

Why does it work?

Alex was asked to solve $2x^2 + 5x + 1 = 0$, and Morgan was asked to solve $ax^2 + bx + c = 0$

Alex's "complete the square" way

First I divided both sides of the equation by the coefficient of x^2 . Then I subtracted the constant term from both sides.

I completed the square by adding $\left(\frac{b}{2a}\right)^2$ to both sides of the equation.

I simplified on both sides.

I simplified further on the right-hand side. I found a common denominator for the two fractions on the right, then added them together.

I took the square root on either side..

I simplified and solved for x .



$$2x^2 + 5x + 1 = 0$$

$$x^2 + \frac{5}{2}x + \frac{1}{2} = 0$$

$$x^2 + \frac{5}{2}x = -\frac{1}{2}$$

$$x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = -\frac{1}{2} + \left(\frac{5}{4}\right)^2$$

$$\left(x + \frac{5}{4}\right)^2 = -\frac{1}{2} + \frac{25}{16}$$

$$\left(x + \frac{5}{4}\right)^2 = \frac{-8}{16} + \frac{25}{16}$$

$$\left(x + \frac{5}{4}\right)^2 = \frac{17}{16}$$

$$\sqrt{\left(x + \frac{5}{4}\right)^2} = \sqrt{\frac{17}{16}}$$

$$x + \frac{5}{4} = \pm \sqrt{\frac{17}{16}}$$

$$x + \frac{5}{4} = \pm \frac{\sqrt{17}}{\sqrt{16}}$$

$$x + \frac{5}{4} = \pm \frac{\sqrt{17}}{4}$$

$$x = \pm \frac{\sqrt{17}}{4} - \frac{5}{4}$$

$$x = \frac{-5 \pm \sqrt{17}}{4}$$

Morgan's "complete the square and derive the quadratic formula" way

First I divided both sides of the equation by the coefficient of x^2 . Then I subtracted the constant term from both sides.

I completed the square by adding $\left(\frac{b}{2a}\right)^2$ to both sides of the equation.

I simplified on both sides.

I simplified further on the right-hand side. I found a common denominator for the two fractions on the right, then added them together.

I took the square root on either side..

I simplified and solved for x .



$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{-4ac + b^2}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \pm \frac{\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

* How did Alex solve the equation? How did Morgan solve the equation?

* What does Morgan's answer look like to you?

* What are some similarities and differences between Alex's and Morgan's problems?

* What are some similarities and differences between Alex's and Morgan's ways?

* What happens when you substitute the values for a , b , and c from Alex's equation into Morgan's answer?

* Where does the quadratic formula come from?