallel to the y -axis (or is the y -axis).
- Find the vertex of the graph of $y = 2x^2 + 4x - 2$. $(-1, -4)$
- Find the axis of symmetry of $y = -3x^2 + 3x + 1$. $x = \frac{1}{2}$

Independent Practice

In Exercises 7–12, decide whether the graph of the equation opens up or down. Then find the coordinates of the vertex.

- $y = 2x^2 + 4$ Up, $(0, 4)$
- $y = -5x^2$ Down, $(0, 0)$
- $y = -x^2 + 12$ Down, $(0, 12)$
- $y = 3x^2 - 2x + 4$ Up, $(\frac{1}{3}, \frac{10}{3})$
- $y = 5x^2 - x + 10$ Up, $(\frac{1}{10}, \frac{29}{10})$
- $y = -x^2 + 1x + 12$ Down, $(\frac{1}{2}, \frac{25}{4})$

In Exercises 13–18, find the coordinates of the vertex and the equation of the axis of symmetry. See below.

- $y = 3x^2 + 2x + 4$
- $y = 2x^2 + 3x + 6$
- $y = -4x^2 - 4x + 8$
- $y = 3x^2 - 9x - 12$
- $y = 2x^2 + 7x - 21$
- $y = -x^2 + 4x + 16$

In Exercises 19–36, sketch the graph of the equation. Label the vertex. See Additional Answers.

- $y = x^2 + x + 2$
- $y = -x^2 + 2x - 1$
- $y = -2x^2 + 6x - 9$
- $y = 2x^2 - 3x + 4$
- $y = 6x^2 - 3x + 4$
- $y = 5x^2 + 4x - 5$
- $y = 4x^2 - x + 6$
- $y = -3x^2 - x + 7$
- $y = -5x^2 + 2x - 2$
- $y = 6x^2 - 4x - 1$
- $y = -3x^2 - 5x + 3$
- $y = -2x^2 - 3x + 2$
- $y = x^2 + 6x + 5$
- $y = -4x^2 - 3x + 6$
- $y = -\frac{1}{2}x^2 - 3x + 4$
- $y = \frac{1}{3}x^2 + 3x - 2$
- $y = -2x^2 + \frac{1}{3}x - 1$
- $y = 3x^2 - \frac{1}{2}x + 4$

You've Got to Have the Right Angle In Exercise 37, use the information given in Example 3.

37. Natalya Lisovskaya's winning throw in the shot put was at a 45° angle. If the shot had been thrown at a 40° angle or 50° angle, would it have gone farther? Explain. See margin.

Throw at 40° angle: $y = -0.0125x^2 + 0.84x + 5$
 Throw at 50° angle: $y = -0.0177x^2 + 1.19x + 5$

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- $x = -\frac{1}{3}; (-\frac{1}{3}, \frac{11}{3})$
- $x = -\frac{3}{4}; (-\frac{3}{4}, \frac{29}{8})$
- $x = -\frac{1}{2}; (-\frac{1}{2}, 9)$
- $x = \frac{3}{2}; (\frac{3}{2}, -\frac{29}{2})$
- $x = -\frac{7}{2}; (-\frac{7}{2}, -21\frac{1}{2})$

2. Describe the two models for vertical motion. *See top of page 474.*

3. State the values of a , b , and c from the standard form of the equation $5 = 6 + 9x - x^2$. $a = -1$, $b = 9$, $c = 1$

4. Solve $x^2 + x - 2 = 0$. $1, -2$

5. Sketch the graph of $y = x^2 + x - 2$ and label the x -intercepts. *See Additional Answers.*

6. Describe the relationship between the x -intercepts found in Exercise 5 and the solutions found in Exercise 4. *They are the same.*

Independent Practice

In Exercises 7–10, write the quadratic equation in standard form.

7. $-3x^2 + 5x = 9$ $-3x^2 + 5x - 9 = 0$

8. $5 - 2x + x^2 = 0$ $x^2 - 2x + 5 = 0$

9. $-4 + 3x + x^2 = 5$ $x^2 + 3x - 9 = 0$

10. $9x - 7x^2 = 16$ $-7x^2 + 9x - 16 = 0$

In Exercises 11–14, find the value of $b^2 - 4ac$ for the equation.

11. $2x^2 - 3x - 1 = 0$ 17

12. $4x^2 + 4x + 1 = 0$ 0

13. $3x^2 - 2x - 5 = 0$ 64

14. $x^2 - 11x + 30 = 0$ 1

In Exercises 15–20, use the quadratic formula to solve the equation.

15. $4x^2 - 13x + 3 = 0$ $3, \frac{1}{4}$

16. $3y^2 + 11y + 10 = 0$ $-\frac{5}{3}, -2$

17. $2x^2 + 7x + 3 = 0$ $-\frac{1}{2}, -3$

18. $x^2 - 6x + 7 = 0$

19. $5y^2 + 2y - 2 = 0$

20. $2x^2 + 4x - 3 = 0$

21. $\frac{-10 + \sqrt{70}}{6} \approx -0.27$

22. $\frac{-10 - \sqrt{70}}{6} \approx -3.06$

In Exercises 21–26, solve the quadratic equation by the most convenient method (finding square roots or the quadratic formula). Explain why you chose your method.

21. $6x^2 + 20x + 5 = 0$

22. $t^2 = 27$

23. $x^2 - 625 = 0$ $25, -25$

24. $4x^2 - 49 = 0$ $\frac{7}{2}, -\frac{7}{2}$

25. $-2x^2 + 6x + 1 = 0$

26. $x^2 + 14x + 49 = 0$ -7

In Exercises 27–32, find the x -intercepts of the graph of the equation.

27. $y = x^2 + 2x + 15$ *None*

28. $y = x^2 - 6x - 7$ $7, -1$

29. $y = x^2 + x - 20$ $4, -5$

30. $y = x^2 + 8x + 12$ $-2, -6$

31. $y = x^2 + x - \frac{3}{4}$ $\frac{1}{2}, -\frac{3}{2}$

32. $y = x^2 + \frac{2}{3}x - 2$ $\frac{2}{3}, -3$

25. $\frac{3 + \sqrt{11}}{2} \approx 3.16, \frac{3 - \sqrt{11}}{2} \approx -0.16$

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19. $\frac{-1 + \sqrt{11}}{5} \approx 0.46, \frac{-1 - \sqrt{11}}{5} \approx -0.86$

20. $\frac{-2 + \sqrt{10}}{2} \approx 0.58, \frac{-2 - \sqrt{10}}{2} \approx -2.58$

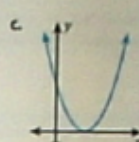
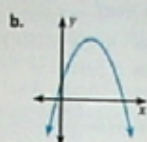
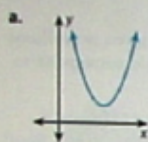
1. Write the quadratic formula and circle the part that is called the $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ discriminant.
2. Explain how the discriminant can be used to determine the number of solutions of $ax^2 + bx + c = 0$. See margin.
3. Find the discriminant of $3x^2 - 2x - 5 = 0$. How many solutions does this equation have? 64, 2

In Exercises 4–6, match the discriminant with the graph.

4. $b^2 - 4ac = 3$

5. $b^2 - 4ac = 0$

6. $b^2 - 4ac = -2$



Independent Practice

In Exercises 7–12, decide how many solutions the equation has.

7. $2x^2 + 3x - 2 = 0$ 16

8. $x^2 - 2x + 4 = 0$ None -42

9. $-2x^2 + 4x - 2 = 0$ $d = 0$

10. $-\frac{1}{2}x^2 + x + 3 = 0$ 7

11. $5x^2 - 2x + 3 = 0$ None -56

12. $3x^2 - 6x + 3 = 0$ 1 $d = 0$

13. **Spelunking** No, the discriminant is negative. You and a friend are spelunking (exploring caves) in a section of the Onondaga Cave in Missouri. The two of you are standing beneath a ledge that is 15 feet high. Your friend can throw a grappling hook upward with an initial velocity of 30 feet per second. Will the grappling hook reach the ledge when it is thrown? Explain.

14. **Spelunking, Part II** While you and your friend are attempting to reach the ledge, some other spelunkers join you. They see the trouble you are having and suggest that your friend stand on a foot-high rock to throw the hook. Would this help? Explain.

Yes, the discriminant would then be positive.

Onondaga Cave, in Leasburg, Missouri



► **CRITICAL THINKING** about the Lesson

1. Use the Zero-Product Property to complete the statement. If $ab = 0$, then $\boxed{a = 0 \text{ or } b = 0}$.
2. Solve the equation: $(x - 2)(x + 1) = 0$.
 $2, -1$
3. Solve the equation: $3x^2 + 4x = 0$. $0, -\frac{4}{3}$
4. Which two numbers satisfy the statement, "The sum of a number and its square is zero."? $0, -1$
5. **True or False?** If $(5x - 1)(x + 3) = 1$, then $5x - 1 = 1$ or $x + 3 = 1$. Explain.
False. The product of any number and its reciprocal is 1; so neither factor has to equal 1.
6. **True or False?** If $(x + 3)(x - 3) = 0$, then $x + 3 = 0$ or $x - 3 = 0$. Explain.
True. A product cannot equal 0, unless one of the factors is 0.

Independent Practice

In Exercises 7–10, solve the equation.

7. $(x + 1)(x + 2) = 0$ $-1, -2$
8. $(x - 3)(x + 7) = 0$ $3, -7$
9. $(x + 3)(x + 4) = 0$ $-3, -4$
10. $(x + 6)(x - 5) = 0$ $-6, 5$

In Exercises 11–16, solve the equation by factoring.

11. $x^2 + 5x - 6 = 0$ $-6, 1$
12. $3x^2 + 11x - 4 = 0$ $\frac{1}{3}, -4$
13. $2x^2 + 5x + 3 = 0$ $-\frac{3}{2}, -1$
14. $6x^2 + 13x + 5 = 0$ $-\frac{1}{2}, -\frac{5}{3}$
15. $3x^2 + 7x + 2 = 0$ $-\frac{1}{3}, -2$
16. $12x^2 - 5x - 3 = 0$ $\frac{3}{4}, -\frac{1}{3}$

In Exercises 17–24, match the equation with its solutions.

- | | | | |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 17. $x^2 - 5x + 6 = 0$ f | 18. $x^2 + 5x + 6 = 0$ b | 19. $x^2 - 7x + 6 = 0$ g | 20. $x^2 + 7x + 6 = 0$ a |
| 21. $x^2 - 5x - 6 = 0$ d | 22. $x^2 + 5x - 6 = 0$ c | 23. $x^2 + x - 6 = 0$ e | 24. $x^2 - x - 6 = 0$ h |
| a. $-1, -6$ | b. $-2, -3$ | c. $1, -6$ | d. $-1, 6$ |
| e. $2, -3$ | f. $2, 3$ | g. $1, 6$ | h. $-2, 3$ |

In Exercises 25–33, solve the equation by finding square roots, by the quadratic formula, or by factoring.

25. $x(x - 9) = 0$ $0, 9$
26. $2y(y + 6) = 0$ $0, -6$
27. $y^2 - 7y + 6 = -6$ $3, 4$
28. $x^2 - 12 = -3$ ± 3
29. $x^2 - 8x = -16$ 4
30. $x^2 + 4x + 7 = 3$ -2
31. $4x^2 + 2x = 0$ $0, -\frac{1}{2}$
32. $4y^2 - 18y = 0$ $0, \frac{9}{2}$
33. $x^2 - 12x + 40 = 4$ 6

In Exercises 34 and 35, multiply both sides of the equation by an appropriate power of ten to obtain integer coefficients. Then solve by factoring.

34. $0.8x^2 + 3.2x + 2.4 = 0$ $-1, -3$
35. $0.23x^2 - 0.54x + 0.16 = 0$ $\frac{8}{23}, 2$

CRITICAL THINKING ABOUT THE LESSON

$2 + \sqrt{12}, 2 - \sqrt{12}; \text{no difference}$

1. Which is a perfect square trinomial?

a. $x^2 - 8x + 8$ b. $x^2 - 8x + 16$

c. $x^2 - 8x + 64$

3. What term must be added to $x^2 + 6x$ to create a perfect square trinomial? 92. Solve $x^2 - 4x = 8$ by completing the square. Solve the same equation by the quadratic formula. Explain the difference in the results.

4. Name the five methods for solving a quadratic equation. See chart on page 550.

Independent Practice

In Exercises 5–10, find the term that must be added to the expression to create a perfect square trinomial.

5. $x^2 - 18x$ 81

6. $x^2 + 6x$ 9

7. $x^2 + 12x$ 36

8. $x^2 - 10x$ 25

9. $x^2 - 7x$ $\frac{49}{4}$

10. $x^2 - 5x$ $\frac{25}{4}$

In Exercises 11–28, solve the equation by completing the square. 16. $-\frac{1}{2}, -\frac{17}{2}$ 25. $-\frac{1}{3}, -1$ 21, 3

11. $x^2 + 10x - 11 = 0$ 1, -11

12. $x^2 + 14x - 15 = 0$ 1, -15

13. $y^2 - 24y + 63 = 0$

14. $y^2 - 8y + 12 = 0$ 6, 2

15. $t^2 + 3t - \frac{7}{4} = 0$ $-\frac{1}{2}, -\frac{7}{2}$

16. $y^2 + 9y + \frac{17}{4} = 0$

17. $x^2 - \frac{2}{3}x - 3 = 0$

18. $x^2 + \frac{4}{5}x - 1 = 0$

19. $x^2 + x - 1 = 0$

20. $1 + x - x^2 = 0$

21. $4y^2 + 4y - 9 = 0$

22. $3x^2 - 24x - 5 = 0$

23. $2x^2 - 6x - 15 = 5$ 5, -2

24. $5x^2 - 20x - 20 = 5$ 5, -1

25. $3x^2 + 4x + 4 = 3$

26. $4x^2 + 6x - 6 = 2$

27. $x^2 + 2x = 2$

28. $x^2 - 2x = 2$

17–22, 26, 27. See margin.

$1 + \sqrt{3}, 1 - \sqrt{3}$

In Exercises 29–43, use the most convenient method to solve the equation. Explain why you made your choice. 29, 34, 36, 37. See margin.

29. $x^2 - 3x - 1 = 0$

30. $4x^2 - 12 = 0$ $\sqrt{3}, -\sqrt{3}$

$-3 + \frac{\sqrt{120}}{2}, -3 - \frac{\sqrt{120}}{2}$

32. $4x^2 - 25 = 0$ $\frac{5}{2}, -\frac{5}{2}$

33. $x^2 + 7x + 10 = 0$ -5, -2

31. $y^2 + 6y - 24 = 0$

35. $3x^2 - 5x = 0$ 0, $\frac{5}{3}$

36. $y^2 + 2y - 26 = 0$

34. $u^2 + 5u + 2 = 0$

38. $4x^2 + 4x + 1 = 0$ $-\frac{1}{2}$

39. $7x^2 - 14x = 0$ 0, 2

37. $9x^2 + 10x - 4 = 0$

41. $8x^2 - 10x + 3 = 0$ $\frac{3}{4}, \frac{1}{2}$

42. $7x^2 - 14 = 0$ $\sqrt{2}, -\sqrt{2}$

40. $4x^2 - 13x + 3 = 0$

44. **Money in the Bank** At your seventh grade graduation, you and your twin sister each received \$200. You each deposited the money in savings accounts that compound interest annually. Two years later your sister's deposit has grown by \$28.98. Your account is in a different bank that pays an interest rate that is 1% more than your sister receives. What is your balance after two years? \$233.28

43. $y^2 + 20y + 10 = 0$

$-10 + \frac{\sqrt{380}}{2}, -10 - \frac{\sqrt{380}}{2}$

40. 3, $\frac{1}{4}$

Answers

$$17. \frac{1}{3} + \frac{\sqrt{28}}{3}, \frac{1}{3} - \frac{\sqrt{28}}{3}$$

$$18. -\frac{2}{5} + \frac{\sqrt{29}}{5}, -\frac{2}{5} - \frac{\sqrt{29}}{5}$$

$$19. -\frac{1}{2} + \frac{\sqrt{5}}{2}, -\frac{1}{2} - \frac{\sqrt{5}}{2}$$

$$20. \frac{1}{2} + \frac{\sqrt{5}}{2}, \frac{1}{2} - \frac{\sqrt{5}}{2}$$

$$21. -\frac{1}{2} + \frac{\sqrt{10}}{2}, -\frac{1}{2} - \frac{\sqrt{10}}{2}$$

$$22. 4 + \frac{\sqrt{159}}{3}, 4 - \frac{\sqrt{159}}{3}$$

$$26. -\frac{3}{4} + \frac{\sqrt{41}}{4}, -\frac{3}{4} - \frac{\sqrt{41}}{4}$$

$$27. -1 + \sqrt{3}, -1 - \sqrt{3}$$

$$29. \frac{3}{2} + \frac{\sqrt{13}}{2}, \frac{3}{2} - \frac{\sqrt{13}}{2}$$

$$34. -\frac{5}{2} + \frac{\sqrt{17}}{2}, -\frac{5}{2} - \frac{\sqrt{17}}{2}$$

$$36. -1 + \sqrt{27}, -1 - \sqrt{27}$$

$$37. -\frac{5}{9} + \frac{\sqrt{244}}{18}, -\frac{5}{9} - \frac{\sqrt{244}}{18}$$