

Polynomials and Factoring

LESSONS

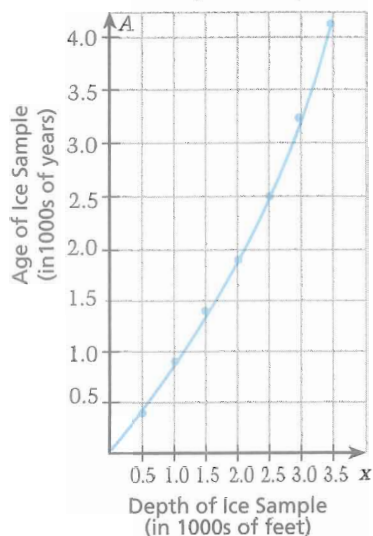
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This brilliant pattern of colors is formed by ice crystals photographed in polarized light.



Real Life Climatology

Dating Ice Samples



To study Earth's climate, scientists are drilling thousands of feet into Greenland's ice sheet at a research site called GISP2. Core samples of ice are removed and analyzed. Samples taken near the surface were formed recently, but samples from great depths were formed thousands of years ago. The *polynomial* model

$$A = \frac{9}{16}(x^3 - 7x^2 + 33x + 12)^2$$

relates the age of the ice, A (in years), with the depth of the core sample, x (in 1000's of feet). The ice crystals undergo changes in size, shape, and orientation with increasing depth. Researchers use polarized light to measure the changes in the ice crystals.

(Source: Popular Science, 1991.)

Depth, x	0.5	1.0	1.5	2.0	2.5	3.0	3.5
Age, A	406	856	1357	1892	2478	3164	4028

10.1

Adding and Subtracting Polynomials

What you should learn:

Goal 1 How to add and subtract polynomials

Goal 2 How to use polynomials as models in real-life settings

Why you should learn it:

You can use polynomials as models for a majority of real-life situations such as the performance of an industry over an interval of time.

Goal 1 Adding and Subtracting Polynomials

An expression whose terms are of the form ax^k , where k is a non-negative integer, is a **polynomial in one variable** or simply a **polynomial**.

The integer k is the **degree** of ax^k . The term ax has a degree of one, and the **constant term** a has a degree of zero. The **degree of a polynomial** is the largest degree of its terms. Polynomials are usually written in **standard form**, which means that the terms are written in descending order, from the largest degree to the smallest degree.

The number a is the **coefficient** of the term ax^k . When a polynomial is written in standard form, the coefficient of its first term is the **leading coefficient** of the polynomial.

Example 1 Identifying Coefficients of a Polynomial

Identify the coefficients of $-4x^2 + x^3 + 3$.

Solution Write the polynomial in standard form. Account for each degree, even if you must include zero coefficients.

$$-4x^2 + x^3 + 3 = (1)x^3 + (-4)x^2 + (0)x + 3$$

The coefficients are 1, -4 , 0, and 3. The leading coefficient is 1.

Polynomials that have only one term are **monomials**. Polynomials that have two terms are **binomials**. Polynomials that have three terms are **trinomials**.

Example 2 Classifying Polynomials

Polynomial	Degree	Classified by Degree	Classified by Terms
a. 6	0	Constant	Monomial
b. $-2x$	1	Linear	Monomial
c. $3x + 1$	1	Linear	Binomial
d. $-x^2 + 2x - 5$	2	Quadratic	Trinomial
e. $4x^3 - 8x$	3	Cubic	Binomial
f. $2x^4 - 7x^3 - 5x + 1$	4	Quartic	Polynomial

To add two polynomials, add the coefficients of like terms.
You can use a horizontal or vertical format.

Example 3 Adding Polynomials Horizontally

Add $2x^2 + x - 5$ and $x^2 + x + 6$.

Solution

$$\begin{aligned}(2x^2 + x - 5) + (x^2 + x + 6) \\&= (2x^2 + x^2) + (x + x) + (-5 + 6) \\&= 3x^2 + 2x + 1\end{aligned}$$

Example 4 Using a Vertical Format to Add Polynomials

Use a vertical format to find the sum.

$$(5x^3 + 2x^2 - x + 7) + (3x^2 - 4x + 7) + (-x^3 + 4x^2 - 8)$$

Solution Align the like terms of the polynomials.

$$\begin{array}{r}5x^3 + 2x^2 - x + 7 \\ 3x^2 - 4x + 7 \\ -x^3 + 4x^2 - 8 \\ \hline4x^3 + 9x^2 - 5x + 6\end{array}$$

To subtract a polynomial, you must subtract *each* of its terms.

Example 5 Subtracting Polynomials Horizontally

Subtract $2x^2 - x - 4$ from $3x^2 - 5x + 3$.

Solution

$$\begin{aligned}(3x^2 - 5x + 3) - (2x^2 - x - 4) \\&= 3x^2 - 5x + 3 - 2x^2 + x + 4 \\&= (3x^2 - 2x^2) + (-5x + x) + (3 + 4) \\&= x^2 - 4x + 7\end{aligned}$$

One of the most common mistakes in algebra is to forget to subtract *each* term when subtracting one expression from another. Here is an example.

$$(x^2 - 2x + 3) - (x^2 + 2x - 2) \neq x^2 - 2x + 3 - x^2 + 2x - 2$$

Wrong Signs
↓ ↓

Example 6 Using a Vertical Format to Subtract Polynomials

Use a vertical format to find the difference.

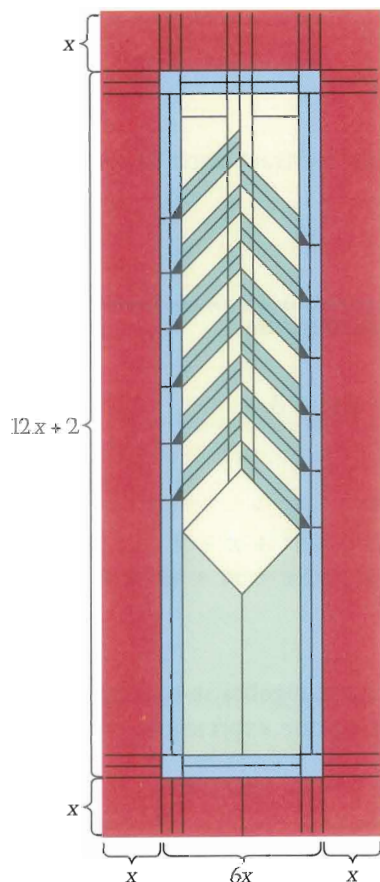
$$(-2x^3 + 5x^2 - x + 8) - (-2x^3 + 3x - 4)$$

Solution To subtract each term, you *add its opposite*. One way is to multiply each “subtracted” term by -1 and then add.

$$\begin{array}{r} (-2x^3 + 5x^2 - x + 8) \\ -(-2x^3 + 3x - 4) \rightarrow \\ \hline -2x^3 + 5x^2 - x + 8 \\ 2x^3 - 3x + 4 \\ \hline 5x^2 - 4x + 12 \end{array}$$

Goal 2 Using Polynomials in Real-Life Modeling

Real Life Stained Glass



Example 7 Finding the Area of a Border

You are designing a stained-glass window. The border is made of red glass and is x inches wide. The interior uses other colors of glass. Write a polynomial expression for the amount of red glass you will use. Then use the expression to find how much red glass would be used for a 2-inch border and for a 3-inch border.

Solution

Verbal Model

$$\text{Area of border} = \text{Total area} - \text{Area of interior}$$

Labels

$$\begin{array}{ll} \text{Total width} = 8x & (\text{inches}) \\ \text{Total height} = 14x + 2 & (\text{inches}) \\ \text{Total area} = 8x(14x + 2) = 112x^2 + 16x & (\text{sq. in.}) \\ \text{Interior width} = 6x & (\text{inches}) \\ \text{Interior height} = 12x + 2 & (\text{inches}) \\ \text{Interior area} = 6x(12x + 2) = 72x^2 + 12x & (\text{sq. in.}) \end{array}$$

Equation

$$\begin{aligned} \text{Area of border} &= (112x^2 + 16x) - (72x^2 + 12x) \\ &= (112x^2 - 72x^2) + (16x - 12x) \\ &= 40x^2 + 4x \end{aligned}$$

If the border is 2 inches wide, then the area of red glass is

$$40(2^2) + 4(2) = 168 \text{ square inches.}$$

If the border is 3 inches wide, then the area of red glass is

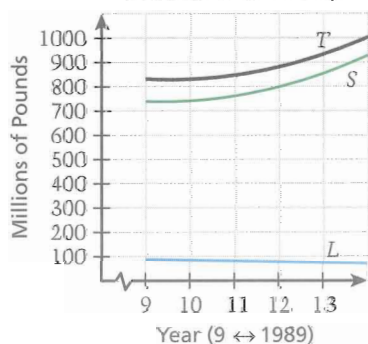
$$40(3^2) + 4(3) = 372 \text{ square inches.}$$

Lobster pots on a dock at West Tremont, Maine. Maine harvests more lobster than any other state.



Real Life Fishing Industry

Annual Consumption of Lobster and Shrimp



Example 8 Using Polynomial Models

The number of pounds of shrimp, S , and lobster, L , consumed by Americans from 1989 to 1993 can be modeled by

$$S = 8.9t^2 - 166.9t + 1522.2 \quad \text{Shrimp (millions of pounds)}$$

$$L = -3.4t + 121.4 \quad \text{Lobster (millions of pounds)}$$

where $t = 9$ represents 1989. Find a model that represents the total amount, T , of shrimp and lobster consumed from 1989 to 1993. Estimate the total amount consumed in 1994.

Solution By adding the two models, you can obtain the following model for the total consumption of shrimp and lobster.

$$T = 8.9t^2 - 170.3t + 1643.6 \quad \text{Total (millions of pounds)}$$

Using this model, and substituting $t = 14$, you can estimate the 1994 consumption to be

$$T = 8.9(14^2) - 170.3(14) + 1643.6 = 1003.8 \text{ million pounds.}$$

(Source: U.S. National Oceanic Atmospheric Administration)

Communicating about ALGEBRA

► SHARING IDEAS about the Lesson

Use Terminology To “do” algebra, you must know how to use the language.

- The prefixes *poly*, *mono*, *bi*, and *tri* mean many, one, two, and three, respectively. Explain how these prefixes are used in *polygon*, *monopoly*, *binocular*, and *tricycle*.
- Give an example of two polynomials of degree 3 whose sum is a polynomial of degree 2.

EXERCISES

Guided Practice

CRITICAL THINKING about the Lesson

1. Describe a polynomial in one variable.
2. Name the terms of $-3x^3 - 2x^2 + 4x - 5$.
3. Name the coefficients in $-7x^3 + 12x - 31$.
4. Write $15y - 6 + 10y^3 - 3y^2$ in standard form.
5. What is the degree of $2x^2 - 4x^3 + 7$?
6. Subtract $(2x^2 - 4x + 1)$ from $(x^2 + 8)$.

Independent Practice

In Exercises 7–10, classify the polynomial by degree and by number of terms.

7. $-5x - 4$
8. -7
9. $16 - 4x + 3x^2$
10. $3x^2 + 6x + 1$

In Exercises 11–16, add the polynomials. (Use a horizontal format.)

11. $x^2 - 3$; $3x^2 + 5$
12. $-3y + 2$; $y^2 + 3y + 2$
13. $2x^2 + 3x + 1$; $x^2 - 2x + 2$
14. $2x^2 - x + 3$; $3x^2 - 4x + 7$
15. $12x^3 + 2x^2 - 4$; $9x^2 + 3x - 8$
16. $-4x^3 - 2x^2 + x - 5$; $2x^3 + 3x + 4$

In Exercises 17–20, add the polynomials. (Use a vertical format.)

17. $2z - 8z^2 - 3$; $z^2 + 5z$
18. $6x^2 + 5$; $3 - 2x^2$
19. $5x^4 - 2x + 7$; $-3x^4 + 6x^2 - 5$
20. $4x^2 - 7x + 2$; $-x^2 + x - 2$

In Exercises 21–24, subtract the second polynomial from the first. (Use a horizontal format.)

21. $z^3 + z^2 + 1$; z^2
22. 10 ; $u^2 + 5$
23. $2x^2 + 3x - 4$; $x^2 + x - 1$
24. $3x^3 - 4x^2 + 3$; $x^3 + 3x^2 - x - 4$

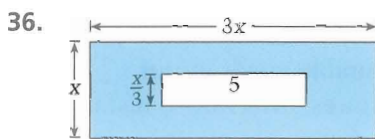
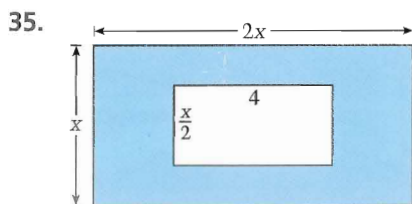
In Exercises 25–28, subtract the second polynomial from the first. (Use a vertical format.)

25. $10x^3 + 15$; $17x^3 - 4x + 5$
26. $y^2 + 3y^4$; $y^5 - y^4$
27. $-2x^3 + 5x^2 - x + 8$; $-2x^3 + 3x - 4$
28. $3x^2 + 7x - 6$; $3x^2 + 7x$

In Exercises 29–34, perform the indicated operations. Use either a horizontal or vertical format and explain why you chose the method you used.

29. $(6x - 5) - (8x + 15) + (3x - 4)$
30. $(2x^2 + 1) + (x^2 - 2x + 1) - (2x^2 + 8)$
31. $-(x^3 - 2) + (4x^3 - 2x) - (2x^2 + 3)$
32. $-(5x^2 - 1) - (-3x^2 + 5) - (x^2 - x)$
33. $2(t^2 + 5) - 3(t^2 + 5) + 5(t^2 + 5)$
34. $-10(u + 1) + 8(u - 1) - 3(u + 6)$

Geometry In Exercises 35 and 36, find the area of the shaded region.

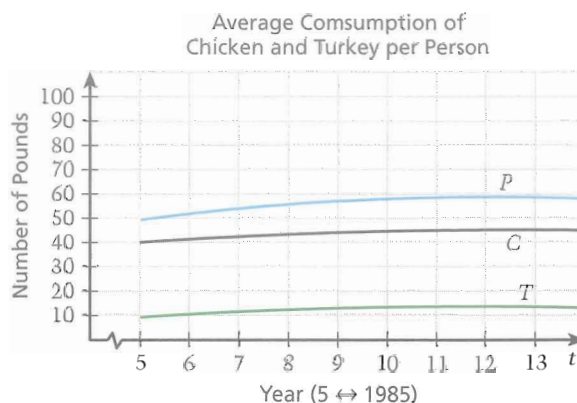


37. **Chicken or Turkey?** For 1985 through 1993, the average numbers of pounds of chicken, C , and turkey, T , consumed per American can be modeled by

$$C = -0.09t^2 + 2.26t + 31.07$$

$$T = -0.10t^2 + 2.34t + 0.30$$

where $t = 5$ represents 1985. Find a model for the average number of pounds, P , of chicken and turkey consumed from 1985 to 1993. (Source: U.S. Department of Agriculture)

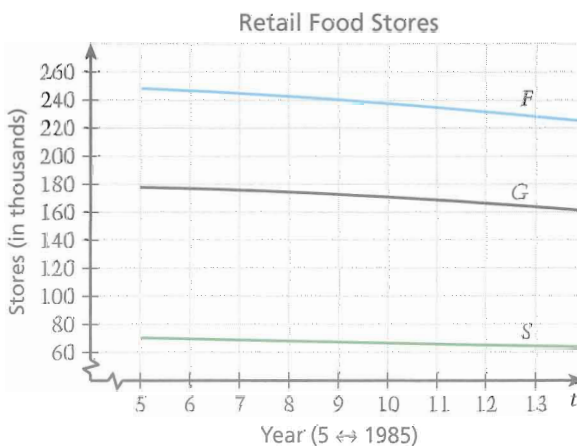


38. **Food Stores** For 1985 through 1993, the average numbers (in thousands) of grocery stores, G (including convenience stores), and specialty food stores, S , in the United States can be modeled by

$$G = -0.12t^2 + 0.41t + 178.87$$

$$S = 0.004t^2 - 0.786t + 74.688$$

where $t = 5$ represents 1985. Find a model that represents the total number of retail food stores, F , from 1985 to 1993. (Source: National Restaurant Association)



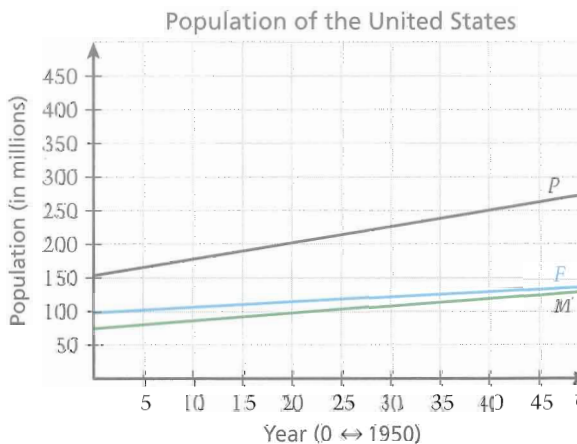
39. **American Men and Women** For 1950 through 1995, the total population, P , and male population, M , of the United States can be modeled by

$$P = 2427.9t + 153,308 \quad (\text{in } 1000\text{'s})$$

$$M = 1160.7t + 75,707 \quad (\text{in } 1000\text{'s})$$

where $t = 0$ represents 1950. Find a model that represents the female population, F , of the United States from 1950 to 1995.

(Source: U.S. Bureau of the Census)

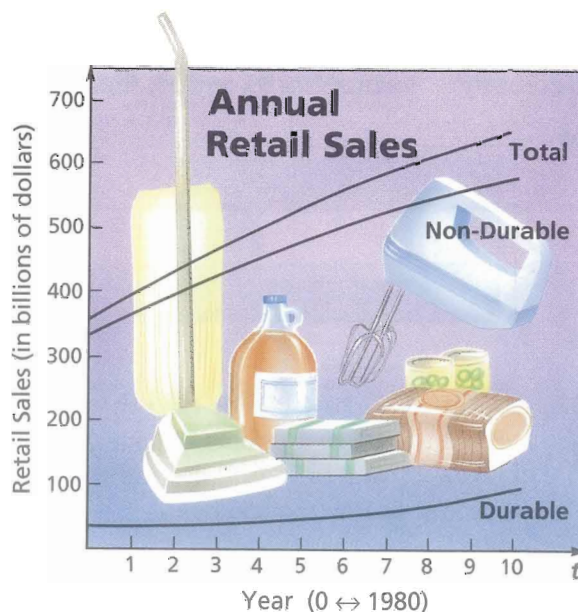


40. **U.S. Retail Sales** Retail stores are classified as durable-goods stores (car dealers, hardware stores, furniture stores, jewelers) and nondurable-goods stores (department stores, drugstores, restaurants, grocery stores, gas stations, general merchandise stores). For 1980 through 1990, the total sales for retail stores, R , and for durable-goods stores, D , in the United States can be modeled by

$$R = -0.21t^2 + 31.6t + 357.4 \text{ (in billions \$)}$$

$$D = 0.26t^2 + 2.9t + 34.3 \text{ (in billions \$)}$$

where $t = 0$ represents 1980. Find a model that represents the sales for non-durable-goods stores, N .



Integrated Review

In Exercises 41–44, simplify the expression.

41. $8y - 2x + 7x - 10y$

42. $\frac{5}{6}x - \frac{2}{3}x + 8$

43. $\frac{1}{2}x + \frac{3}{2}y - 3y + 4x$

44. $-2y + 4x - 4x + 6y$

Discrete Math In Exercises 45–48, perform the indicated matrix operation.

45. $\begin{bmatrix} x^2 - 4x & -3x \\ 7x & 2x^2 + 4 \end{bmatrix} + \begin{bmatrix} x^2 + 4x & 2x \\ -7 + 2x & -5x^2 \end{bmatrix}$

46. $\begin{bmatrix} 4 & -6x \\ 5x^2 + x & 2x \end{bmatrix} + \begin{bmatrix} 3x^2 - 1 & 4x \\ 2x & x - 1 \end{bmatrix}$

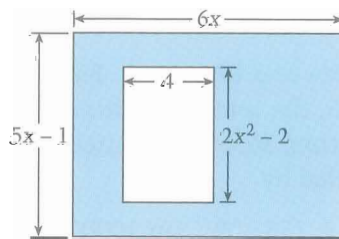
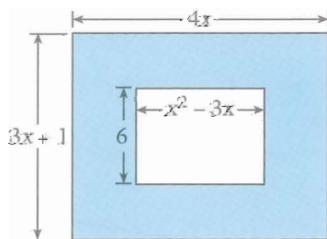
47. $\begin{bmatrix} 6x - 5 & 10 \\ x^3 & 4y^2 - 5y \end{bmatrix} - \begin{bmatrix} 8 - 14x & 3x \\ -7y^3 & 6y - 2 \end{bmatrix}$

48. $\begin{bmatrix} 7x^3 & x^2 - 3x \\ 2x - 1 & 4 - 2x \end{bmatrix} - \begin{bmatrix} -2x^2 & 2x^2 + x \\ 3 & x^2 + 5x \end{bmatrix}$

Geometry In Exercises 49 and 50, find the value(s) for x .

49. Area of shaded region is 260 square units.

50. Area of shaded region is 84 square units.



Exploration and Extension

51. What must you add to $4x^2 + x - 5$ to get $x^2 + 5x + 1$?
 52. What must you add to $x^2 - 7x + 3$ to get $-3x^2 + 2x - 1$?
 53. What must you subtract from $3x^2 + 7x - 9$ to get $12x^2 - x + 6$?

10.2

Multiplying Polynomials

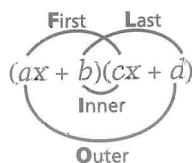
What you should learn:

Goal 1 How to multiply two polynomials using the Distributive Properties and the FOIL method

Goal 2 How to use polynomial multiplication in real-life settings

Why you should learn it:

You can model many situations with polynomial expressions that have to be multiplied, such as a rate times a quantity.



Goal 1 Multiplying Polynomials

You already know the simplest type of polynomial multiplication—multiplying a polynomial by a monomial.

$$(3x)(2x^2 - 5x + 3) = (3x)(2x^2) - (3x)(5x) + (3x)(3) \\ = 6x^3 - 15x^2 + 9x$$

To multiply two binomials, you can use both left and right Distributive Properties.

$$(3x - 2)(2x + 7) = 3x(2x + 7) - 2(2x + 7) \\ = (3x)(2x) + (3x)(7) - (2)(2x) - 2(7) \\ = 6x^2 + 21x - 4x - 14$$

Product of
First terms

Product of
Outer terms

Product of
Inner terms

Product of
Last terms

$$= 6x^2 + 17x - 14$$

With practice, you will be able to multiply two binomials in a single step using the **FOIL pattern**.

Example 1 Multiplying Binomials (Distributive Property)

Multiply: $(x - 3)$ by $(x + 2)$.

Solution

$$(x + 2)(x - 3) \\ = x(x - 3) + 2(x - 3) \quad \text{Right Distributive Property} \\ = x^2 - 3x + 2x - 6 \quad \text{Left Distributive Property} \\ = x^2 - x - 6 \quad \text{Combine like terms.}$$

Example 2 Multiplying Binomials (FOIL Pattern)

Multiply: $(3x + 4)(2x + 1)$.

Solution

$$(3x + 4)(2x + 1) = \overbrace{6x^2}^F + \overbrace{3x}^O + \overbrace{8x}^I + \overbrace{4}^L \\ = 6x^2 + 11x + 4$$

To multiply two polynomials, remember that *each term of one polynomial must be multiplied by each term of the other polynomial*. This can be done using either a vertical or a horizontal format. In either case, it is best to begin by writing each polynomial in standard form.

Example 3 Multiplying Polynomials (Vertical Format)

Multiply: $(x - 2)(5 + 3x - x^2)$.

Solution Begin by aligning like terms in columns.

$$\begin{array}{r}
 -x^2 + 3x + 5 \quad \text{Standard form} \\
 \times x - 2 \quad \text{Standard form} \\
 \hline
 2x^2 - 6x - 10 \quad \leftarrow -2(-x^2 + 3x + 5) \\
 -x^3 + 3x^2 + 5x \quad \leftarrow x(-x^2 + 3x + 5) \\
 \hline
 -x^3 + 5x^2 - x - 10 \quad \text{Add like terms.}
 \end{array}$$

Example 4 Multiplying Polynomials (Horizontal Format)

Multiply: $(4x^2 - 3x - 1)(2x - 5)$.

Solution

$$\begin{aligned}
 &(4x^2 - 3x - 1)(2x - 5) \\
 &= 4x^2(2x - 5) - 3x(2x - 5) - (1)(2x - 5) \\
 &= 8x^3 - 20x^2 - 6x^2 + 15x - 2x + 5 \\
 &= 8x^3 - 26x^2 + 13x + 5
 \end{aligned}$$



Example 5 An Area Model for Multiplying Polynomials

Use an area model (or algebra tiles) to show that

$$(x + 2)(2x + 1) = 2x^2 + 5x + 2.$$

Solution Think of a rectangle whose sides have lengths $x + 2$ and $2x + 1$. The area of this rectangle is

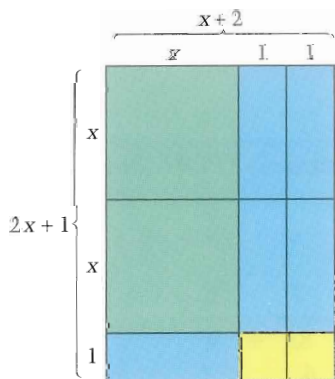
$$(x + 2)(2x + 1). \quad \text{Area} = (\text{width})(\text{length})$$

Another way to find the area is to add the areas of the rectangular parts. There are two squares whose sides are x , five rectangles whose sides are x and 1 , and two squares whose sides are 1 . The total area of these nine rectangles is

$$2x^2 + 5x + 2. \quad \text{Area} = \text{sum of rectangular areas}$$

Because each method must produce the same area, you can conclude that

$$(x + 2)(2x + 1) = 2x^2 + 5x + 2.$$



For many American dairy farmers, the 1980's were not good years. Although milk production per cow increased through computer management and breeding, the price of milk paid to dairy farmers decreased.



Goal 2

Polynomial Multiplication in Real Life

Example 6 The Plight of the Dairy Farmer

Dairies measure milk in 100-pound units. (A gallon of milk weighs about 8 pounds.) For 1980 through 1990, the average annual milk production, M , per dairy cow can be modeled by $M = 3t + 115$, where M is measured in hundreds of pounds and $t = 0$ represents 1980. The prices, p , paid to dairy farmers can be modeled by $p = -0.25t + 14.25$, where p is price in dollars per hundred pounds. Find a model for average annual revenue per cow. What can you conclude from the model?

Solution

Verbal Model

$$\text{Revenue per cow} = \text{100 pounds per cow} \cdot \text{Price per 100 pounds}$$

Labels

$$\text{Revenue per cow} = R$$

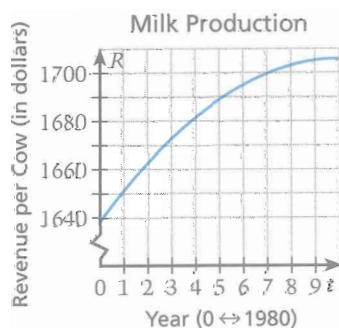
$$\text{100 pounds per cow} = M = 3t + 115$$

$$\text{Price per 100 pounds} = p = -0.25t + 14.25$$

Equation

$$R = (3t + 115)(-0.25t + 14.25) \\ = -0.75t^2 + 14t + 1638.75$$

From the graph, you can see that the average revenue per cow peaked in about 1989. For many dairy farmers, the increased revenue was not sufficient to offset the increasing costs of machinery, fertilizers, and feed. (Source: U.S. Dept. of Agriculture)



Communicating about ALGEBRA

Cooperative Learning

SHARING IDEAS about the Lesson

Work with a Partner and Make a Model Construct area models similar to Example 5 for each polynomial product.

A. $(x + 3)(x + 2)$

B. $(x + 1)(3x + 1)$

EXERCISES

Guided Practice

CRITICAL THINKING about the Lesson

1. Show how the Distributive Properties can be used to multiply $(2x - 3)$ and $(x + 4)$.
2. Multiply: $(x + 1)(x^2 - x + 1)$. Explain your use of the Distributive Property.
3. Multiply: $(x - 3)(2x + 5)$.
4. What does FOIL represent?

Independent Practice

In Exercises 5–10, multiply.

5. $(3x - 7)(-2x)$
6. $3x^2(5x - x^3 + 2)$
7. $(-x)(2x^2 - 3x)$
8. $2x(3x^2 - 4x + 1)$
9. $4x^2(5x^3 - 2x^2 + x)$
10. $-x^2(6x^3 - 14x + 9)$

In Exercises 11–16, use the FOIL pattern to multiply.

11. $(3x - 2)(5x + 7)$
12. $(3x + 5)(2x + 1)$
13. $(x - 4)(x + 4)$
14. $(2x - 3)(x + 3)$
15. $(x - 5)(2x + 10)$
16. $(3x - 5)(2x + 1)$

In Exercises 17–22, use an area model (or algebra tiles) to multiply.

17. $(x + 1)(x + 5)$
18. $(x + 2)(x + 6)$
19. $(x + 1)(x + 2)$
20. $(3x + 1)(2x + 2)$
21. $(x + 2)(2x + 3)$
22. $(2x + 1)(x + 3)$

In Exercises 23–28, use the Distributive Property to multiply.

23. $(x - 3)(3x + 1)$
24. $(2x + 1)(3x + 1)$
25. $(3x^2 + x - 5)(2x - 1)$
26. $(2x^2 - 7x + 1)(4x + 3)$
27. $(x^2 + 9)(x^2 - x - 4)$
28. $(x + 3)(x^2 - 6x + 2)$

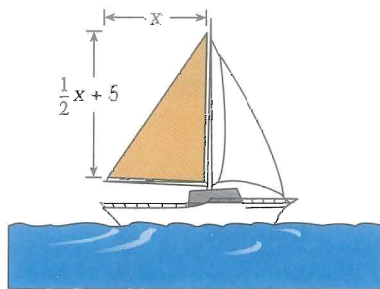
In Exercises 29–37, multiply.

29. $(x + 3)(x - 4)$
30. $(2x - 1)(x + 9)$
31. $(2x - 5)(x + 6)$
32. $(3x - 4)(\frac{1}{3}x + 1)$
33. $(x + \frac{6}{5})(4x - 5)$
34. $(x + \frac{1}{4})(x - \frac{5}{4})$
35. $(\frac{1}{2}x + 3)(\frac{1}{2}x - 2)$
36. $(-3x^2 + x - 1)(x + 3)$
37. $(x^2 + 4x - 9)(x - 4)$

38. **Area of a Sail** The base of a triangular sail is x feet and its height is $\frac{1}{2}x + 5$ feet. Find an expression for the area, A , of the sail.

39. Use the expression in Exercise 38 to complete the table.

Base, x	5	6	7	8	9	10
Area, A	?	?	?	?	?	?



Crop Circles In Exercises 40 and 41, use the following information.

In recent years, people in Britain were puzzled by the overnight appearance of "crop circles." Many were surrounded by concentric rings. Those shown below appeared in a wheat field in 1987 about three miles from Stonehenge, England.

40. The area (in square feet) of the circular ring of grain shown in the upper right corner of the photo is given by

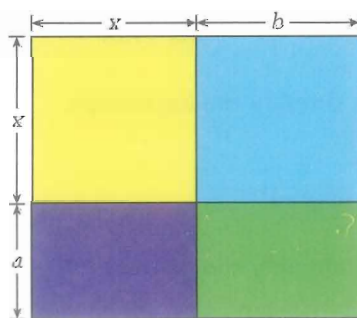
$$A = \pi(4x + 28)(4x - 28).$$

Simplify this expression. Substitute $x = 9$ to obtain the area of this circular ring in square feet.

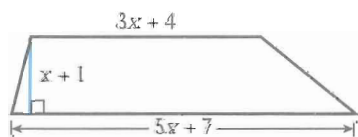
41. If you doubled the value of x , would the value of A double? Explain.



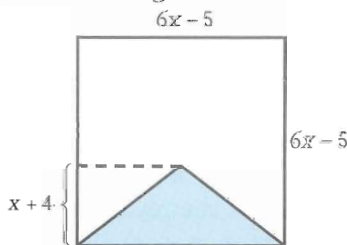
42. **FOIL Pattern** Find the area of the rectangle in two different ways: by multiplying length times width *and* by adding the areas of the smaller rectangles. Explain how this is related to the FOIL pattern for finding the product $(x + a)(x + b)$.



43. **College Entrance Exam Sample** $(x + 9)(x + 2) = \boxed{?}$
 a. $x^2 + 18$ b. $11x$ c. $x^2 + 11$ d. $x^2 + 11x + 18$ e. $9(x + 2) + 2(x + 9)$
44. **Geometry** Find an expression for the area of the trapezoid. (Area = $\frac{1}{2}h(b_1 + b_2)$)



45. **Geometry** Find an expression for the area of the triangle.



46. **Gross Domestic Product (GDP)** For 1980 through 1990, each American's dollar share, S , of the GDP can be modeled by

$$S = 900t + 11,800 \quad \text{dollar share per American}$$

where $t = 0$ represents 1980. The population, P (in millions), of the United States during the same time can be modeled by

$$P = 228 + 2.2t. \quad \text{number of Americans}$$

Find a model for the *total* Gross Domestic Product, T , from 1980 to 1990. Then use the three models to complete this table.

Year, t (1980 = 0)	0	5	10
Per capita GDP, S (dollars per American)	?	?	?
Population, P (millions of Americans)	?	?	?
Total GDP, T (millions of dollars)	?	?	?

(Source: Organization for Economic Cooperation and Development, Paris, France)

Integrated Review

In Exercises 47–50, rewrite the product in exponential form.

47. $(-3)(-3)(-3)(-3)$ 48. $(x)(x)(x)$ 49. $(-2)(-2)(x)(x)(x)$ 50. $(4)(4)(y)(y)$

In Exercises 51–54, rewrite the expression as repeated multiplication.

51. $(\frac{4}{5})^4$ 52. $(4.5)^5$ 53. $(x^2)^3$ 54. $(y^2)^2$

In Exercises 55–58, simplify the expression.

55. $2(x - 4) + 5x$ 56. $4(3 - y) + 2(y + 1)$
 57. $-3(z - 2) - (z - 6)$ 58. $(u - 2) - 3(2u + 1)$

In Exercises 59–62, simplify the expression.

59. $\frac{x^4}{3x^{-3}}$ 60. $\frac{6x^{-1}}{2x}$ 61. $\frac{5x^2y^0}{xy^3}$ 62. $\frac{x^3y^3}{x^0y}$

Exploration and Extension

In Exercises 63 and 64, multiply.

63. $(x + 1)(2x - 3)(x - 7)$ 64. $(3x + 7)(x + 4)(2x - 3)$

65. **Patterns** Multiply: $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$. Find a pattern in your results. Use the pattern to guess the result of multiplying $(x - 1)$ and $(x^4 + x^3 + x^2 + x + 1)$. Then verify your guess by multiplying.

Mixed REVIEW

- What is 22% of \$4,242.00?
- What is the reciprocal of x^2y ? (2.7)
- Solve $6^{x+3} = (3 + 3)^2$. (8.2)
- Solve $x^2 + 4x + 4 = 0$. (9.4)
- Evaluate $x^2 - 5x - 7$ when $x = -1$. (1.3)
- Evaluate $(0.9)(3.2)^t$ when $t = 2$. (1.3)
- Is 3 a solution of $3x + 2^x \geq 15$? (1.5)
- Is $(5, -3)$ a solution of $x + 2 = |-3y|$? (2.1)
- Write 0.00794 in scientific notation. (8.4)
- Find the slope of $2x - 5y = 10$. (4.3)
- What is $\frac{1}{4}$ of $\frac{2}{3}$?
- What is the reciprocal of $-\frac{1}{2}$? (2.7)
- Solve $3x^2 - 2x - 1 = 0$. (9.4)
- Solve $y^{x+3} = y^7$ for x . (8.1)
- Evaluate $|x + 2| + 3x$ when $x = -4$. (2.1)
- Evaluate $(7.29)(6.2)^{-t}$ when $t = 1$. (8.2)
- Is -4 a solution of $-y + 7y^2 - 14 < 18$? (1.5)
- Is $(-4, -1)$ a solution of $2^x < (2^y)^2$? (8.1)
- Write 3.29×10^4 in decimal form. (8.4)
- Find the intercepts of $y = 3x - 2$. (4.3)

Milestones

LEONHARD EULER (1707–1783)

1650

1700

Euler born

1750

Euler at
St. Petersburg
Academy of Science

Euler heads Academy of Science, Berlin

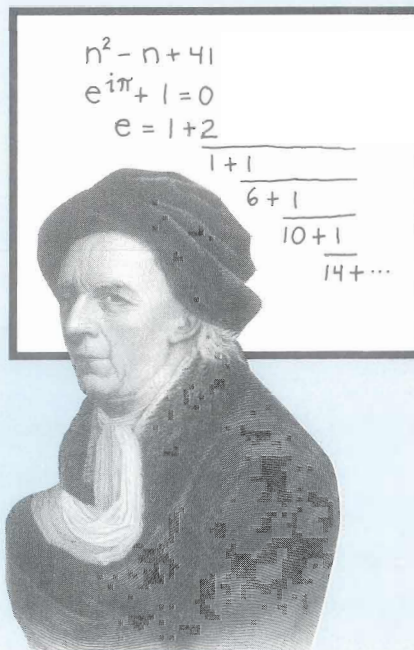
1800

1850

When Leonhard Euler entered the University of Basel in Switzerland, he began studying for the clergy but, encouraged by one of his teachers, soon switched to mathematics. He received his master's degree at the age of sixteen. His arrival in Russia in 1727 coincided with the death of Czarina Catherine I and the beginning of a regime unfavorable to scientific pursuits. Later political change brought a more favorable climate for research, and he began teaching at the newly founded St. Petersburg Academy of Science.

Euler made significant contributions to the study of number theory, geometry, and calculus. A prolific writer, he published 866 papers on mathematical topics. He introduced the symbol i for imaginary numbers, e for the base of natural logarithms, and $f()$ for functions. He was one of the first to adopt the use of the symbol π . He was also a pioneer in the field of topology. Although totally blind the last 17 years of his life, he continued his work in mathematics. An unassuming man, he said, "The path that I followed will be of some help perhaps."

Name a significant event in the history of the United States that occurred during Euler's lifetime.



Leonhard Euler

10.3

Multiplying Polynomials: Two Special Cases

What you should learn:

Goal 1 How to use special-product patterns for the product of a sum and difference and for the square of a binomial

Goal 2 How to use special-product patterns in real-life models

Why you should learn it:

You can multiply some polynomials very efficiently if they have a special form.

Goal 1 Using Special-Product Patterns

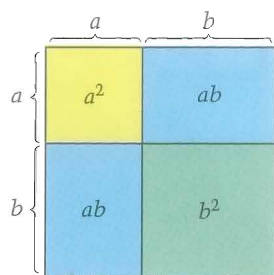
Some binomial products occur so frequently in algebra that it is worth remembering special patterns for them. For instance, the product $(x + 3)(x - 3)$ is called the *product of the sum and difference of two terms*. This special product has no “middle term.”

$$\begin{aligned}(x + 3)(x - 3) &= x^2 - 3x + 3x - 9 && \text{“Sum and difference”} \\ &= x^2 - 9 && \text{No middle term}\end{aligned}$$

Another common type of product is the *square of a binomial*. In its pattern, the middle term is always twice the product of the two terms in the binomial.

$$\begin{aligned}(2x + 5)^2 &= (2x + 5)(2x + 5) && \text{Square of a binomial} \\ &= 4x^2 + 10x + 10x + 25 \\ &= 4x^2 + 20x + 25 && \text{Middle term is } 2(2x)(5).\end{aligned}$$

These two patterns are generalized in the following summary. When you use these special-product patterns, remember that a and b can be numbers, variables, or even variable expressions.



Connections
Geometry

Special Products

Sum and Difference Pattern

Example

$$(a + b)(a - b) = a^2 - b^2 \quad (3x - 4)(3x + 4) = 9x^2 - 16$$

Square of a Binomial Pattern

Example

$$\begin{aligned}(a + b)^2 &= a^2 + 2ab + b^2 && (x + 4)^2 = x^2 + 2(x)(4) + 4^2 \\ & && = x^2 + 8x + 16 \\ (a - b)^2 &= a^2 - 2ab + b^2 && (x - 6)^2 = x^2 - 2(x)(6) + 36 \\ & && = x^2 - 12x + 36\end{aligned}$$

You don't need to write the steps that can be performed with mental math.

The square-of-a-binomial pattern $(a + b)^2 = a^2 + 2ab + b^2$ can be verified with the area model shown at the left. The two rectangles with areas ab produce the middle term $2ab$.

Example 1 The Product of the Sum and Difference of Two Terms

Multiply: $(5t - 2)(5t + 2)$.

Solution This special product represents the sum and difference of two terms. The product has the form $(a - b)(a + b) = a^2 - b^2$.

$$\begin{aligned}(5t - 2)(5t + 2) &= (5t)^2 - 2^2 \\ &= 25t^2 - 4\end{aligned}$$

Example 2 Squaring a Binomial

Multiply: $(2x - 7)^2$.

Solution This special product represents the square of a binomial. The product has the form $(a - b)^2 = a^2 - 2ab + b^2$. Note that the *middle* term of the product is twice the product of the two terms of the binomial.

$$\begin{aligned}(2x - 7)^2 &= (2x)^2 - 2(2x)(7) + 7^2 \\ &= 4x^2 - 28x + 49\end{aligned}$$

Goal 2

Using Special Products in Real-Life Models



Example 3 Finding an Area

Find an expression for the area of the blue region shown at the left. Then evaluate the expression when x is equal to 2 inches; to 3 inches; and to 4 inches.

Solution

Verbal Model

$$\begin{array}{|c|c|c|c|c|} \hline \text{Area of} & & \text{Area of} & & \text{Area of} \\ \text{blue region} & = & \text{entire square} & - & \text{green region} \\ \hline \end{array}$$

Labels

$$\begin{array}{|c|c|c|} \hline \text{Area of blue region} = A & & (\text{sq. in.}) \\ \text{Area of entire region} = (x + 3)^2 & & (\text{sq. in.}) \\ \text{Area of green region} = (x + 1)(x - 1) & & (\text{sq. in.}) \\ \hline \end{array}$$

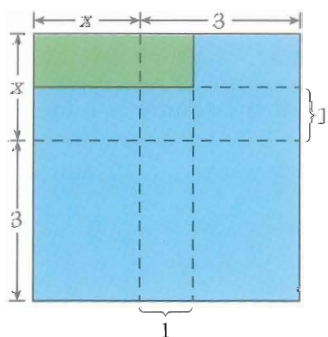
Equation

$$\begin{aligned}A &= (x + 3)^2 - (x + 1)(x - 1) \\ &= (x^2 + 6x + 9) - (x^2 - 1) \\ &= x^2 + 6x + 9 - x^2 + 1 \\ &= 6x + 10\end{aligned}$$

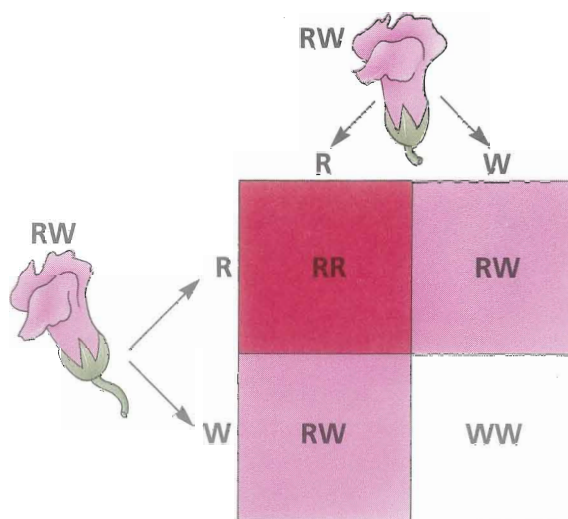
When $x = 2$ inches, $A = 6(2) + 10 = 22$ square inches.

When $x = 3$ inches, $A = 6(3) + 10 = 28$ square inches.

When $x = 4$ inches, $A = 6(4) + 10 = 34$ square inches.



A pink snapdragon has two genes—one white (W) and one red (R)—that determine its color. This Punnett square illustrates the possible results of crossing two pink snapdragons. Since each parent snapdragon passes along only one gene for color to an offspring (and pollination is at random), the snapdragon offspring will be approximately 25% red (RR), 50% pink (RW), and 25% white (WW).



Example 4 Using Punnett Squares

Show how the product of two binomials can be used to model the Punnett square shown above.

Solution Each of the two pink “parent” snapdragons have half red genes and half white genes. You can model the genetic makeup of each parent as $50\% R + 50\% W = 0.5R + 0.5W$. When the two parents are crossed, the genetic makeup of the offspring can be modeled by the product $(0.5R + 0.5W)^2$.

$$\begin{aligned}
 (0.5R + 0.5W)^2 &= (0.5R)^2 + 2(0.5R)(0.5W) + (0.5W)^2 \\
 &= \underbrace{0.25R^2}_{\text{Red}} + \underbrace{0.5RW}_{\text{Pink}} + \underbrace{0.25W^2}_{\text{White}}
 \end{aligned}$$

Thus, 25% of the offspring should be red, 50% should be pink, and 25% should be white. ■

Communicating about ALGEBRA

► SHARING IDEAS about the Lesson

Extend the Concept Is the trinomial the square of a binomial? Explain your reasoning.

A. $x^2 - 4x + 4$

B. $2x^2 - 6x + 9$

C. $4x^2 + 24x + 36$

EXERCISES

Guided Practice

CRITICAL THINKING about the Lesson

- True or False?** The product of $(a - b)$ and $(a - b)$ is $a^2 - b^2$. Explain.
- Find the missing term:
 $(a + b)^2 = a^2 + \boxed{?} + b^2$.
- Write two expressions for the area of a square whose sides are each $x - 4$.
- Give an example of each of the three types of special products in this lesson.

Independent Practice

In Exercises 5–10, use an area model (or algebra tiles) to write the square as a trinomial.

- $(x + 2)^2$
- $(x + 3)^2$
- $(2n + 1)^2$
- $(3a + 2)^2$
- $(2x + 2)^2$
- $(3x + 1)^2$

In Exercises 11–16, write the square as a trinomial.

- $(n + 6)^2$
- $(x + 4)^2$
- $(2x + 1)^2$
- $(2m - 3)^2$
- $(3t - 2)^2$
- $(x - 9)^2$

In Exercises 17–22, multiply.

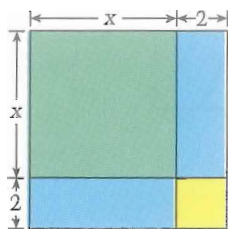
- $(x + 5)(x - 5)$
- $(x - 2)(x + 2)$
- $(2x - 2)(2x + 2)$
- $(5x - 6)(5x + 6)$
- $(a + 2b)(a - 2b)$
- $(4x - 7y)(4x + 7y)$

In Exercises 23–28, write the square as a trinomial.

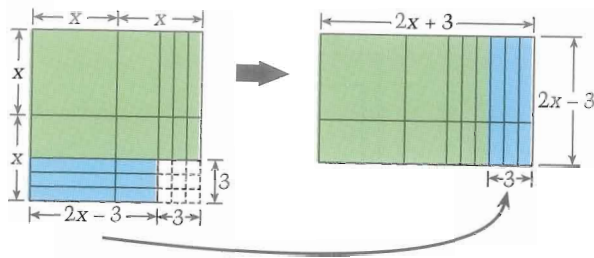
- $(x + 6)^2$
- $(x + 10)^2$
- $(a - 2)^2$
- $(2x - 5)^2$
- $(2x - 5y)^2$
- $(4s + 3t)^2$

Area Model (or Algebra Tiles) In Exercises 29 and 30, write two different expressions for the area of the figure. Describe the special-product pattern that is represented.

29.



30.



31. **Albino Tigers** In tigers, the normal color gene, C , is *dominant* and the albino color, A , is *recessive*. This means that tigers whose color genes are CC , CA , or AC will have normal coloring. A tiger whose color genes are AA will be an albino. The Punnett square at the right shows the possible results of crossing two tigers that have recessive albino genes. What percent of the offspring of two such tigers will be the normal color? What percent will be albino? Use the model

$$\begin{aligned}(0.5C + 0.5A)^2 \\ = 0.25CC + 0.5CA + 0.25AA\end{aligned}$$

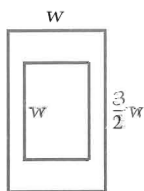
to answer the question.

32. **Frizzle-Feathered Chickens** In chickens, neither the normal-feathered gene, F , nor the frizzle-feathered gene, f , is dominant. This means that chickens whose feather genes are FF will have normal feathers. Chickens with Ff or fF will have mildly frizzled feathers. Chickens with ff will have extremely frizzled feathers. The Punnett square at the right shows the possible results of crossing two chickens with mildly frizzled feathers. What percent of the offspring of two such chickens will have normal feathers? What percent will have mildly frizzled feathers? What percent will have extremely frizzled feathers? Use the model

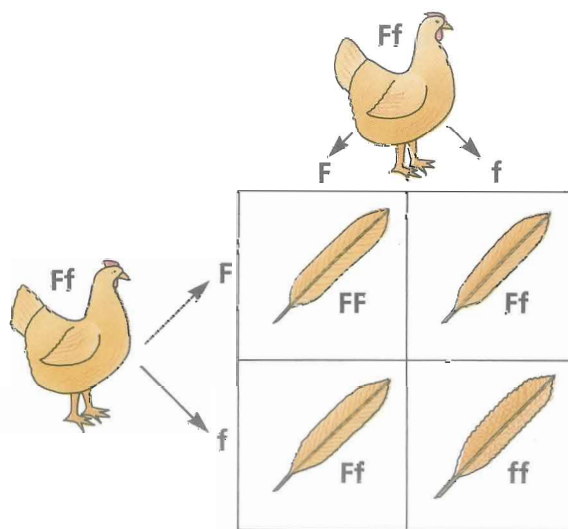
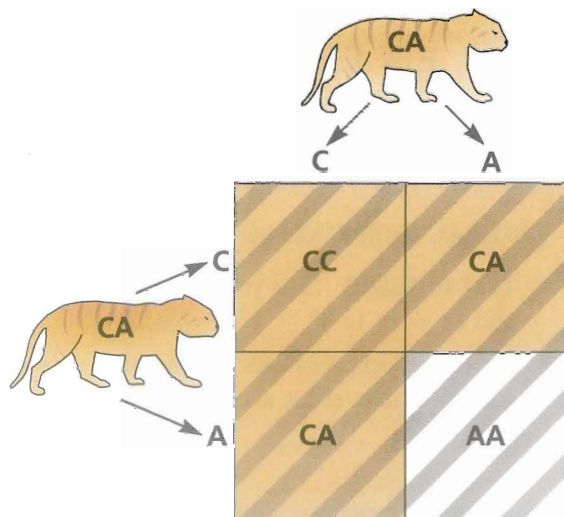
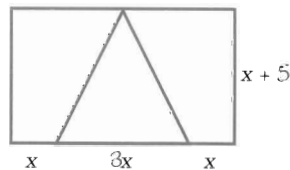
$$\begin{aligned}(0.5F + 0.5f)^2 \\ = 0.25FF + 0.5Ff + 0.25ff\end{aligned}$$

to answer the question.

33. **Geometry** The ratio of the height and width of the smaller rectangle is equal to the ratio of the height and width of the larger rectangle. Find expressions for the perimeters and areas of both.



34. **Geometry** Find the area of the rectangle and the area of the triangle.



35. Compounded Interest After two years, an investment of \$1000 compounded annually at an interest rate, r , will grow to the amount $1000(1 + r)^2$ in dollars. Write this product as a trinomial.

36. Compounded Interest After two years, an investment of \$2000 compounded annually at an interest rate of 6% will have a dollar balance of

$$2000(1 + 0.06)^2 = 2000 + 2000(0.12) + 2000(0.0036).$$

Evaluate each side of this equation and compare your results.

37. Common Errors Verify that $(a + b)^2 \neq a^2 + b^2$ by letting $a = 3$, $b = 4$, and evaluating both expressions.

38. Common Errors Verify that $(a - b)^2 \neq a^2 - b^2$ by letting $a = 5$, $b = 4$, and evaluating both expressions.

Integrated Review

In Exercises 39–44, evaluate the expression.

39. $(x + 5)^2$ when $x = 4$

40. $(2x - 4)^2$ when $x = 1$

41. $(-3x + 5)^2$ when $x = 0$

42. $(x + 3)^2$ when $x = -6$

43. $(\frac{1}{4}x + 9)^2$ when $x = -40$

44. $(6x - 5)^2$ when $x = \frac{1}{3}$

In Exercises 45–50, simplify the expression.

45. $\frac{x^2y}{xy^2}$

46. $\frac{4x^6y^8}{2y^2}$

47. $\frac{9x^5y^3}{3x^2y}$

48. $4x - (x + 5)(x - 5)$

49. $(4 - x)^2 + 8$

50. $(x + 2)^2 - x^2 + 9$

51. College Entrance Exam Sample If $x = -3$, then $(x + 3)^2 - 3x = \boxed{?}$
a. -6 b. -3 c. 0 d. 6 e. 9

52. College Entrance Exam Sample For what positive value of x does $\frac{4}{x} = \frac{x}{16}$?
a. 2 b. 8 c. 20 d. 32 e. 64

53. Estimation—Powers of Ten Which of the following is the best estimate for the capacity of an automobile's fuel tank?
a. 1.5 gallons b. 15 gallons
c. 150 gallons d. 1500 gallons

54. Estimation—Powers of Ten Which of the following is the best estimate for the number of fourteen-year-olds in the United States?
a. 4×10^3 b. 4×10^4
c. 4×10^5 d. 4×10^6

Exploration and Extension

55. Find the product of $(a + b)(a + b)(a - b)$ by two methods.
a. First step: Find the product of the sum and difference of two terms.
b. First step: Find the square of the binomial. (Compare your results.)
c. Which method do you prefer? Explain.

56. Geometry Use an area model to find the product: $(a + b + c)^2$.

10.4

Factoring: Special Products

What you should learn:

Goal 1 How to factor polynomials that have a monomial factor, that are the difference of two squares, and that are perfect-square trinomials

Goal 2 How to use factoring in real-life models

Why you should learn it:

You can use factoring to help answer questions about polynomial models such as those used for geometric situations.

Goal 1 Factoring Polynomials

In this lesson and in the next lesson, you will learn strategies for factoring polynomials. **Factoring** is the *reverse* process of multiplying.

Part of factoring is being able to find the **greatest common factor** of two or more terms. For example, $2x$ is the greatest common factor of the terms of $(4x^2 + 6x)$. Writing this polynomial as the product $2x(2x + 3)$ illustrates one type of factoring. The factor $2x$ is a **monomial factor**. The Distributive Property is used to “factor out” a monomial factor.

Polynomial	Monomial Factor	Factored Form
$3x + 6$	3	$3(x + 2)$
$5x^2 - 15x$	$5x$	$5x(x - 3)$
$4x^2 + 6x + 8$	2	$2(2x^2 + 3x + 4)$

Each of the special product patterns that you studied in Lesson 10.3 can be used to factor polynomials.

Factoring Special Products

Difference of Two Squares Pattern

$$a^2 - b^2 = (a + b)(a - b) \qquad 9x^2 - 16 = (3x + 4)(3x - 4)$$

Example

Perfect Square Trinomial Pattern

$$\begin{aligned} a^2 + 2ab + b^2 &= (a + b)^2 & x^2 + 8x + 16 &= (x + 4)^2 \\ a^2 - 2ab + b^2 &= (a - b)^2 & x^2 - 12x + 36 &= (x - 6)^2 \end{aligned}$$

Example

Note that *perfect square trinomials* come in two forms. The coefficient of the middle term can be positive or negative.

To recognize the **difference of two squares**, look for coefficients that are squares and for variables that are raised to the second power.

Polynomial	Difference of Squares	Factored Form
$x^2 - 4$	$x^2 - 2^2$	$(x + 2)(x - 2)$
$4x^2 - 25$	$(2x)^2 - 5^2$	$(2x + 5)(2x - 5)$
$25 - 49x^2$	$5^2 - (7x)^2$	$(5 + 7x)(5 - 7x)$

To recognize a **perfect square trinomial**, remember that the first and last terms must be positive and perfect squares, a^2 and b^2 , and the middle term must be twice the product of a and b .

Example 1 Factoring Perfect Square Trinomials

- $x^2 - 4x + 4 = x^2 - 2(2x) + 2^2$
 $= (x - 2)^2$
- $16y^2 + 24y + 9 = (4y)^2 + 2(4y)(3) + 3^2$
 $= (4y + 3)^2$
- $9t^2 - 30t + 25 = (3t)^2 - 2(3t)(5) + 5^2$
 $= (3t - 5)^2$

Before trying to apply one of the special product patterns for factoring, factor out any common monomial factor.

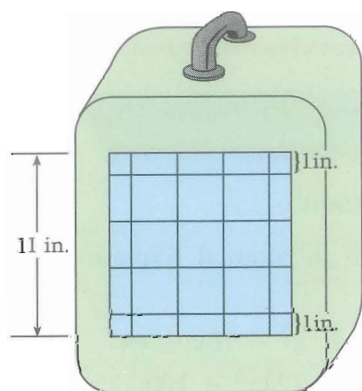
Example 2 Removing a Common Monomial Factor First

- $3x^2 - 27 = 3(x^2 - 9)$
 $= 3(x + 3)(x - 3)$
- $3x^2 - 30x + 75 = 3(x^2 - 10x + 25)$
 $= 3(x - 5)^2$
- $16y^3 + 80y^2 + 100y = 4y(4y^2 + 20y + 25)$
 $= 4y(2y + 5)^2$

Goal 2

Using Factoring in Real-Life Models

Connections Geometry



Example 3 Finding Dimensions

You are shopping for a travel kennel. Your new puppy can just barely squeeze through a square opening that is $3\frac{1}{2}$ inches on each side. Are the grille openings in the kennel at the left too large to hold your puppy?

Solution Let each of the large grille openings be x inches on each side. The grille, then, forms 9 large squares with an area of x^2 each, 12 rectangles with an area of x each, and 4 small squares with an area of 1 each. By factoring,

$$9x^2 + 12x + 4 = (3x + 2)^2$$

you find that each side of the grille is $3x + 2$. By solving the equation $3x + 2 = 11$, you find x to be 3 inches. So your pup will be secure.



Finding integers that represent the sides of a right triangle is an ancient problem. Right-triangle triples, or Pythagorean triples, such as 3, 4, 5 ($3^2 + 4^2 = 5^2$) and 5, 12, 13 ($5^2 + 12^2 = 13^2$) occur in Babylonian cuneiform tablets.

Example 4 Right-Triangle Triples

Here is a technique for finding right-triangle triples.

Solution

1. Choose a positive integer and square it: $7^2 = 49$.
2. Factor into two odd factors or two even factors:
 $7^2 = (1)(49)$.
3. Write the product as the difference of two squares:

$$(1)(49) = (25 - 24)(25 + 24) = 25^2 - 24^2.$$

Average of factors
Average of factors
↓
↓
↑
↑
Difference between average and either factor
Difference between average and either factor

4. Write in Pythagorean form:

$$7^2 = 25^2 - 24^2$$

$$7^2 + 24^2 = 25^2 \quad \text{Pythagorean form, } a^2 + b^2 = c^2$$

The triple is $a = 7$, $b = 24$, and $c = 25$. ■

Communicating about ALGEBRA

► SHARING IDEAS about the Lesson

Recognize Patterns Factor each polynomial. Explain your steps and reasoning.

A. $9t^2 - 49$

B. $x^2 - 16x + 64$

C. $4y^2 + 36y + 81$

D. $121x^2 - 100$

EXERCISES

Guided Practice

CRITICAL THINKING about the Lesson

- Describe the relationship between multiplying polynomials and factoring a polynomial.
- Factor out the greatest common monomial factor: $3x^3 - 6x^2 + 9$.
- Show how the Distributive Property can be used to factor $2x(2x - 3) + 3(2x - 3)$.
- Give an example of each of the three special-product factoring patterns in the lesson.

Independent Practice

In Exercises 5–10, find the greatest common factor of the given terms.

- $6x^5, 30x^4, 12x^3$
- $7x^3, 28x, 14x^4$
- $24x^3, 32x^2$
- $99x^6, 45x^3$
- $16x^2y, 84xy^2, 36x^2y^2$
- $10xy^2, 25x^2y^2, 80x^2y$

In Exercises 11–19, factor out the greatest common monomial factor.

- $2x^2 - 4$
- $3x + 6$
- $4a - 12$
- $14z^3 + 21$
- $24x^2 - 18$
- $-a^3 - 4a$
- $21u^2 - 14u$
- $36y^4 + 24y^2$
- $4x^2 - 8x + 8$

In Exercises 20–28, factor the expression.

- $x^2 - 64$
- $y^2 - 144$
- $2x^2 + 16x + 32$
- $9x^2 - 30xy + 25y^2$
- $4y^2 + 20yz + 25z^2$
- $u^2 - \frac{1}{16}$
- $v^2 - \frac{9}{25}$
- $81 - (z + 5)^2$
- $3(x - 3)^2 - 12$

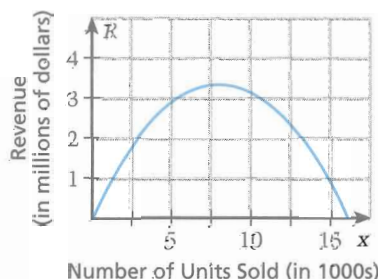
In Exercises 29 and 30, use the following information.

A manufacturer of television sets has modeled the revenue, R , for Model TXX to be

$$R = 800x - 0.05x^2 = xp$$

where x is the number of units sold, and p is the price of Model TXX. The graph of the model is shown at the right.

Revenue from Sale of Model TXX



- Economists use a principle that states that "as the price decreases, the demand increases." Explain how this principle applies to the graph of the price model.
- If you were the manager of the marketing department, what price would you recommend for each television set? Explain your reasoning.

Right-Triangle Triples In Exercises 31–33, use the technique described in Example 4 to find the indicated right-triangle triple. (Exercise 33 has two solutions.)

31. $4^2 + b^2 = c^2$ 32. $6^2 + b^2 = c^2$ 33. $8^2 + b^2 = c^2$

34. **A Jade Pi** The *jade pi* (pronounced “jade bee”) at the right has an inside radius of 5 centimeters and an outside radius of 30 centimeters. Find the area of one of the flat surfaces of the pi.



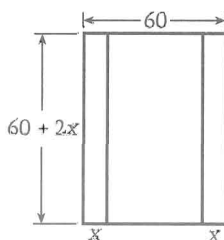
Early Chinese emperors used ring-shaped disks of jade in ceremonies to appeal to celestial spirits. The circular shape symbolized Heaven.

35. **Chemical Reaction** The rate of change of some chemical reactions can be modeled by

$$kQx - kx^2$$

where Q is the amount of the original substance, x is the amount of the substance formed, and k is a constant of proportionality. Factor this rate-of-change expression.

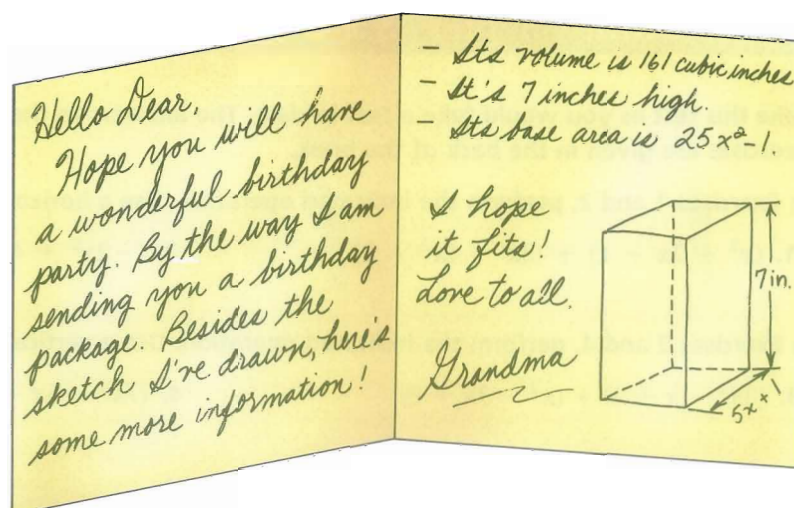
36. **Marching Band** Your band is asked to be in a local parade. It is allotted 3200 square feet in the parade and must stay at least x feet from each curb. Your director decides that rows will be 4 feet apart and each row will contain 8 band members. Can all of the 160 members of the band be in the parade?



It takes musicianship, teamwork, and plenty of practice to maintain correct marching ranks, while still producing a well-balanced sound.

37. Package from Grandma

A week before your birthday you receive this card from your grandmother, a retired math teacher. Will her package fit in your mailbox, which is 9 inches by 5 inches by 4 inches, or will it be returned to the post office, where you will have to pick it up later?



Integrated Review

In Exercises 38–43, multiply.

38. $12x(2x - 3)$

39. $7y(4 - 3y)$

40. $t(t^2 + 1) - t(t^2 - 1)$

41. $2z(z + 5) - 7(z + 5)$

42. $(11 - x)(11 + x)$

43. $(6r + 5s)(6r - 5s)$

In Exercises 44–49, solve the equation.

44. $(x + 2)^2 - x^2 = 8$

45. $(x - 1)^2 + 2x = 4$

46. $(x - 4)^2 + 8x = 32$

47. $(x - 6)^2 - x^2 = 0$

48. $(x + 3)(x - 3) + 5 = 0$

49. $(x - 4)(x + 4) = -7$

50. **Up, Up, and Away** The height (in feet) of a bottle rocket is modeled by

$$h = -16t^2 + 57t$$

where t is the time in seconds. Find the height of the rocket after 2 seconds.

51. **Pythagorean Theorem** The lengths of the sides of a right triangle are $a = 5$, b , and $c = 13$. What is b ?

Exploration and Extension

Geometry In Exercises 52 and 53, find the perimeter of the rectangle and square. The lengths of the sides are obtained by factoring the expression for the area.

52. Area = $x^2 - 49$



53. Area = $x^2 + 6x + 9$



Mid-Chapter SELF-TEST

Take this test as you would take a test in class. The answers to the exercises are given in the back of the book.

In Exercises 1 and 2, perform the indicated operation. Use a horizontal format. (10.1)

1. $(x^2 + 2x - 1) + (3x - 4x^2 + 2)$

2. $(-2x^2 + 4x - 5) - (3 - 7x + x^2)$

In Exercises 3 and 4, perform the indicated operation. Use a vertical format. (10.1)

3. $(3x^2 - x + 2) + (x^2 - 3x + 2)$

4. $(7x^2 - 5x + 10) - (3x^2 + 2x + 5)$

In Exercises 5–8, multiply. (10.2)

5. $(3x + 2)(3x + 5)$

6. $(4x - 5)(2x + 10)$

7. $(x + 1)(2x^2 - 3x + 2)$

8. $(2x + 1)(5x^2 + 7x - 3)$

In Exercises 9–12, multiply using the FOIL pattern. (10.2)

9. $(x + 2)(x - 3)$

10. $(x - 3)(x - 6)$

11. $(2x + 3)(4 - x)$

12. $(3 + x)(4 + x)$

In Exercises 13–16, use the special-product patterns to multiply. (10.3)

13. $(2x + 3)^2$

14. $(4x - 5)^2$

15. $(2x - 6)(2x + 6)$

16. $(x - 7)(x + 7)$

In Exercises 17–22, factor the polynomial. (10.4)

17. $4x^2 - 49$

18. $3x^2 - 108$

19. $x^2 - 2x + 1$

20. $7x^2 + 42x + 63$

21. $9x^2 + 12x + 4$

22. $x^2 - 169$

23. From 1970 to 1990, the number of pieces, F , of first-class U.S. mail can be modeled by

$$F = 50,262 + 99.2t^2 \text{ (in millions)}$$

where $t = 0$ represents 1970. The number of pieces, K , of all other types of U.S. mail can be modeled by

$$K = 33,045 + 140.7t^2 \text{ (in millions)}$$

Find a model for the number of pieces, M , of all types of U.S. mail.

(Source: Statistical Abstract, 1990) (10.1)

24. Three storage cubes have sides of $(x + 1)$, $(x + 3)$, and $(x + 5)$.

Write an expression for the total volume of the three cubes. (10.2)

25. You are making a tick-tack-toe board for your cousin's birthday. Each square will have a side of x inches, and the board will have a 1-inch-wide border. Write a polynomial for the area of the board. (10.3)

26. You want to frame a 16-inch by 20-inch watercolor for your parents' anniversary. The frame is x inches wide on all four sides. Write a polynomial for the area of the framed picture. (10.2)

10.5

Factoring Quadratic Trinomials

What you should learn:

Goal 1 How to factor a quadratic trinomial or recognize that it cannot be factored

Goal 2 How to use factoring in real-life models

Why you should learn it:

You can use factoring as an efficient step in solving some types of mathematical problems.

Goal 1 Factoring a Quadratic Trinomial

Let's look at quadratic trinomials of the form $x^2 + bx + c$. Try covering the factored forms in the left column. Can you find the factored form from the trinomial form?

Factored Form	F	O	I	L	Trinomial Form
$(x + 2)(x + 4)$	$=$	x^2	$+ 4x$	$+ 2x$	$+ 8 = x^2 + 6x + 8$
$(x - 3)(x - 2)$	$=$	x^2	$- 2x$	$- 3x$	$+ 6 = x^2 - 5x + 6$
$(x - 1)(x + 4)$	$=$	x^2	$+ 4x$	$- x$	$- 4 = x^2 + 3x - 4$

To see how to factor this type of quadratic trinomial, consider the following.

$$\begin{array}{rcl}
 (x + r)(x + s) & = & x^2 + \overbrace{(r + s)x}^{\text{Sum of terms}} + \overbrace{rs}^{\text{Product of terms}} \\
 & = & x^2 + \underbrace{\quad}_{b}x + \underbrace{\quad}_{c}
 \end{array}$$

Notice that you must find factors of c whose sum is b . If c is positive, the factors must have *like* signs. If c is negative, the factors must have *unlike* signs.

Example 1 Factoring When c Is Positive

Factor $x^2 + 3x + 2$.

Solution For this trinomial, $b = 3$ and $c = 2$.

$$\begin{aligned}
 x^2 + 3x + 2 &= (x + \boxed{?})(x + \boxed{?}) && \text{Think: You need factors of 2 whose sum is 3.} \\
 &= (x + 1)(x + 2) && 2 = (1)(2), 1 + 2 = 3
 \end{aligned}$$

Example 2 Factoring When c Is Positive

Factor $x^2 - 5x + 6$.

Solution For this trinomial, $b = -5$ and $c = 6$.

$$\begin{aligned}
 x^2 - 5x + 6 &= (x + \boxed{?})(x + \boxed{?}) && \text{Think: You need factors of 6 whose sum is } -5. \\
 &= (x - 2)(x - 3) && 6 = (-2)(-3), \\
 & && -2 - 3 = -5
 \end{aligned}$$

Example 3 Factoring When c Is Negative

Factor $x^2 - 2x - 8$.

Solution For this trinomial, $b = -2$ and $c = -8$.

$$\begin{aligned}x^2 - 2x - 8 &= (x + \boxed{?})(x + \boxed{?}) \\ &= (x + 2)(x - 4)\end{aligned}$$

Think: You need factors of -8 whose sum is -2 .
 $-8 = (2)(-4)$, $2 - 4 = -2$ ■

To factor a trinomial whose leading coefficient is not 1, consider the following pattern.

$$ax^2 + bx + c = (\boxed{?}x + \boxed{?})(\boxed{?}x + \boxed{?})$$

Factors of a Factors of c

The goal is to find a combination of factors of a and c so that the outer and inner products add to the middle term bx .

Example 4 Factoring When Leading Coefficient Is Not 1

Factor $6x^2 + 7x - 5$.

Solution For this trinomial, $a = 6$, $b = 7$, and $c = -5$. Use a guess-and-check strategy to find the binomial factors.

Guess	Check
$(x + 1)(6x - 5) = 6x^2 + x - 5$	
$(x - 1)(6x + 5) = 6x^2 - x - 5$	
$(2x + 1)(3x - 5) = 6x^2 - 7x - 5$	
$(2x - 1)(3x + 5) = 6x^2 + 7x - 5$	← Correct factoring. ■

It is important to remember that *many* quadratic trinomials cannot be factored with integer coefficients. A quadratic trinomial $ax^2 + bx + c$ will factor (using integer coefficients) only if the *discriminant* $b^2 - 4ac$ is a perfect square.

Example 5 Using the Discriminant

Show that $2x^2 + 3x - 6$ cannot be factored.

Solution For this trinomial, $a = 2$, $b = 3$, and $c = -6$. Since the discriminant

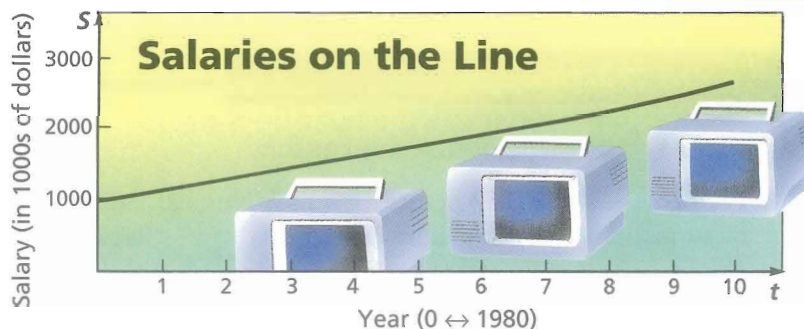
$$b^2 - 4ac = 3^2 - 4(2)(-6) = 57$$

is not a perfect square, the trinomial cannot be factored (using integer coefficients). ■

Problem Solving
Guess and Check

Goal 2
Using Factoring in Real-Life Models

For 1980 to 1990, the annual salary, S (in 1000s of dollars), paid by a manufacturer to its assembly-line workers is given by the quadratic model $S = 3t^2 + 130t + 1000$, where $t = 0$ represents 1980.


Example 6 Finding the Salary per Employee

In 1980, the above manufacturer had 100 assembly-line workers. Each year the number of workers increased by 3. Find a model for the workers' average annual salary from 1980 to 1990.

Solution Because the number of workers increased by 3 each year, the number of workers in year t is given by the linear model $3t + 100$.

Problem Solving
Write an Equation

Verbal Model	Total salary = Number of workers \times Average salary		
Labels	Year = t		(0 is 1980)
	Total salary = $3t^2 + 130t + 1000$		(\$1000s)
	Number of workers = $3t + 100$		(workers)
	Average salary = W		(\$1000s per worker)
Equation	$3t^2 + 130t + 1000 = (3t + 100)W$		

By factoring, you can see that

$$3t^2 + 130t + 1000 = (3t + 100)(t + 10)$$

which means the average salary was $W = t + 10$. From this model, you can see that the average salary was \$10,000 in 1980. For the next 10 years, it increased by \$1000 each year. ■

Communicating about ALGEBRA

SHARING IDEAS about the Lesson

Recognize Patterns Factor each polynomial. Explain your steps and reasoning.

A. $x^2 + 7x + 12$ B. $a^2 - 6a + 5$ C. $3n^2 + 4n - 4$

EXERCISES

Guided Practice

CRITICAL THINKING about the Lesson

- Factor $x^2 - 4x + 3$. When testing possible factorizations, why is it unnecessary to test $(x - 1)(x + 3)$ and $(x + 1)(x - 3)$?
- Factor $x^2 + 2x - 3$. When testing possible factorizations, why is it unnecessary to test $(x - 1)(x - 3)$ and $(x + 1)(x + 3)$?
- What is the discriminant of $ax^2 + bx + c$?
- If the discriminant of $ax^2 + bx + c$ is 35, can the trinomial be factored with integer coefficients? Explain.

Independent Practice

In Exercises 5–10, choose the correct factorization. (If neither is correct, find the correct factorization.)

- | | | |
|--|---|---|
| 5. $x^2 + x - 20$
a. $(x - 4)(x + 5)$
b. $(x + 4)(x - 5)$ | 6. $x^2 + 8x + 16$
a. $(x + 2)(x + 8)$
b. $(x + 4)(x + 4)$ | 7. $x^2 - 10x + 24$
a. $(x - 6)(x - 4)$
b. $(x - 12)(x + 2)$ |
| 8. $3x^2 - 7x - 6$
a. $(x - 3)(3x + 2)$
b. $(x + 3)(3x - 2)$ | 9. $6x^2 - 7x - 5$
a. $(6x + 1)(x - 5)$
b. $(2x + 1)(3x - 5)$ | 10. $2x^2 - 7x - 9$
a. $(x - 1)(2x + 9)$
b. $(2x - 1)(x + 9)$ |

In Exercises 11–28, factor the trinomial.

- | | | |
|----------------------|-----------------------|-----------------------|
| 11. $x^2 + 3x - 4$ | 12. $x^2 - 5x + 6$ | 13. $x^2 + 3x - 18$ |
| 14. $y^2 - 16y - 36$ | 15. $x^2 - 10x + 24$ | 16. $x^2 + 13x + 22$ |
| 17. $x^2 + 15x + 50$ | 18. $y^2 + 30y + 216$ | 19. $y^2 - 35y + 300$ |
| 20. $t^2 - 4t - 21$ | 21. $3x^2 + 8x + 5$ | 22. $6x^2 + 5x - 4$ |
| 23. $2x^2 - x - 21$ | 24. $3x^2 + 11x + 10$ | 25. $48 - 16y + y^2$ |
| 26. $32 + 12x + x^2$ | 27. $2x^2 - x - 6$ | 28. $5 + 34x - 7x^2$ |

In Exercises 29–34, use the discriminant to decide whether the polynomial can be factored with integer coefficients. If it can be factored, then find the factors.

- | | | |
|-----------------------|------------------------|-----------------------|
| 29. $12x^2 - 11x + 3$ | 30. $2x^2 - 5x - 12$ | 31. $6x^2 - 10x + 4$ |
| 32. $10x^2 - 9x + 6$ | 33. $14x^2 - 19x - 40$ | 34. $24x^2 + 3x - 11$ |
35. **Geometry** The area of a rectangle is given by $A = x^2 + 4x - 5$. Find expressions for possible lengths and widths of the rectangle.
36. **Geometry** The area of a circle is given by $A = \pi(4x^2 + 12x + 9)$. Find an expression for the radius of the circle.

The Art of Africa In Exercises 37–46, use the following information.

The letters of the alphabet from A to Z (excluding W and X) are represented by nonzero integers from -12 to 12 . Copy and complete the table by factoring the polynomials in Exercises 37–42. Then use the table to match the coded words in Exercises 43–46 with one of the pieces of African art.

Letter	A	B	C	D	E	F	G	H	I	J	K	L
Code Number	1	?	?	?	9	?	?	?	2	?	?	?
Letter	M	N	O	P	Q	R	S	T	U	V	Y	Z
Code Number	3	?	?	?	6	?	?	?	11	?	?	?

37. $8x^2 - 35x + 12 = (Ax + B)(Cx + D)$

38. $90x^2 - 143x + 56 = (Ex + F)(Gx + H)$

39. $10x^2 - 57x + 54 = (Ix + J)(Kx + L)$

40. $12x^2 - 43x + 10 = (Mx + N)(Ox + P)$

41. $42x^2 - 89x + 22 = (Qx + R)(Sx + T)$

42. $132x^2 - 199x + 60 = (Ux + V)(Yx + Z)$

43.

2	-12	4	-11	12		3	1	7	5
---	-----	---	-----	----	--	---	---	---	---

44.

8	1	-11	-12	9	-3		7	-2	4	4	-6
---	---	-----	-----	---	----	--	---	----	---	---	----

45.

-1	1	-2	-2	9	-11	-10	9	-3		-12	9	7	7	9	-6
----	---	----	----	---	-----	-----	---	----	--	-----	---	---	---	---	----

46.

3	1	7	1	2		-10	9	8	5	-6	1	8	9
---	---	---	---	---	--	-----	---	---	---	----	---	---	---

a. Ivory Coast



b. Central Africa



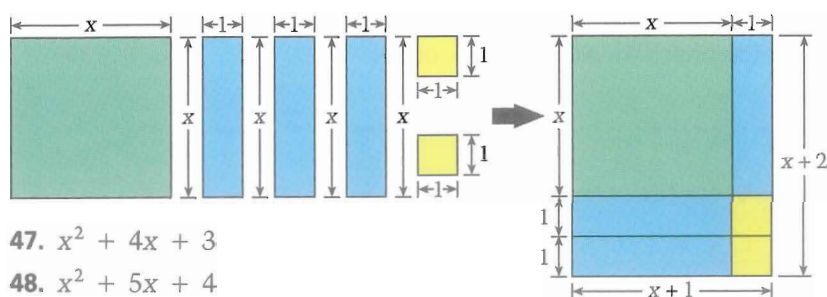
c. Central Africa



d. Kenya



Algebra Tiles In Exercises 47 and 48, factor the trinomial. Use an area model to illustrate your result. Use the following area model for $x^2 + 3x + 2 = (x + 1)(x + 2)$ as a sample.



47. $x^2 + 4x + 3$

48. $x^2 + 5x + 4$

49. **Hot Dog Stand** You run a hot-dog stand for the adult softball league. Your revenue, R (in dollars), each week can be given by

$$R = \frac{1}{100}(-40t^2 + 740t + 1200)$$

where t represents the week with $t = 0$ for the first week. In the first week, you sold 12 hot dogs and each week after that the number of sales increased by 8. Find a model for the price of a hot dog over the 8-week season. Use the model to find the price of a hot dog during each week of the season.

50. **Summer Basketball** During the summer, the number, B , of people who played intramural basketball each week is given by the model

$$B = t^2 + 11t + 30$$

where t represents the week, with $t = 0$ for the first week. In the first week, six teams were in the league. Each week, for five weeks, a new team joined the league. Find a model for the average number of members per team for the six-week season. Use the model to find the average number of players on each team during each week.

Integrated Review

In Exercises 51–53, multiply.

51. $-2y(y + 1)$

52. $(x + 4)^2$

53. $(v - 1)(v - 6)$

In Exercises 54–56, solve the equation.

54. $3(x + 2) = 5$

55. $-1(4x - 5) = x$

56. $5(y - 3) = -1(y + 6)$

Exploration and Extension

In Exercises 57–60, factor the polynomial as the product of *linear* factors.

57. $2x^3 - 5x^2 - 3x$

58. $48x^3 - 2x^2 - 20x$

59. $x^4 - 5x^2 + 4$

60. $x^4 - 13x^2 + 36$

10.6

Solving Quadratic Equations by Factoring

What you should learn:

Goal 1 How to use factoring to solve a quadratic equation

Goal 2 How to use factoring with real-life models

Why you should learn it:

You can use factoring to help efficiently solve some quadratic equations.

Problem Solving
Solve a Simpler Problem

Goal 1 Solving by Factoring

The **Zero-Product Property** states that if the product of two factors is zero, then one (or both) of the factors must be zero.

Zero-Product Property

If the product $ab = 0$, then $a = 0$ or $b = 0$.

This property connects factoring to solving equations. For instance, to solve the equation $(x - 2)(x + 3) = 0$, you can use the Zero-Product Property to conclude that either $x - 2 = 0$ or $x + 3 = 0$.

Set first factor equal to 0.

$$\begin{aligned}x - 2 &= 0 \\x &= 2\end{aligned}$$

Set second factor equal to 0.

$$\begin{aligned}x + 3 &= 0 \\x &= -3\end{aligned}$$

The equation $(x - 2)(x + 3) = 0$ has the solutions 2 and -3 .

Factoring and the Zero-Product Property allow you to solve a quadratic equation by converting it into two *linear* equations, which you already know how to solve. This is a common strategy of algebra—to break down a problem into simpler parts, each solved by previously learned methods.

Example 1 Solving a Quadratic Equation by Factoring

Solve $x^2 - x - 12 = 0$.

Solution

$$\begin{aligned}x^2 - x - 12 &= 0 \\(x + 3)(x - 4) &= 0 && \text{Factor.} \\x + 3 = 0 \text{ or } x - 4 = 0 && \text{Zero-Product Property} \\x = -3 \text{ or } x = 4 && \text{Solve for } x.\end{aligned}$$

The equation has two solutions: -3 and 4 . Check these solutions in the original equation. ■

To use the Zero Product Property, the quadratic equation should be written in the standard form, $ax^2 + bx + c = 0$.

Example 2 Rewriting in Standard Form First

Solve $3x^2 + 5x = 12$.

Solution

$$\begin{array}{ll} 3x^2 + 5x = 12 & \text{Rewrite original equation.} \\ 3x^2 + 5x - 12 = 0 & \text{Write in standard form.} \\ (3x - 4)(x + 3) = 0 & \text{Factor.} \\ 3x - 4 = 0 & \text{Set factors equal to 0.} \\ \text{or } x + 3 = 0 & \\ x = \frac{4}{3} & \\ \text{or } x = -3 & \text{Solve for } x. \end{array}$$

The equation has two solutions: $\frac{4}{3}$ and -3 . Check these solutions in the original equation. ■

In Examples 1 and 2, each equation had two solutions. Some quadratic equations have only one solution. This occurs when the quadratic is a perfect square trinomial.

Example 3 A Quadratic Equation with One Solution

Solve $x^2 - 6x + 11 = 2$.

Solution

$$\begin{array}{ll} x^2 - 6x + 11 = 2 & \text{Rewrite original equation.} \\ x^2 - 6x + 9 = 0 & \text{Write in standard form.} \\ (x - 3)^2 = 0 & \text{Factor.} \\ x - 3 = 0 & \text{Set factor equal to 0.} \\ x = 3 & \text{Solve for } x. \end{array}$$

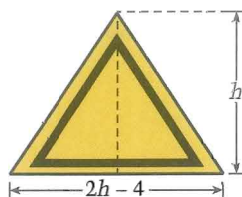
The equation has only one solution: 3. Check this solution in the original equation. ■

Be sure you see that the Zero Product Property can only be applied to a product that is equal to zero. For instance, you *cannot* conclude from the equation $x(x - 1) = 6$ that $x = 6$ or $x - 1 = 6$. Instead, you must first write the equation in standard form and then factor the left side.

$$\begin{array}{l} x(x - 1) = 6 \\ x^2 - x = 6 \\ x^2 - x - 6 = 0 \\ (x - 3)(x + 2) = 0 \end{array}$$

From the factored form, the solutions are 3 and -2 .

Problem Solving
Write an Equation



Example 4 Finding the Dimensions of a Sign

A triangular sign has a base that is to be 4 feet less than twice its height. A local zoning ordinance restricts the area of signs to no more than 24 square feet. Find the base and height of the largest triangular sign that meets the zoning ordinance.

Solution Since the base is to be 4 feet less than twice the height, let h be the height and label the base as $2h - 4$.

Verbal Model

$$\frac{1}{2} \times \text{Base} \times \text{Height} = \text{Area}$$

Labels

$$\text{Height} = h \quad (\text{feet})$$

$$\text{Base} = 2h - 4 \quad (\text{feet})$$

$$\text{Area} = 24 \quad (\text{square feet})$$

Equation

$$\frac{1}{2}(2h - 4)(h) = 24$$

$$h^2 - 2h = 24$$

$$h^2 - 2h - 24 = 0$$

$$(h + 4)(h - 6) = 0$$

$$h = -4 \quad \text{or} \quad h = 6$$

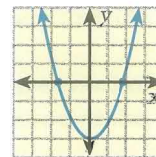
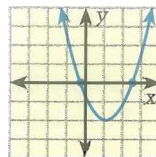
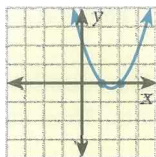
The equation has two solutions: -4 and 6 . But a negative height makes no sense, so you can conclude that the height of the sign is 6 feet, and the base is $2(6) - 4 = 8$ feet. ■

Communicating about ALGEBRA

► **SHARING IDEAS** about the Lesson

Choose a Method You have now studied three ways to solve quadratic equations: by finding square roots, by the quadratic formula, and by factoring. Solve each equation by the method you think is most efficient. Discuss the reasons for your choices and explain how the graphs can be used as a check of the solutions.

A. $x^2 - 3x + 2 = 0$ B. $x^2 - 2x - 1 = 0$ C. $x^2 - 3 = 0$



EXERCISES

Guided Practice

CRITICAL THINKING about the Lesson

1. Use the Zero-Product Property to complete the statement. If $ab = 0$, then $\boxed{?}$.
2. Solve the equation: $(x - 2)(x + 1) = 0$.
3. Solve the equation: $3x^2 + 4x = 0$.
4. Which two numbers satisfy the statement, "The sum of a number and its square is zero."?
5. **True or False?** If $(5x - 1)(x + 3) = 1$, then $5x - 1 = 1$ or $x + 3 = 1$. Explain.
6. **True or False?** If $(x + 3)(x - 3) = 0$, then $x + 3 = 0$ or $x - 3 = 0$. Explain.

Independent Practice

In Exercises 7–10, solve the equation.

7. $(x + 1)(x + 2) = 0$
8. $(x - 3)(x + 7) = 0$
9. $(x + 3)(x + 4) = 0$
10. $(x + 6)(x - 5) = 0$

In Exercises 11–16, solve the equation by factoring.

11. $x^2 + 5x - 6 = 0$
12. $3x^2 + 11x - 4 = 0$
13. $2x^2 + 5x + 3 = 0$
14. $6x^2 + 13x + 5 = 0$
15. $3x^2 + 7x + 2 = 0$
16. $12x^2 - 5x - 3 = 0$

In Exercises 17–24, match the equation with its solutions.

- | | | | |
|------------------------|------------------------|------------------------|------------------------|
| 17. $x^2 - 5x + 6 = 0$ | 18. $x^2 + 5x + 6 = 0$ | 19. $x^2 - 7x + 6 = 0$ | 20. $x^2 + 7x + 6 = 0$ |
| 21. $x^2 - 5x - 6 = 0$ | 22. $x^2 + 5x - 6 = 0$ | 23. $x^2 + x - 6 = 0$ | 24. $x^2 - x - 6 = 0$ |
| a. $-1, -6$ | b. $-2, -3$ | c. $1, -6$ | d. $-1, 6$ |
| e. $2, -3$ | f. $2, 3$ | g. $1, 6$ | h. $-2, 3$ |

In Exercises 25–33, solve the equation by finding square roots, by the quadratic formula, or by factoring.

- | | | |
|---------------------|----------------------|--------------------------|
| 25. $x(x - 9) = 0$ | 26. $2y(y + 6) = 0$ | 27. $y^2 - 7y + 6 = -6$ |
| 28. $x^2 - 12 = -3$ | 29. $x^2 - 8x = -16$ | 30. $x^2 + 4x + 7 = 3$ |
| 31. $4x^2 + 2x = 0$ | 32. $4y^2 - 18y = 0$ | 33. $x^2 - 12x + 40 = 4$ |

In Exercises 34 and 35, multiply both sides of the equation by an appropriate power of ten to obtain integer coefficients. Then solve by factoring.

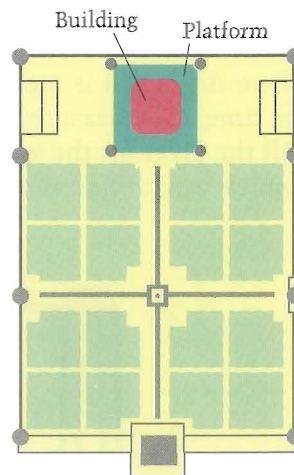
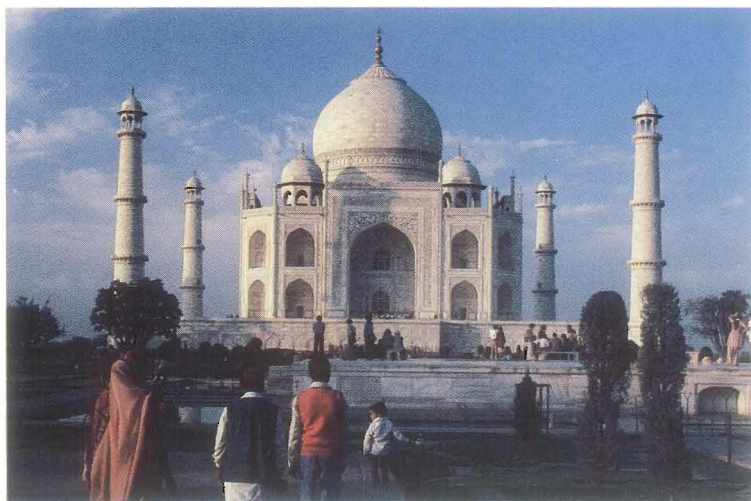
34. $0.8y^2 + 3.2x + 2.4 = 0$
35. $0.23x^2 - 0.54x + 0.16 = 0$

36. **How to Weigh an Elephant Seal** The rectangular scale in the photo is 8 feet longer than it is wide. The scale has an area of 33 square feet. What are the dimensions of the scale?
37. **A California Raisin** During the breeding season, the male elephant seal in the photo lost 38% of his weight. His weight, w (in hundreds of pounds), at the beginning of the season was a solution of $w^2 - 38w + 361 = 0$. What did he weigh at the end of the season?



To weigh this male elephant seal, a biologist at the University of California lured him to cross a special scale by pulling a model female seal named Raisin. (Source: Discover, 1991)

The Taj Mahal In Exercises 38 and 39, use the following information.



The Taj Mahal was built by the Indian ruler Shah Jahan as a tomb for his wife who died in 1629. The building is constructed of white marble and sits on a platform of red sandstone. Each corner of the platform has a minaret (tower) that is 133 feet high.

38. The platform is about 140 feet wider than the main building. The total area of the platform is about 102,400 square feet. Find the dimensions of the platform and the base of the building. (Assume each is a square.)
39. Which of the following is the best estimate of the dimensions of the entire courtyard shown in the diagram?
- a. 640 feet \times 1280 feet b. 960 feet by 1280 feet c. 640 feet \times 1600 feet

Integrated Review

In Exercises 40–45, multiply and simplify.

40. $(2x + 6)(\frac{1}{2}x - 3)$

41. $(x + 9)(4x - 3)$

42. $36 - (x + 2)(x - 3)$

43. $3(x - \frac{1}{3})(3x + 7)$

44. $4(x + 3)(3x - 1) + 1$

45. $5 + (2x - \frac{1}{2})(\frac{1}{2}x + 1)$

In Exercises 46–51, decide whether the number is a solution of the equation.

46. $x^2 - 3x + 4 = 0$; 2

47. $x^2 + 5x + 4 = 0$; -1

48. $\frac{1}{3}x^2 + 2x - 36 = 0$; 6

49. $2x^2 - 3x - 4 = 0$; 2

50. $4x^2 + 3x - 27 = 0$; -3

51. $\frac{1}{2}x^2 - x + 8 = 0$; 4

In Exercises 52–57, use the quadratic formula to solve the equation.

52. $x^2 + 4x + 3 = 0$

53. $x^2 - 7x + 9 = 0$

54. $2x^2 - 5x + 3 = 0$

55. $3x^2 - 11x - 24 = 0$

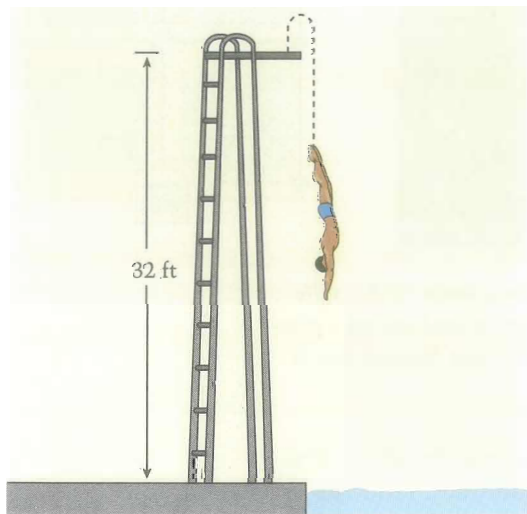
56. $5x^2 - 16x - 12 = 0$

57. $9x^2 - 17x + 6 = 0$

58. **Height of a Diver** A diver jumps from a diving board that is 32 feet above the water. The height of the diver is given by

$$\text{Height} = -16(t - 2)(t + 1)$$

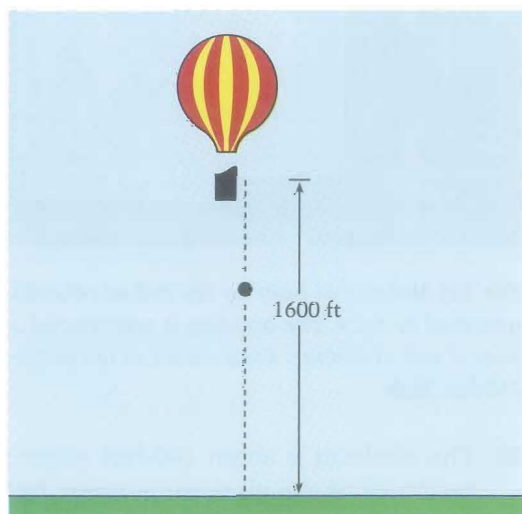
where the height is measured in feet, and the time, t , is measured in seconds. When will the diver hit the water? Can you see a quick way to find the answer? Explain.



59. **Balloon Drop** An object is dropped from a hot-air balloon 1600 feet above the ground. The height of the object is given by

$$\text{Height} = -16(t - 10)(t + 10)$$

where the height is measured in feet, and the time, t , is measured in seconds. When will the object hit the ground? Can you see a quick way to find the answer? Explain.

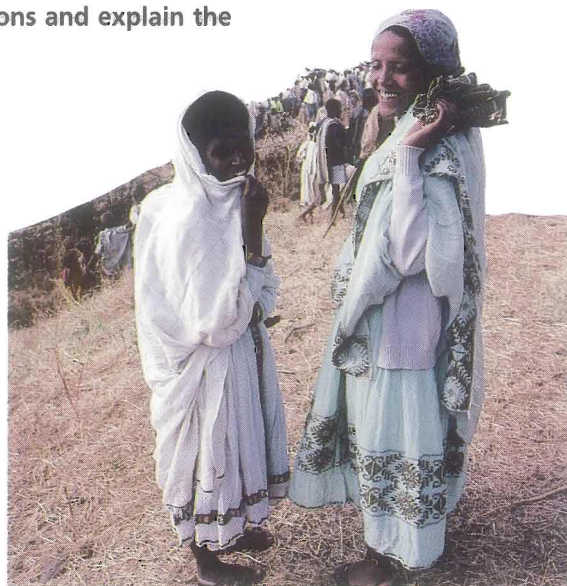


60. **Graphing Calculator** Sketch $y = x^2 + 5x - 6$ and $y = (x + 6)(x - 1)$ on the same screen. What do you notice?
61. **Graphing Calculator** Sketch $y = x^2 - 7x - 8$ and $y = (x - 8)(x + 1)$ on the same screen. What do you notice?

Exploration and Extension

Tutoring a Friend In Exercises 62–65, you are tutoring a friend and want to create some quadratic equations that can be solved by factoring. Find a quadratic equation that has the given solutions and explain the procedure you used to obtain the equation.

62. 4 and -3 63. 5 and 5
 64. -2 and -5 65. 0 and -1
66. Let a and b be real numbers such that $a \neq 0$. Find the solutions of $ax^2 + bx = 0$.
67. Let a be a nonzero real number. Find the solutions of $ax^2 - ax = 0$.
68. **Ethiopian Weavers** In every square inch of the cotton fabric used for a shammash, the warp (lengthwise threads) intersects the weft (crosswise threads) 5000 times. The density (number of threads per inch) of the weft threads is twice that of the warp threads. How many weft threads are in each inch? How many warp threads are in each inch?



These Ethiopian women are wearing a fine cotton fabric known as a shammash, a national costume that is often edged with bright trimmings.

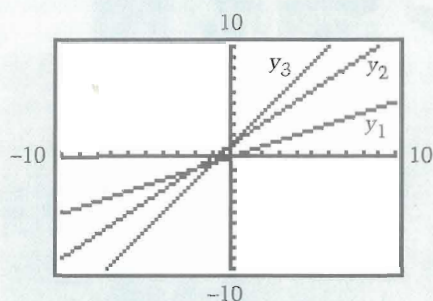
Mixed REVIEW

- Simplify $x^2y^{-1} \div 3x \cdot y^2$. (8.3)
- Find the quotient of $3x^2$ and $9x$. (8.3)
- Solve $4m + 2n = 3$ for m . (3.6)
- Write $2 \cdot x \cdot x \cdot x \cdot x$ in exponential form. (1.3)
- Evaluate $(2 \times 10^{-3}) \cdot (4 \times 10^4)$. (8.4)
- Solve $|x + 3| \leq 4$. (4.8)
- What are the square roots of 64? (9.1)
- Sketch the graph of $2x + y = 6$. (4.4)
- Is -6 a solution of $2^{-x} < -7x$? (8.2)
- Solve the system. $\begin{cases} x - 7y = -17 \\ 3x + y = -7 \end{cases}$ (7.2, 7.3)
- Simplify $(p^2q^4 \div 2p) \cdot (4p^{-2} \div q^{-2})$. (8.2)
- Find the sum of $2p$ and $-4p$. (2.6)
- Solve $ab + a = 5$ for a . (3.6)
- Write 0.00012 in scientific notation. (8.4)
- Evaluate $(3.2 \times 10^6) \div (1.6 \times 10^{-1})$. (8.4)
- Solve $2|3 - x| > 1$. (4.8)
- What are the square roots of x^2 ? (9.1)
- Sketch the graph of $2|x + 2| + 3 = 0$. (4.7)
- Is $(-1, -2)$ a solution of $y = -|2x|$? (2.1)
- Solve the system. $\begin{cases} 4x + 3y = -1 \\ 5x - 6y = 28 \end{cases}$ (7.2, 7.3)



USING A GRAPHING CALCULATOR

Graphing calculators can be used to obtain a graphic interpretation of polynomial addition or subtraction. In the first graph below, notice that for any x -value, the sum of y_1 and y_2 is equal to y_3 . In the second graph, for any x -value, the difference of y_1 and y_2 is equal to y_3 . Keystroke instructions for doing this on a TI-82, Casio fx-9700GE, and Sharp EL-9300C are listed on page 743.



y_3 is the sum of y_1 and y_2 .

First Graph

$$y_1 = \frac{1}{2}x$$

$$y_2 = x + 1$$

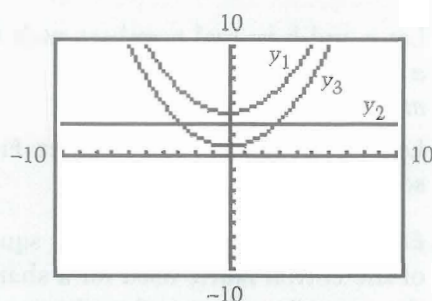
$$y_3 = \frac{3}{2}x + 1$$

Second Graph

$$y_1 = \frac{1}{4}x^2 + 4$$

$$y_2 = 3$$

$$y_3 = \frac{1}{4}x^2 + 1$$



y_3 is the difference of y_1 and y_2 .

Real Life Agriculture

Comparing Grain Consumptions From 1980 through 1994, the annual amount y (in pounds) of wheat and rice eaten by Americans can be modeled as follows.

$$y_1 = 2t + 114 \quad \text{Wheat}$$

$$y_2 = 0.75t + 8 \quad \text{Rice}$$

In the model $t = 0$ represents 1980. Use a graphing calculator to graph each model. Then, on the same screen, sketch the graph of the sum of the two models. What does the sum represent?

Exercises

In Exercises 1–3, let y_3 be the sum of y_1 and y_2 . Sketch the graph of all three equations.

1. $y_1 = x - 1$, $y_2 = \frac{1}{2}x + 2$

2. $y_1 = x^2 + x$, $y_2 = 1$

3. $y_1 = \frac{1}{2}x^2 - 1$, $y_2 = \frac{1}{2}x^2 + 1$

In Exercises 4–6, let y_3 be the difference of y_1 and y_2 . Sketch the graph of all three equations.

4. $y_1 = 2x + 1$, $y_2 = x + 1$

5. $y_1 = x^2 + 3$, $y_2 = \frac{1}{2}x^2$

6. $y_1 = \frac{1}{3}x^2 + x$, $y_2 = 1$

10.7

Solving Quadratic Equations by Completing the Square

What you should learn:

Goal How to solve quadratic equations by completing the square

Why you should learn it:

You can complete the square to help efficiently solve some quadratic equations.

Goal Solving by Completing the Square

In this lesson, you will learn to rewrite a quadratic equation in *completed-square form* by **completing the square**.

LESSON INVESTIGATION



Investigating Completed Square Form

Partner Activity Complete the following solution.

$$x^2 + 8x + 5 = 0$$

Original equation

$$x^2 + 8x = -5$$

Subtract 5 from both sides.

$$x^2 + 8x + ? = -5 + ?$$

Add ? to both sides.

$$(x + ?)^2 = ?$$

Perfect square form

$$x = ?$$

Solutions

In the third step, how do you know what quantity to add to both sides? Check your conjecture by solving $x^2 - 6x + 5 = 0$ in two ways: once by factoring and once by completing the square.

Study Tip

In Example 1, notice in the third step that the quantity that is added to both sides is the square of one half the coefficient of x .

Example 1 Completing the Square: Leading Coefficient is 1

Solve $x^2 - 6x + 7 = 0$ by completing the square.

Solution

$$x^2 - 6x + 7 = 0$$

Rewrite original equation.

$$x^2 - 6x = -7$$

Subtract 7 from both sides.

$$x^2 - 6x + (-3)^2 = -7 + 9$$

Add $(-3)^2$, or 9, to both sides.

$$(x - 3)^2 = 2$$

Binomial squared

$$x - 3 = \pm\sqrt{2}$$

Find square roots.

$$x = 3 \pm \sqrt{2}$$

Solve for x .

The equation has two solutions: $3 + \sqrt{2}$ and $3 - \sqrt{2}$. ■

If the leading coefficient of the quadratic is not 1, you should divide both sides of the equation by this coefficient *before* completing the square.

Example 2 **Completing the Square: Leading Coefficient Is Not 1**

Solve $2x^2 - x - 2 = 0$ by completing the square.

Solution

$$\begin{array}{ll}
 2x^2 - x - 2 = 0 & \text{Rewrite original equation.} \\
 2x^2 - x = 2 & \text{Subtract } -2 \text{ from both sides.} \\
 x^2 - \frac{1}{2}x = 1 & \text{Divide both sides by 2.} \\
 x^2 - \frac{1}{2}x + \left(-\frac{1}{4}\right)^2 = 1 + \frac{1}{16} & \text{Add } \left(-\frac{1}{4}\right)^2, \text{ or } \frac{1}{16}, \text{ to both sides.} \\
 \quad \quad \quad \uparrow & \\
 \quad \quad \quad \left(\text{half of } -\frac{1}{2}\right)^2 & \\
 \left(x - \frac{1}{4}\right)^2 = \frac{17}{16} & \text{Binomial squared} \\
 x - \frac{1}{4} = \pm \frac{\sqrt{17}}{4} & \text{Find square roots.} \\
 x = \frac{1}{4} \pm \frac{\sqrt{17}}{4} & \text{Solve for } x.
 \end{array}$$

The equation has two solutions, $x = \frac{1}{4} + \frac{\sqrt{17}}{4}$ and $x = \frac{1}{4} - \frac{\sqrt{17}}{4}$. As a check, you can use the quadratic formula on the original equation. ■

You have studied five different methods for solving quadratic equations.

Summary of Methods for Solving $ax^2 + bx + c = 0$

Method	Lesson	Comments
Finding Square Roots	9.2	Efficient way to solve $ax^2 + c = 0$.
Graphing	9.3	Can be used for <i>any</i> quadratic equation, but gives only approximate solutions.
Using Quadratic Formula	9.4	Can be used for <i>any</i> quadratic equation. Always gives exact solutions.
Factoring	10.5	Efficient way to solve equation <i>if</i> quadratic can be factored easily.
Completing the Square	10.6	Can be used for <i>any</i> quadratic equation, but is best suited for quadratics with $a = 1$ and b an even number.



A deposit of \$6000 was put into a savings account that paid annual interest of $r\%$, compounded yearly. After two years, the balance in the account was \$6933.75.



Example 3 Finding the Annual Interest Rate

Find the annual interest rate for the above deposit.

Solution

Known
Formula

$$A = P(1 + r)^t$$

Labels

Number of years $= t = 2$ (years)

Principal (original deposit) $= P = 6000$ (dollars)

Balance $= A = 6933.75$ (dollars)

Annual interest rate $= r$ (decimal form)

Equation

$$6933.75 = 6000(1 + r)^2$$

$$1.156 = (1 + r)^2$$

$$1.075 = 1 + r$$

$$0.075 = r$$

Quadratic model

Divide both sides by 6000.

Take positive square roots.

Subtract 1 from both sides.

The annual interest rate is 7.5%. (Note that the only operation that was necessary to write this equation in completed square form was to divide both sides by 6000.)

Communicating about ALGEBRA

SHARING IDEAS about the Lesson

Choose a Method Solve each equation by the most efficient method. Explain the reasons for your choices.

A. $x^2 - 6x + 8 = 0$ B. $x^2 - 6 = 0$ C. $x^2 - 6x = 0$

D. $x^2 - 6x - 8 = 0$ E. $2(x - 3)^2 = 12$ F. $2x^2 - 6x - 8 = 0$

EXERCISES

Guided Practice

CRITICAL THINKING about the Lesson

- Which is a perfect square trinomial?
a. $x^2 - 8x + 8$ b. $x^2 - 8x + 16$
c. $x^2 - 8x + 64$
- Solve $x^2 - 4x = 8$ by completing the square. Solve the same equation by the quadratic formula. Explain the difference in the results.
- What term must be added to $x^2 + 6x$ to create a perfect square trinomial?
- Name the five methods for solving a quadratic equation.

Independent Practice

In Exercises 5–10, find the term that must be added to the expression to create a perfect square trinomial.

- | | | |
|----------------|---------------|----------------|
| 5. $x^2 - 18x$ | 6. $x^2 + 6x$ | 7. $x^2 + 12x$ |
| 8. $x^2 - 10x$ | 9. $x^2 - 7x$ | 10. $x^2 - 5x$ |

In Exercises 11–28, solve the equation by completing the square.

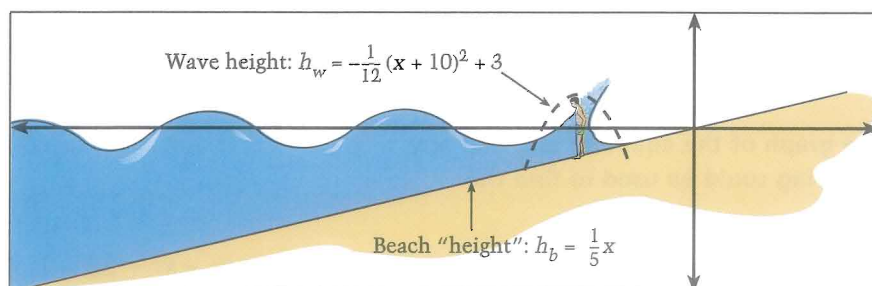
- | | | |
|----------------------------------|----------------------------------|-----------------------------------|
| 11. $x^2 + 10x - 11 = 0$ | 12. $x^2 + 14x - 15 = 0$ | 13. $y^2 - 24y + 63 = 0$ |
| 14. $y^2 - 8y + 12 = 0$ | 15. $t^2 + 3t - \frac{7}{4} = 0$ | 16. $y^2 + 9y + \frac{17}{4} = 0$ |
| 17. $x^2 - \frac{2}{3}x - 3 = 0$ | 18. $x^2 + \frac{4}{5}x - 1 = 0$ | 19. $x^2 + x - 1 = 0$ |
| 20. $1 + x - x^2 = 0$ | 21. $4y^2 + 4y - 9 = 0$ | 22. $3x^2 - 24x - 5 = 0$ |
| 23. $2x^2 - 6x - 15 = 5$ | 24. $5x^2 - 20x - 20 = 5$ | 25. $3x^2 + 4x + 4 = 3$ |
| 26. $4x^2 + 6x - 6 = 2$ | 27. $x^2 + 2x = 2$ | 28. $x^2 - 2x = 2$ |

In Exercises 29–43, use the most convenient method to solve the equation. Explain why you made your choice.

- | | | |
|--------------------------|-------------------------|--------------------------|
| 29. $x^2 - 3x - 1 = 0$ | 30. $4x^2 - 12 = 0$ | 31. $y^2 + 6y - 24 = 0$ |
| 32. $4x^2 - 25 = 0$ | 33. $x^2 + 7x + 10 = 0$ | 34. $u^2 + 5u + 2 = 0$ |
| 35. $3x^2 - 5x = 0$ | 36. $y^2 + 2y - 26 = 0$ | 37. $9z^2 + 10z - 4 = 0$ |
| 38. $4x^2 + 4x + 1 = 0$ | 39. $7x^2 - 14x = 0$ | 40. $4x^2 - 13x + 3 = 0$ |
| 41. $8x^2 - 10x + 3 = 0$ | 42. $7x^2 - 14 = 0$ | 43. $y^2 + 20y + 10 = 0$ |

44. **Money in the Bank** At your seventh grade graduation, you and your twin sister each received \$200. You each deposited the money in savings accounts that compound interest annually. Two years later your sister's deposit has grown by \$28.98. Your account is in a different bank that pays an interest rate that is 1% more than your sister receives. What is your balance after two years?

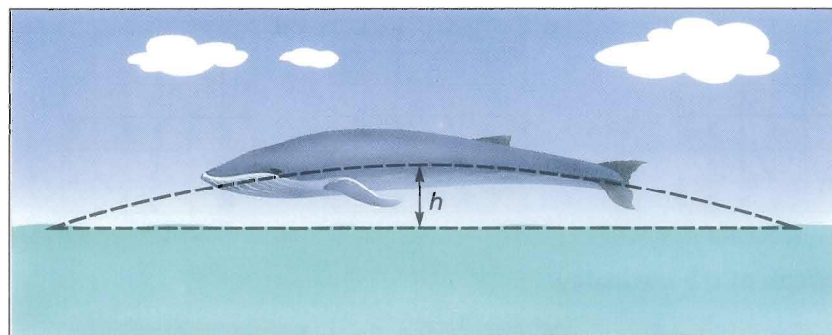
45. **Splash!** You and Jared are playing in the surf at the beach. Jared is 5 feet 4 inches tall and is standing at the point where the wave crests, as shown below. Did the wave go over his head? Explain.



46. **Whale Watching** At the beach you see an 80-foot whale jump above water. The path followed by the whale is given by the model

$$h = -\frac{1}{400}x^2 + \frac{4}{5}x - 52$$

where h is the height (in feet) above the sea and x is the horizontal distance (in feet) traveled by the whale. Sketch a graph of this equation. For how many horizontal feet did the whale travel over the water before reaching its maximum height?



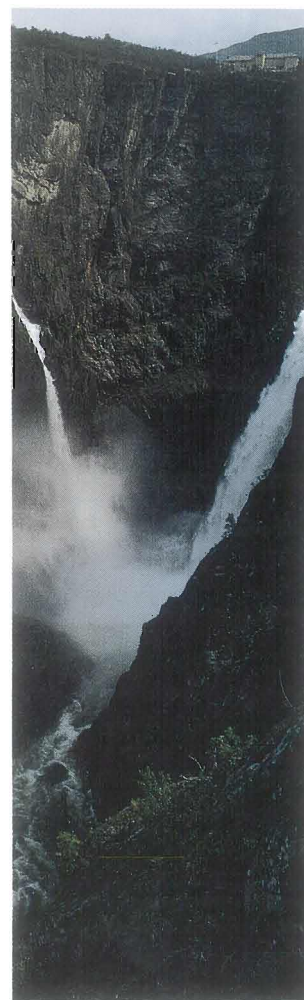
To attain a maximum thrust, a whale beats its tail down when a wave moves water upward (the front face of the wave) and beats its tail up when the wave moves water downward (the back face of the wave).

47. **Waterfall** The Vettisfoss waterfall falls over a vertical cliff. The path followed by the water as it flows over the Vettisfoss Falls can be modeled by

$$h = -\frac{1}{30}(x - 10)^2 + 900$$

where h is the height (in feet) above the lower river level, and x measures the horizontal distance (in feet) from the base of the cliff. How far from the base of the cliff does the water hit the river?

48. How long does it take the water to hit the lower river? (Use a vertical motion model. See page 474.)



Vettisfoss Falls, in Norway, is one of the most famous waterfalls in the world. The distance between the lower river and the top of the falls is 900 feet.

Integrated Review

In Exercises 49–54, solve the equation.

49. $x^2 = 16$

50. $x^2 + 3 = 7$

51. $x^2 + 4 = 29$

52. $\frac{1}{7}x^2 + 8 = 15$

53. $2x^2 - 7 = 11$

54. $3x^2 - 8 = 100$

In Exercises 55–60, sketch the graph of the equation and find any x -intercepts. Explain how factoring could be used to find the x -intercepts.

55. $y = x^2 - 6x + 8$

56. $y = -2x^2 - 8x - 6$

57. $y = -x^2 - 2x - 1$

58. $y = x^2 + 8x + 16$

59. $y = 2x^2 - x - 10$

60. $y = x^2 + 5x - 6$

In Exercises 61–66, factor the expression.

61. $3x^2 - 15x - 18$

62. $12x^2 + 46x - 8$

63. $140y^2 + 340y + 120$

64. $-18y^2 + 156y + 54$

65. $12a^2 + 36a + 24$

66. $10x^2 + 15x + 5$

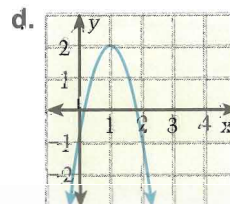
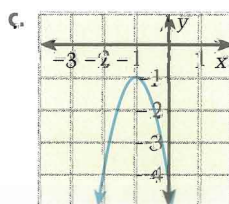
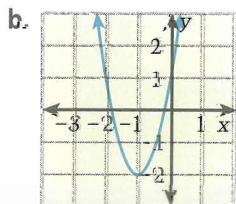
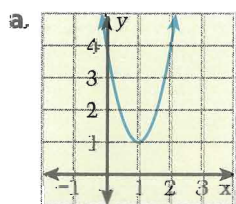
In Exercises 67–70, match the equation with its graph.

67. $y = -3x^2 + 6x - 1$

68. $y = 3x^2 - 6x + 4$

69. $y = 3x^2 + 6x + 1$

70. $y = -3x^2 - 6x - 4$



In Exercises 71–74, sketch the graph of the inequality.

71. $y \geq \frac{1}{2}x^2 - 4x + 6$

72. $y < \frac{1}{3}x^2 + 2x + 1$

73. $y > -x^2 + 8x - 16$

74. $y \leq -x^2 - 4x - 5$

Exploration and Extension

75. **Revenue** The revenue, R , for selling x units of a product is given by

$$R = x(50 - \frac{1}{2}x).$$

How many units must be sold to produce a revenue of \$1218?

76. **Revenue** The revenue, R , for selling x units of a product is given by

$$R = x(100 - \frac{1}{10}x).$$

How many units must be sold to produce a revenue of \$990?

77. **College Entrance Exam Sample** If x and y are positive integers,

$$x^2 + y^2 = 25, \text{ and } x^2 - y^2 = 7, \text{ then } y = \boxed{?}.$$

- a. 3 b. 4 c. 5 d. 9 e. 16

Chapter Summary

What did you learn?

Skills

1. Classify a polynomial by degree and by number of terms. (10.1)
2. Add and subtract polynomials. (10.1)
3. Multiply polynomials
 - using the Distributive Property. (10.2)
 - using the FOIL pattern. (10.2)
 - using the sum and difference pattern. (10.3)
 - using the square of a binomial pattern. (10.3)
4. Factor a polynomial
 - using the difference of two squares pattern. (10.4)
 - using the perfect square pattern. (10.4)
 - using the guess-and-check strategy. (10.5)
5. Solve a quadratic equation
 - by factoring. (10.6)
 - by completing the square. (10.7)
6. Use quadratic models in real-life problems. (10.1–10.7)
7. Recognize sum, difference, and product models in real life. (10.1–10.7)

Strategies

Exploring Data

Why did you learn it?

Polynomials were among the first mathematical models to be used. By now, you can see why—they are relatively simple and yet they can model a great variety of real-life situations. In this chapter, you learned that some models are related to each other by addition, subtraction, or multiplication. For instance, a model for the total revenue, R , can be obtained by multiplying the models for the price per unit, p , and the number, x , of units sold. In other words, $R = xp$.

How does it fit into the bigger picture of algebra?

Polynomials are among the most commonly used models for real-life situations. You were already familiar with three types of polynomial models: $y = a$ (constant models), $y = ax + b$ (linear models), and $y = ax^2 + bx + c$ (quadratic models). In this chapter, you learned how to add and subtract polynomials. You also learned how to multiply polynomials and how to “undo” multiplication by a process called *factoring*. This chapter has many connections with the mathematics you studied in Chapter 9. For instance, in Lessons 10.6 and 10.7 you learned two additional methods—factoring and completing the square—for solving a quadratic equation.

Chapter REVIEW

In Exercises 1–8, classify the polynomial by degree and by terms. (10.1)

1. $x^2 - 1$
2. $3x^2 + 2x - 2$
3. 121
4. $4x$
5. $x^4 - x^2 + 2x + 3$
6. $49x - 2$
7. $8x^3 - 27$
8. $2x^3 + 4x^2 - 5x + 6$

In Exercises 9–20, perform the indicated operation. Use a horizontal format. (10.1, 10.2)

9. $(x + 2 - x^2) + (3x^2 + 4x + 5)$
10. $(4x^3 + x^2 - 1) + (2 - x - x^2)$
11. $(15 + 3x - x^2) + (4x^2 - 2x - 10)$
12. $(3x + 2 - x^2) + (4 - x - x^3)$
13. $(x^2 + 3x - 1) - (4x^2 - 5x + 6)$
14. $(x^2 + 9x + 2) - (5 + 8x - 3x^2)$
15. $(3x^2 - 2x + 4) - (-x^3 + 2x - 6)$
16. $(x^3 + 5x^2 - 4x) - (3x^2 - 6x + 2)$
17. $(x - 5)(x - 10)$
18. $(2x + 2)(x + 4)$
19. $(6 + x)(x^2 - 2x + 3)$
20. $(7 - x)(3x^2 + 2x - 6)$

In Exercises 21–32, perform the indicated operation. Use a vertical format. (10.1, 10.2)

21. $(6x^2 + 2x - 1) + (x^2 - 2)$
22. $(x - 2) + (4x^2 - 7x + 5)$
23. $(-x^2 - x + 2) + (x^2 + 2x - 4)$
24. $(x^2 + 3x + 5) + (3x^2 - 4x + 6)$
25. $(x^2 - 3) - (4x^2 - 3x + 2)$
26. $(x^2 + 3x - 7) - (-2x^2 + x + 14)$
27. $(x^2 - 4x + 2) - (6x^2 + 4x - 3)$
28. $(10x^2 + 3x - 4) - (5x^2 + 2x - 6)$
29. $(x - 2)(3x^2 + 4x - 1)$
30. $(10 - x)(x^2 + x + 1)$
31. $(2x + 2)(4x^2 - 6x + 2)$
32. $(4 + 3x)(1 - 4x + 6x^2)$

In Exercises 33–36, multiply. (10.3)

33. $(x + 15)(x - 15)$
34. $(3x + 2)(3x - 2)$
35. $(x + 2)^2$
36. $(5x - 6)^2$

In Exercises 37–46, use the discriminant to determine whether the polynomial can be factored. If possible, factor the polynomial. (10.4, 10.5)

37. $x^2 - 2x - 15$
38. $x^2 + 3x - 70$
39. $x^2 - 64$
40. $4x^2 + 25$
41. $x^2 - 8x + 8$
42. $9x^2 + 12x + 4$
43. $x^2 + 10x + 25$
44. $x^2 - 8x + 16$
45. $4x^2 - 32x + 60$
46. $3x^2 + 21x + 30$

In Exercises 47–54, solve the equation. (10.6)

47. $x^2 - 21x + 108 = 0$

48. $x^2 - 8x - 240 = 0$

49. $-x^2 + 30x - 200 = 0$

50. $-15x^2 + 45x + 150 = 0$

51. $36x^2 - 49 = 0$

52. $x^2 + 26x + 169 = 0$

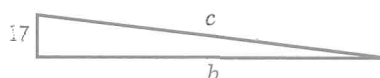
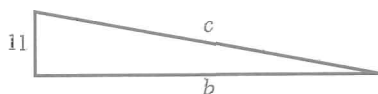
53. $x^2 - 14x + 36 = 0$

54. $x^2 + 10x - 3 = 0$

Right-Triangle Triple In Exercises 55 and 56, find the right-triangle triple. (10.4)

55. $11^2 + b^2 = c^2$

56. $17^2 + b^2 = c^2$



57. **Tossing a Ball** A ball is tossed into the air from a height of 10 feet with an initial velocity of 12 feet per second. Find the time, t (in seconds), for the object to reach the ground by solving the equation

$$-16t^2 + 12t + 10 = 0.$$

58. **Summer Business** Your friend's weekly revenue, R (in dollars), from her tie-dye T-shirt business can be modeled by

$$R = -2t^2 + 37t + 60$$

where t represents the week of sales, with $t = 0$ for the first week.

In the first week, 3 T-shirts were sold. After that, the sales increased by 2 T-shirts per week. Did the price of T-shirts remain constant during the 8-week summer season? Explain.

Huffing and Puffing In Exercises 59 and 60, use the following information.

Porcupine fish, members of the puffer fish family, range between 10 and 20 inches in length. When in danger, the body of the fish puffs up by taking in water or air. (The tail, which is about 3 inches long, does not puff up.)

Porcupine Fish:
Deflated



Inflated



59. The volume of a "puffed-up" porcupine fish can be modeled by

$$V = \left(\frac{1}{6}\right)\pi(x - 3)^3$$

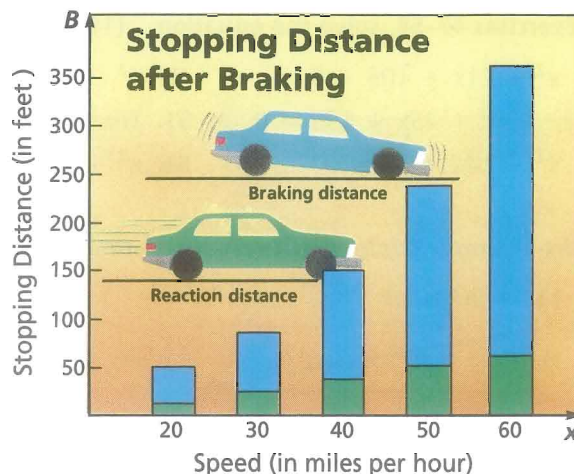
where x is the total length of the fish in inches. Write the right side of this equation in standard polynomial form.

60. Approximate the volume of an 11-inch porcupine fish.

Stopping Distance In Exercises 61–64, use the following information.

The stopping distance of an automobile traveling at x miles per hour is the sum of the distance the car travels during the driver's reaction time *and* the distance the car travels after the brakes are applied. The distance, R , in feet, traveled during the driver's reaction time is approximately $R = 1.1x$. The distance, B , in feet, traveled after the brakes are applied is $B = 0.14x^2 - 4.43x + 58.4$ approximately.

(These models are based on normal road conditions.)



61. Find a model for the (total) stopping distance, S (in feet), of an automobile that is traveling x miles per hour.
62. Estimate the total stopping distance for an automobile that is traveling 15 miles per hour; 30 miles per hour; 55 miles per hour.
63. Are stopping distances and speeds related by a linear model? Explain. What does this tell you about the amount of distance you need to allow for stopping when traveling at various speeds?
64. The recommended safe *following distance* on highways under normal road conditions is given by $F = 1.8x$. How many feet should be between cars traveling at 55 miles per hour?

Registered Cars In Exercises 65 and 66, use the following information.

From 1940 to 1990, the number, C , of registered cars in the United States can be modeled by $C = 20,500t^2 + 1,460,100t + 25,942,500$, where $t = 0$ represents 1940. (Source: *Motor Vehicle Facts and Figures '90*, Federal Highway Administration)

65. During which year were 129,874,900 cars registered?
66. According to this model, how many cars will be registered in 1995?



Chapter TEST

In Exercises 1–12, perform the indicated operations.

1. $(x^2 + 3x - 1) + (4x^2 + 2)$
2. $(x^4 + 3x^2 + 2) + (2x^4 - 3x^2 + 6)$
3. $(5x^2 - 2x + 1) - (7x + 10)$
4. $(5x^3 + 2x - 4) - (4x^3 + 2x^2 - 5x)$
5. $(9x + 2)(9x - 2)$
6. $(5x - 4)(5x + 4)$
7. $(x - 14)^2$
8. $(3x + 5)^2$
9. $(x + 2)(3x + 5)$
10. $(2x - 1)(13x + 5)$
11. $(x - 6)(4x^2 + 3x - 5)$
12. $(4x^3 - 6x + 7)(x + 1)$

In Exercises 13–20, factor the expression.

13. $x^2 - 144$
14. $36x^2 - 25$
15. $x^2 - 12x + 36$
16. $x^2 + 10x + 25$
17. $3x^2 + 2x - 1$
18. $5x^2 - 3x - 2$
19. $x^3 + 2x^2 + x$
20. $2x^2 - 28x + 96$

In Exercises 21–26, solve the equation.

21. $x^2 + 4x + 4 = 0$
22. $x^2 - 7x + 6 = 0$
23. $x^2 - 5x - 150 = 0$
24. $x^2 + 6x - 91 = 0$
25. $12x^2 + 15x + 3 = 0$
26. $4x^2 - 10x - 36 = 0$

In Exercises 27–30, solve the equation by completing the square.

27. $x^2 - 4x + 1 = 0$
28. $x^2 + 6x - 9 = 0$
29. $x^2 + 20x + 3 = 0$
30. $x^2 - 2x - 5 = 0$

31. Find a right-triangle triple such that $5^2 + b^2 = c^2$.
32. The length of a bedroom is 3 feet less than twice its width. The area of the bedroom is 135 square feet. What are the dimensions of the room?
33. A deposit of \$5000 was put into a savings account paying an annual interest of $r\%$, compounded yearly. After 2 years, the balance in the account was \$5,644.53. What was the rate of interest?
34. The bed of a pond can be modeled by $25y = 2x^2 - 20x + 1$, where x and y are measured in meters and the x -axis matches the water level of the pond. What is the width of the pond?

