

# Investigation 3

## ACE

### Assignment Choices



#### Problem 3.1

Core 1, 2, 12

Other Connections 13–19, 20–26; Extensions 40; unassigned choices from previous problems

#### Problem 3.2

Core 3–9, 28

Other Connections 27, 29–31; Extensions 41–45; unassigned choices from previous problems

#### Problem 3.3

Core 10, 11

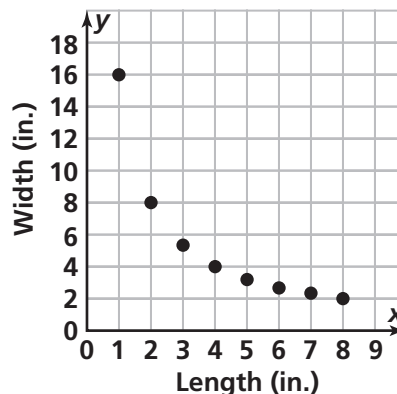
Other Connections 32–39; Extensions 46–48; unassigned choices from previous problems

**Adapted** For suggestions about adapting ACE exercises, see the *CMP Special Needs Handbook*.  
**Connecting to Prior Units** 12: *Moving Straight Ahead, Covering and Surrounding*; 13–18, 20–25: *Accentuate the Negative*; 27–28: *Data About Us*; 30–35: *Moving Straight Ahead*; 36–39: *Comparing and Scaling*

## Applications

1. a. (Figure 2)

b. Rectangles With an Area of 16 in.<sup>2</sup>



c. As length increases, width decreases at a decreasing rate.

d.  $w = \frac{16}{\ell}$  (or  $w\ell = 16$ , or  $\ell = \frac{16}{w}$ ); not linear

2. a. Values in table will vary. See Figure 3.

Figure 2

Rectangles With an Area of 16 in.<sup>2</sup>

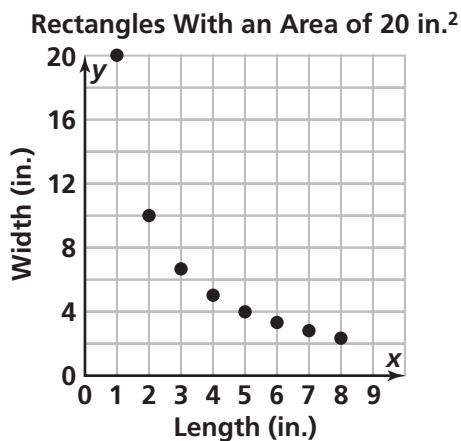
Length (in.)	1	2	3	4	5	6	7	8
Width (in.)	16	8	$\frac{16}{3}$	4	$\frac{16}{5}$	$\frac{16}{6}$	$\frac{16}{7}$	2

Figure 3

Rectangles With an Area of 20 in.<sup>2</sup>

Length (in.)	1	2	3	4	5	6	7	8
Width (in.)	20	10	$\frac{20}{3}$	5	4	$\frac{10}{3}$	$\frac{20}{7}$	$\frac{5}{2}$

b. Points will vary.



c.  $w = \frac{20}{\ell}$ ; not linear

d. The graphs are similar in shape, but the coordinates of the points are different.

e. The equations have the same form, but the constant is different.

3. a. Answers will vary, but  $y = \frac{96}{x}$ , where  $x$  is the length and  $y$  is the breaking weight, is a reasonable choice.

b. In the equation  $y = \frac{96}{x}$ ,  $x$  (or length) is in the denominator, so, as  $x$  increases,  $y$  (or breaking weight) decreases. This is reasonable because the data show that as the length of a bridge increases, the strength decreases.

4. Not an inverse variation

5. Inverse variation;  $y = \frac{48}{x}$

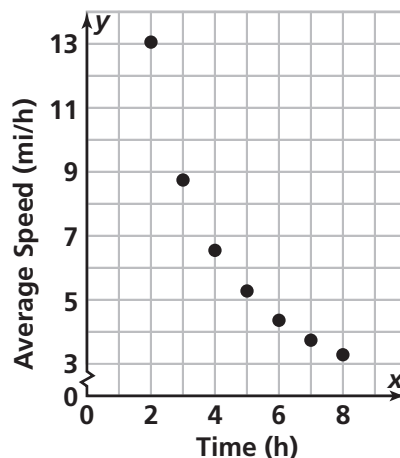
6. Inverse variation;  $y = \frac{100}{x}$

7. Not an inverse variation

8. a. **Marathon Speeds**

Time (h)	Running Speed (mi/h)
2	13.1
3	8.73
4	6.55
5	5.24
6	4.37
7	3.74
8	3.28

**Marathon Speeds**



b.  $s = \frac{26.2}{t}$

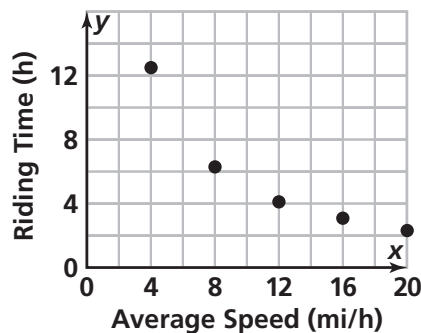
c. From 2–3 hr: decreases by 4.37 mph;  
from 3–4 hr: decreases by 2.18 mph;  
from 4–5 hr: decreases by 1.31 mph

d. For constant change in time, the change in average speed is not constant.

9. a. **Charity Bike Ride**

Time (h)	Riding Speed (mi/h)
4	12.5
8	6.25
12	4.17
16	3.125
20	2.5

**Charity Bike Ride**

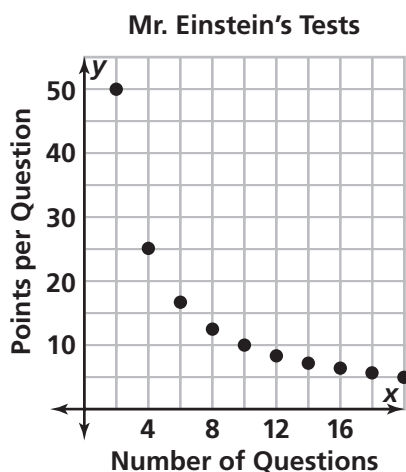


b.  $t = \frac{50}{s}$

- c. From 4–8 mph: decreases by 6.25 hours;  
from 8–12 mph: decreases by 2.08 hours;  
from 12–16 mph: decreases by 1.05 hours
- d. For constant change in average riding speed, the change in time is not constant.

10. a. Mr. Einstein's Tests

Number of Questions	Points per Question
2	50
4	25
6	16.67
8	12.5
10	10
12	8.33
14	7.14
16	6.25
18	5.56
20	5



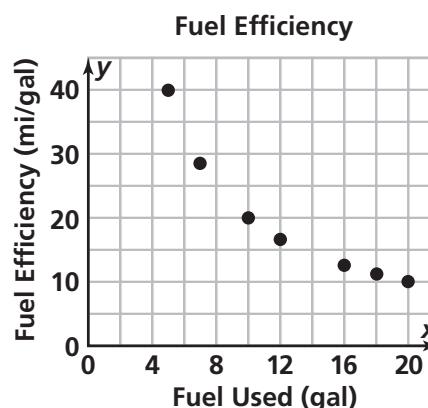
b.  $p = \frac{100}{q}$

- c. Decreases by 25 points per question;  
decreases by 8.32 points per question;  
decreases by 4.17 points per question;  
decreases by 2.5 points per question
- d. For constant change in the number of questions, that change in points per question is not constant.

11. a.

Vehicle Type	Fuel Used (gal)	Fuel Efficiency (mi/gal)
Large Truck	20	10
Large SUV	18	11.11
Limousine	16	12.5
Large Sedan	12	16.67
Small Truck	10	20
Sports Car	12	16.67
Compact Car	7	28.57
Sub-Compact Car	5	40

b.



c.  $e = \frac{200}{u}$

- d. Decreases by 20 mpg;  
decreases by 6.67 mpg;  
decreases by 3.33 mpg
- e. Constant change in fuel used does not lead to constant change in fuel efficiency.

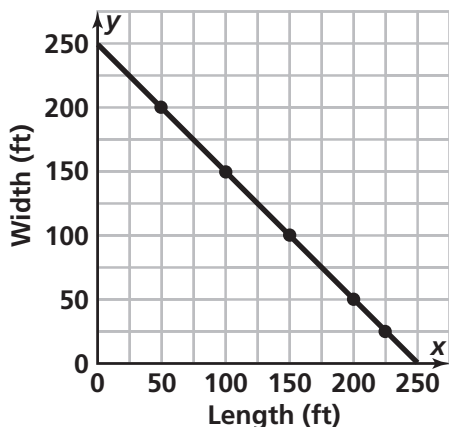
## Connections

12. a. Possible table:

Rectangles With a Perimeter of 500 ft

Length (ft)	50	100	150	200	225
Width (ft)	200	150	100	50	25

b. Rectangles With a Perimeter of 500 ft



c. As length increases, width decreases. The rate of change is constant.

d.  $w = 250 - \ell$  or  $w = \frac{1}{2}(500 - 2\ell)$ . This is linear. The graph is a straight line and the equation has the form  $y = mx + b$ .

13. -2      14. 3      15. -2.5

16. 2.11      17.  $-\frac{7}{3}$       18.  $-\frac{3}{7}$

19. (Figure 4); A number and its additive inverse are the same distance from 0 on the number line. The labeled number line has reflection symmetry.

20.  $\frac{1}{2}$       21.  $-\frac{1}{2}$

22. 2      23.  $\frac{1}{4}$

24.  $\frac{4}{3}$       25.  $\frac{3}{5}$

26. (Figure 5); Numbers greater than 1 have multiplicative inverses between 0 and 1. Numbers less than -1 have multiplicative inverses between -1 and 0.

27. C

28. a. 6.7

b. 7.62

c. The effect was different because the relationship between number of quizzes and average quiz scores is not linear.

29. a. Possible answer:  $y = 5x$  (ACE 19, Investigation 2) and  $d = 50t$  (Problem 3.2 Question C). The ratio  $\frac{y}{x}$  is 5 and the ratio  $\frac{d}{t}$  is 50.

b. The ratio equals  $k$  in both cases.

c. Possible answer:  $y$  changes by  $k$  as  $x$  changes by 1. This pattern results in a straight-line graph with slope  $k$ .

d. With a direct variation the graph is a line, and the equation is of the form  $y = kx$  where the slope of the line equals  $k$ . With an inverse variation, the graph is a curve, and the equation is of the form  $y = \frac{k}{x}$ .

30.  $x = 5$ ;

$$5x - 28 = -3$$

$$5x = 25$$

$$x = 5$$

To solve with a graph, graph  $y = 5x - 28$ , and find the  $x$ -coordinate of the point where  $y = -3$ . To solve with a table, make a table of  $(x, y)$  values for  $y = 5x - 28$ , find the  $x$ -value corresponding to  $y = -3$ .

Figure 4

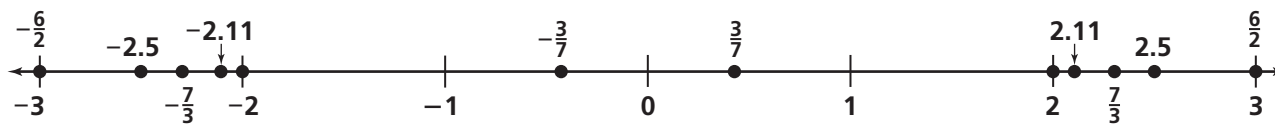
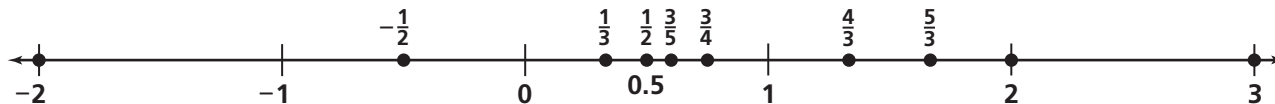


Figure 5



31.  $x = 2$ ;

$$10 - 3x = 7x - 10$$

$$10 = 10x - 10$$

$$20 = 10x$$

$$x = 2$$

To solve with a graph, graph  $y = 10 - 3x$ , and  $y = 7x - 10$ , and find the  $x$ -coordinate of the intersection point. To solve with a table, make a table of  $(x, y)$  values for  $y = 5x - 28$  and  $y = 10 - 3x$ , and find the  $x$ -value for which the  $y$ -values are the same.

32.  $y = -\frac{1}{2}x + 5$

33.  $y = 3x - 4$

34.  $y = -2x + 12$

35.  $y = \frac{1}{6}x + 7$ . To find the slope, take the points (30, 12) and (6, 8) on the line and find the vertical change (4) and the horizontal change (24). Slope is the ratio  $\frac{\text{vertical change}}{\text{horizontal change}} = \frac{4}{24} = \frac{1}{6}$

36. a. About \$0.53;  $\$3.20 \div 6 \approx 0.53$ .

b. About \$5.30;  $0.53 \times 10 = 5.3$

c.  $0.53n$

37. Super Market; Super Market charges about \$0.58 per tomato and Gus's Groceries charges about \$0.67.

38. Super Market; Gus's Groceries charges about \$0.44 per cucumber and Super Market charges \$0.40.

39. Gus's Groceries; Gus's Groceries charges \$0.50 per apple and Super Market charges about \$0.58.

## Extensions

40. a.  $250 \text{ cm}^3$

b. 10 cm by 10 cm by 2.5 cm

c. The surface area of the original prism is  $250 \text{ cm}^2$ . The surface area of the prism in part (b) is  $300 \text{ cm}^2$ . The surface area of the original prism is smaller.

41. Ms. Singh traveled 80 mi in 3 hr, for an average

speed of  $80/3 = 26.67 \text{ mph}$ .

42. a. If  $x$  is the number of tickets sold and  $y$  is the profit, then  $y = 4.5x - 150$ .

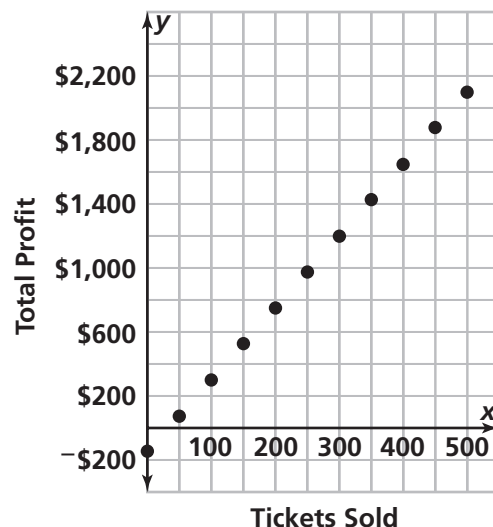
b and d.

### Spring Show Ticket Sales

Tickets Sold	Total Profit	Per-Ticket Profit
0	−150	—
50	\$75	\$1.50
100	\$300	\$3.00
150	\$525	\$3.50
200	\$750	\$3.75
250	\$975	\$3.90
300	\$1,200	\$4.00
350	\$1,425	\$4.07
400	\$1,650	\$4.13
450	\$1,875	\$4.17
500	\$2,100	\$4.20

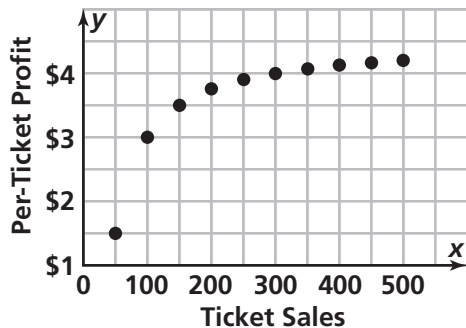
c.

### Spring Show Ticket Sales



d. See table above.

e. Spring Show Ticket Sales



f. The pattern for total profit is linear; the pattern for per-ticket profit is not. The graph for total profit is a straight line; the graph for per-ticket profit is a curve. In the column for total profit, there is a constant difference in values; in the per-ticket profit column, there is not. The per-ticket profit decreases by a smaller and smaller amount as the number of tickets sold increases.

43.  $(3, 16), (12, 4); y = \frac{48}{x}$

44.  $(3, 9), (4, \frac{27}{4}); y = \frac{27}{x}$

45.  $(3, 4), (4, 3); y = \frac{12}{x}$

46. A      47. G      48. A

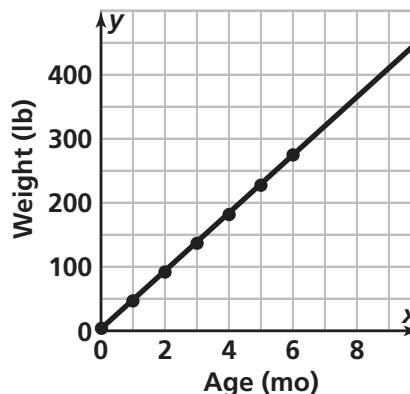
## Possible Answers to Mathematical Reflections

1. a. In an inverse variation, as values of  $x$  increase, values of  $y$  decrease at a decreasing rate.  
 b. For any point on the graph, multiplying the  $x$ -coordinate by the  $y$ -coordinate will yield a constant value. The graph will be a decreasing curve.  
 c.  $y = \frac{k}{x}$ ,  $xy = k$ , or  $x = \frac{k}{y}$ .
2. In a linear relationship, the rate of change is constant. In an inverse variation, the rate of change decreases as the values of the independent variable increase. The graph of a linear relationship is a straight line (increasing or decreasing). The graph of an inverse relationship is a decreasing curve. An equation for a linear relationship can be written in the form  $y = mx + b$ , while the equation for an inverse variation can be written as  $y = \frac{k}{x}$ , or  $xy = k$ .

## Answers to Looking Back and Looking Ahead

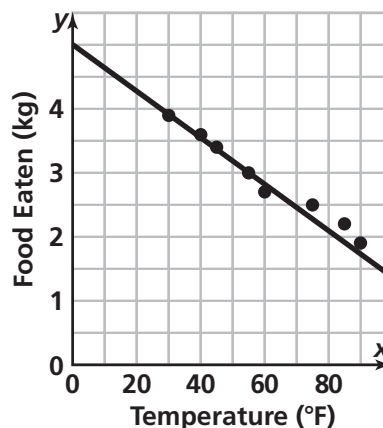
1. a. Line will vary.

Average Growth of Properly-Fed Pig



- b. Possible equation:  $y = 45x + 3$   
 c. Answers will vary. For the equation above, the slope  $m$  suggests that for every month, the weight increases by 45 lb. The  $y$ -intercept  $b$  suggests that the pig was 3 lb when it was born.  
 d. Answers will vary. The equation above predicts that at 3.5 mo, the pig weighed 160 lb. At 7 mo, the pig weighed roughly 318 lb.

2. a. Temperature and Food Eaten



- b. Possible equation:  $y = -0.04x + 5$   
 c. Answers will vary. For the equation above, the slope  $m$  suggests that for every increase of 30°F in temperature, there is a decrease of 1 kg in food eaten. The  $y$ -intercept  $b$  suggests that the goat would eat 5 kg of food if the temperature were 0° F.