

# Investigation 3

## ACE

### Assignment Choices



#### Problem 3.1

Core 1, 2, 4, 24–30

Other Applications 3, 5–8; unassigned choices from previous problems

#### Problem 3.2

Core 9–15, 18, 20

Other Applications 16, 17, 19; Connections 31, 32; Extensions 40–45; unassigned choices from previous problems

#### Problem 3.3

Core 21, 22, 23

Other Connections 33–39; Extensions 46, 47; unassigned choices from previous problems

**Adapted** For suggestions about adapting Exercise 1 and other ACE exercises, see the *CMP Special Needs Handbook*.

**Connecting to Prior Units** 24–26, 32, 34: *Bits and Pieces III*; 33, 38: *Stretching and Shrinking*, 35: *Moving Straight Ahead*; 37: *Comparing and Scaling*; 39: *Bits and Pieces I*

## Applications

### 1. a. Growth of Wolf Population

Year	Wolf Population
0	20
1	24
2	29
3	35
4	41
5	50
6	60

b.  $p = 20(1.2^t)$ , where  $p$  is the population and  $t$  is the number of years

c. About 9 yr

### 2. a. 1.9. Possible explanation:

$$\frac{57}{30} \approx \frac{108}{57} \approx \frac{206}{108} \approx \frac{391}{206} \approx \frac{793}{391} \approx 1.9$$

b. After 10 yr, there would be

$30 \times (1.9^{10}) \approx 18,393$  elk. After 15 yr, there would be  $30 \times (1.9^{15}) \approx 455,434$  elk.

c.  $p = 30(1.9^n)$

d. After 16 yr, there will be 865,324 elk. After 17 yr, there will be 1,644,116 elk. This means that between year 16 and year 17 the population will reach 1 million. Some industrious students might find by guess-and-check that the population exceeds 1 million after 16.225 yr, or approximately 16 yr and 3 mo.

3. Between 1 and 2 years.  $100(1.5) = 150$ , and  $100(1.5)^2 = 225$ . Students may want to use a graph, a table, or guess-and-check to find a more precise answer: 1.71 yr.

4.  $p = 500,000(1.6^n)$ . (Note: This isn't a good model as  $n$  gets large. In fact, in less than 500 years, it predicts there will be more squirrels than atoms in the universe.)

5. D

6. 1.3 yr. (Note: Students are likely to estimate the doubling time.)

7. 3.8 yr. (Note: Students are likely to estimate the doubling time.)

### 8. a. $y = 50(2.2)^x$

x	y
0	50
1	110
2	242
3	532.4
4	1,171.3
5	2,576.8

### $y = 350(1.7)^x$

x	y
0	350
1	595
2	1,011.5
3	1,719.6
4	2,923.2
5	4,969.5

- b. In the first equation, the growth factor is 2.2. In the second, the growth factor is 1.7.
- c. Yes; although the  $y$ -intercept of the first graph is lower, the graph is increasing at a faster rate.
- d. The graphs will cross between  $x = 7$  and  $x = 8$ . Some students might check carefully and find that the graphs cross at around  $x = 7.547$ .

9. a. Maya's Savings Account

Age	Value
0	\$100
1	\$104
2	\$108.16
3	\$112.49
4	\$116.99
5	\$121.67
6	\$126.53
7	\$131.59
8	\$136.86
9	\$142.33
10	\$148.02

b. 1.04

c.  $a = 100(1.04)^n$ , where  $a$  is the amount of money in the account and  $n$  is Maya's age

10. 40%

11. 90%

12. 75%

13. 1.45

14. 1.9

15. 1.31

16. 1.25

17. a. 6 yr. The projected population at that point is 1,340.

b. 6 yr. The projected population at that time is 1,300. (Note: The linear equation  $p = 1,000 + 50x$  models the problem, where  $p$  is the population in year  $x$ . Solving  $1,300 = 1,000 + 50x$  shows that the population will outgrow the facilities in 6 yr. The two growth models can also be compared by looking at tables for

$y = 1,000(1.05)^x$  and  $y = 1,000 + 50x$ . This is particularly easy if a calculator is used to generate the tables. You might ask students to continue to scroll beyond the values for the first 6 yr and see what they discover. Beyond that time, the exponential assumption will produce greater year-to-year growth.)

18. a.

Year	Radios Sold
1	1,000,000
2	1,030,000
3	1,060,900
4	1,092,727
5	1,125,509
6	1,159,274
7	1,194,052

b.

Year	Radios Sold
1	100,000
2	103,000
3	106,090
4	109,273
5	112,551
6	115,927
7	119,405

19. a. About \$8.72 [Note: The related equation is  $p = 7(1.045^t)$  where  $p$  is the price of the ticket and  $t$  is the time in years; when  $t = 5$ ,  $p$  is about \$8.72.]

b. About \$10.87

c. About \$26.22 [Note: Students may round the prices in parts (a)–(c) to \$8.75, \$10.75, and \$26.25 (or \$26), arguing that movie theaters don't generally charge prices like \$8.72 or \$10.87.]

20. 100%

21. F. This results in 72% total growth over the 8 years. G and H each give a total growth of 70%.

22. a. Initial value: \$130; growth rate: 7%; growth factor: 1.07; number of years: 5

b. \$223.36

23. Latanya's mice are reproducing most quickly. Carlos's mice are reproducing most slowly. Expressed as percents, the growth factors are Carlos: 114%; Mila: 125%; and Latanya: 300%.

## Connections

24. \$3,600
25. \$300
26. \$3,325
27. This pattern represents exponential growth because each value is the previous value times a growth factor of 1.1.
28. This pattern represents exponential growth because each value is the previous value times a growth factor of  $\frac{5}{3}$ .
29. This pattern does not represent exponential growth because there is no constant by which each value is multiplied to find the next value. The pattern is, in fact, linear with an addition of  $\frac{5}{3}$  for each term.
30. Answers may vary. A student could argue that the growth factor is approximately 3.2 and be correct. If this were "real world" data, most people (for most purposes) would consider this exponential growth. Another student might say that since there is variation in the growth factor between 3.18 and 3.22, this does not represent exponential growth.
31. a. The bars represent the number of new subscribers for each year.  
b. The curve represents the total number of subscribers each year.  
c. Answers will vary. It is difficult to read exact data from this graph. Although the graph appears to be exponential, it does not demonstrate pure exponential growth. The number of subscribers in 1995 was about 3.4 million, and in 1996, it was about 4.4 million, representing a growth factor of approximately 1.29. Between 1997 and 1998, the growth was from about 5.5 million to about 6.9 million, a growth factor of 1.25.
- d. The growth between these two years is only about 10%—a growth factor of about 1.10. This is significantly smaller than the growth factor in the preceding years.
32. a. 3% raise: \$600; 4% raise: \$800; 6% raise: \$1,200  
b. 3% raise: \$20,600; 4% raise: \$20,800; 6% raise: \$21,200  
c. Possible answer:  $103\% = 100\% + 3\%$ , so 103% of \$20,000 is the same as 100% of \$20,000 plus 3% of \$20,000. This is the same as  $\$20,000 + (3\% \text{ of } \$20,000)$ . Or, because  $103\% = 1.03$ , we can reason as follows:  
 $103\% \text{ of } \$20,000 = 1.03(\$20,000) = 1(\$20,000) + 0.03(\$20,000) = \$20,000 + 0.03(\$20,000) = \$20,000 + (3\% \text{ of } \$20,000)$
33. a. 5 cm; 6 cm<sup>2</sup>.  
b. 5.5 cm; 7.26 cm<sup>2</sup>. [Note: As each linear dimension increases by a factor of 1.1—a 10% increase—the area increases by  $(1.1)^2 = 1.21$ .]  
c. After five enlargements, the length of the diagonal will be about 8.05 cm, and the area of the shaded region will be about 15.56 cm<sup>2</sup>.  
d. Yes, they are similar:  $\frac{3}{4} = \frac{3.3}{4.4} = \frac{3.63}{4.84}$ .
34. She is correct. If the amount the customer pays her this summer is  $s$ , then under her plan, she will earn  $(1.04)s$  the second summer and  $(1.03)(1.04)s$  the third summer, for a total of  $s + (1.04)s + (1.03)(1.04)s$ . Under the customer's plan, her total will be  $s + (1.03)s + (1.04)(1.03)s$ . Her earnings the third summer will be the same under both plans, but because she will make more money the second summer, her total earnings will be greater under her plan.
35. a.  $\$9.00 \times 40 \times 52 = \$18,720$ .  
b.  $a = 360w$  (Note: Some students may include the paid vacation time and write the equation  $a = 360w + 720$ .)  
c. She is trying to figure out how many weeks she needs to work to earn \$9,000. The answer is 25 weeks.

d. **Kim's Salary**

Year	Annual Income
1	\$18,720
2	\$19,282
3	\$19,860
4	\$20,456
5	\$21,070
6	\$21,702
7	\$22,353
8	\$23,023
9	\$23,714
10	\$24,425

e. For the first 6 years, the \$600-per-year raise plan is better. Under the \$600-per-year plan, Kim would earn \$21,720 in year 6 and \$22,320 in year 7. In year 7, the salary for the 3% raise plan would be \$22,353 and from then on would result in greater yearly salaries than the \$600-per-year raise plan. The plan Kim chooses would depend on how many years she anticipates working for this company. (Note: Graphing the equations may not help students answer this question; for  $x$ -values from 0 to 10, both graphs look linear because the exponential growth is very slow for the first 10 years.)

36. 2.5

37. 130%, 1.475,  $\frac{3}{2}$ , 2

38. Answers may vary. Anything less than or equal to 88% (the scale factor that takes  $8\frac{1}{2}$  to  $7\frac{1}{2}$ ) will work.

39. a. Matches: 20% and 1.2; 120% and 2.2; 50% and 1.5; 400% and 5; 2% and 1.02.

No match: 200%, 4, 2.

b. 2%, 20%, 50%, 120%, 200%, 400%

c. 1.02, 1.2, 1.5, 2, 2.2, 4, 5

## Extensions

40. 2,500%. Because the growth factor is 26, the growth rate is  $26 - 1$ , or 25, expressed as a percent, which is 2,500%.

41. a. Using these assumptions, in 2010, the population would be about 305 million.

b. About 70 yr

c. Answers will vary. Students may compare their answers to current information on the U.S. population. Seventy years is a very long time for a model to remain a good predictor. The fact that a 1% rate of increase translates into 2,500,000 additional people the first year and a greater number in each successive year might surprise some students.

d. The actual growth rate for this time period was greater than that predicted by the model in this problem.

42. a. Averaging the ratios gives a growth factor of 1.1.

$$\left( \frac{3.02}{2.76} + \frac{3.33}{3.02} + \frac{3.69}{3.33} + \frac{4.07}{3.69} + \frac{4.43}{4.07} + \frac{4.83}{4.43} + \frac{5.26}{4.83} + \frac{5.67}{5.26} + \frac{6.07}{5.67} \right) \div 9 \approx 1.1$$

b.  $p = 2.76(1.1)^x$ , where  $x$  is the number of 5-year intervals

c.  $p = 2.76(1.1)^8 \approx 5.92$  billion, so the population will double the 1955 population sometime between 1990 and 1995 (the eighth 5-year period).

d. When  $x = 16$ ,  $p = 2.76(1.1)^{16} \approx 12.68$  billion, so the population will double the 2000 population sometime between 2030 and 2035 (the sixteenth 5-year period). (Note: Doubling time is independent of the starting population.)

43.  $p = 300(1.2)^t$ , where  $p$  is population and  $t$  is the year

44.  $p = 579(1.2)^t$ , where  $p$  is population and  $t$  is the year

45.  $v = 2,413(1.03)^t$ , where  $v$  is the value of the investment and  $t$  is the year

46. a. Possible answer: You could evaluate  $((1.5)^2)^3$ ; in other words, multiply  $1.5 \times 1.5 = 2.25$ , then  $2.25 \times 2.25 = 5.0625$ , and then  $5.0625 \times 5.0625 \times 5.0625 = 129.75$ .

b. You have to press the  $\boxed{\times}$  key 4 times to get the answer in the method outlined above.

47. a. \$10,400

b. \$10,406.04

- c. \$10,407.42. (**Note:** This is the exact answer using a growth factor of  $1.00\overline{3} = 1 + \frac{1}{12}(0.04)$ . However, students may round and use a growth factor of 1.003. This gives an answer of \$10,366.00, which is significantly less. In compound growth situations, rounding leads to significantly different answers over time.)
- d. He will earn more if he chooses the account for which interest is compounded monthly. The more often the interest is compounded, the faster the investment grows.

### Possible Answers to Mathematical Reflections

1. a. You can convert the growth rate to a growth factor by adding 100% and changing the result to a decimal. Then, you can use an equation of the form  $p = a \times b^t$ , where  $b$  is the growth factor and  $a$  is the size of the original population. You can also compute the population from one time to the next by finding the percent increase and adding it to the previous value to get the next successive value. If the growth rate is 4% and population after  $n$  yr is  $P$ , then the value after  $n + 1$  yr is  $P + 0.04P$  or  $P(1.04)$ .
- b. A growth rate is the percent growth. If you convert the growth rate to a decimal and add 1, you will get the growth factor. For example, a growth rate of 4% corresponds to a growth factor of 1.04.
2. a. You can use the equation  $p = a \times b^t$ , where  $b$  is the growth factor and  $a$  is the size of the original population. You can also generate the population for each year recursively, by multiplying the population for each year by the growth factor to get the population for the next year.
- b. Figure out the percent change from one year to the next. If you convert the growth factor to a percent and subtract 100%, you will get the growth rate. For example, a growth factor of 1.04 corresponds to a growth rate of 4%.