

# Investigation 4

## ACE

### Assignment Choices



#### Problem 4.1

Core 1, 2, 8

Other Unassigned choices from previous problems

#### Problem 4.2

Core 3–5

Other Connections 9–11; Extension 13; unassigned choices from previous problems

#### Problem 4.3

Core 6, 7

Other Connections 12; unassigned choices from previous problems

**Adapted** For suggestions about adapting Exercise 1 and other ACE exercises, see the *CMP Special Needs Handbook*.

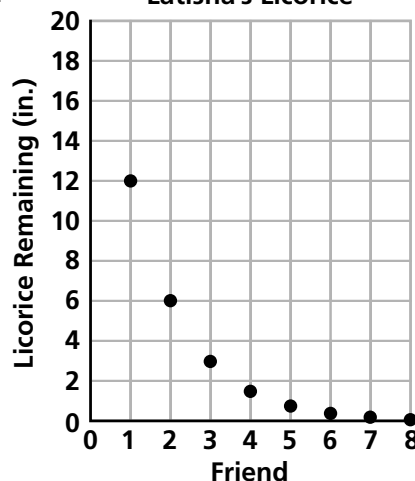
**Connecting to Prior Units** 9: *Moving Straight Ahead*; 10: *Bits and Pieces II*; 11: *Covering and Surrounding* and *Stretching and Shrinking*; 12: *Bits and Pieces I*

## Applications

### 1. a. Latisha's Licorice

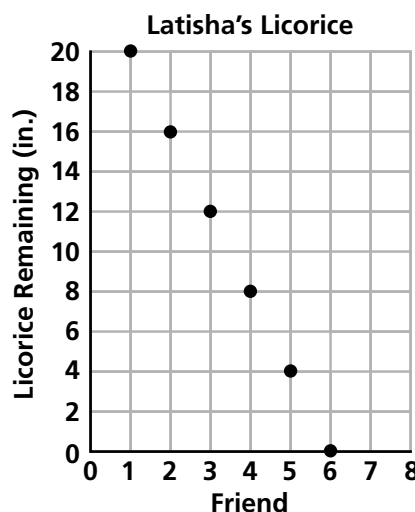
Friend	Licorice Remaining (in.)
1	12
2	6
3	3
4	1.5
5	0.75
6	0.375
7	0.1875
8	0.09375

### b. Latisha's Licorice



### c. Latisha's Licorice

Friend	Licorice Remaining (in.)
1	20
2	16
3	12
4	8
5	4
6	0



- d. The first graph shows exponential decay; Latisha gave away less and less to each friend. The second graph is linear; each of the first six friends received the same amount. In the first graph, Latisha's licorice never runs out. In the second graph, the licorice runs out after 6 friends.

2.

Cuts	Area (in. <sup>2</sup> )
0	324
1	162
2	81
3	40.5
4	20.25
5	10.125
6	5.0625
7	2.53125
8	1.265625
9	0.6328125
10	0.31640625

- a.  $A = 324(\frac{1}{2})^n$   
 b. 9 cuts  
 c. If the paper were at least 4,096 in.<sup>2</sup>, he would be able to make 12 cuts:  
 $1 \cdot 2^{12} = 4,096$ .

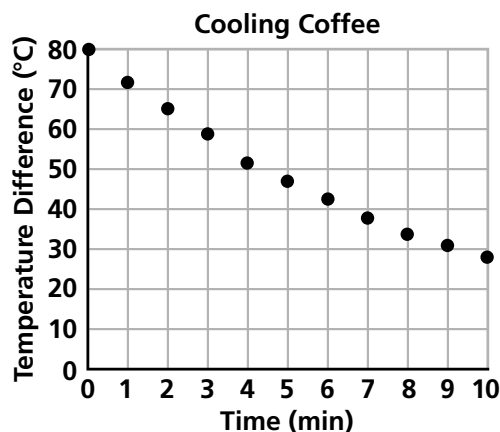
3. a.

Days Since Dose	Penicillin in Blood (mg)
0	300
1	180
2	108
3	64.8
4	38.9
5	23.3
6	14.0
7	8.4

- b.  $d = 300(0.6^m)$   
 c.  $d = 400(0.6^m)$ , assuming the decay factor remains the same

4. Exponential growth because  $2.1 > 1$   
 5. Exponential decay because  $0.5 < 1$   
 6. a. The decay factor is  $\frac{1}{3}$  and the y-intercept is 300.  
 b.  $y = 300(\frac{1}{3})^x$

7. a.

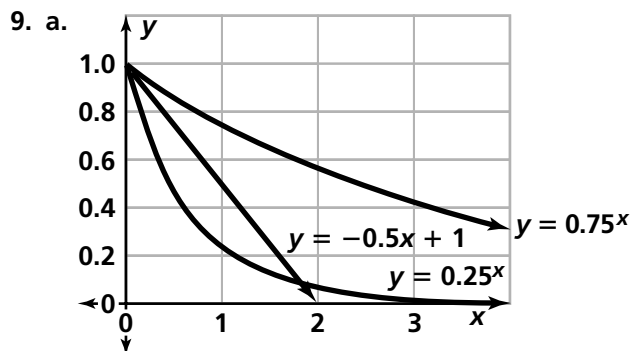


There is a slight curve in the graph, suggesting that the temperature dropped a bit more rapidly just after it was poured. The differences between the first several pairs of temperatures in the table reflect this pattern.

- b. Averaging the ratios between successive temperature differences gives a decay factor of  
 $(0.90 + 0.90 + 0.89 + 0.90 + 0.90 + 0.91 + 0.88 + 0.89 + 0.91 + 0.90) \div 10 \approx 0.90$ .  
 c.  $d = 80(0.90^n)$ , where  $d$  is temperature difference and  $n$  is time in min.  
 d. Theoretically, if the temperature decline followed an exponential pattern, the temperature would never exactly equal room temperature. However, the difference between coffee temperature and room temperature would have been less than 1°C after 42 min:  $d = 80(0.90^{42}) = 0.96^\circ\text{C}$ .

## Connections

8. a. Molecules :  $3.34 \times 10^{22}$   
 b. Red blood cells:  $2.5 \times 10^{13}$   
 c. Earth to sun:  $9.3 \times 10^7$  mi;  $1.5 \times 10^8$  km  
 d. Age of universe:  $1.8 \times 10^{10}$  yr  
 Big Bang temperature:  $1.0 \times 10^{11}^\circ\text{C}$



- b. The three graphs intersect at (0, 1). The graphs of  $y = -0.5x + 1$  and  $y = (0.25)^x$  also intersect at about (1.85, 0.075). In Quadrant II, there is a point of intersection for  $y = -0.5x + 1$  and  $y = (0.75)^x$ .
- c. The graph of  $y = (0.25)^x$  decreases faster than that of  $y = -0.5x + 1$  until about  $x = 0.7$ . The graph of  $y = -0.5x + 1$  decreases the fastest for  $x$ -values greater than 0.7.
- d. Because the graph of  $y = -0.5x + 1$  is a straight line, it is not an example of exponential decay.
- e. The equation  $y = -0.5x + 1$  does not include a variable exponent, so it is not an example of exponential decay.

10. a.

Hop	Location
1	$\frac{1}{2}$
2	$\frac{3}{4}$
3	$\frac{7}{8}$
4	$\frac{15}{16}$
5	$\frac{31}{32}$
6	$\frac{63}{64}$
7	$\frac{127}{128}$
8	$\frac{255}{256}$
9	$\frac{511}{512}$
10	$\frac{1,023}{1,024}$

- b.  $1 - \left(\frac{1}{2}\right)^n$ , or  $\frac{2^n - 1}{2^n}$
- c. No; the numerator is always 1 less than the denominator. This means that the fraction approaches, but never reaches, 1.
11. a. circumference =  $\pi d = 5\pi \approx 15.7$  in.,  
 area =  $\pi r^2 = 6.25\pi \approx 19.6$  in.<sup>2</sup>
- b. (Figure 2) **NOTE:** Students may round answers in different ways and at different stages. This is a good opportunity to have a discussion about rounding.
- c. diameter =  $5(0.9)^n$   
 circumference =  $15.7(0.9)^n$   
 area =  $19.6(0.81)^n$
- d. diameter =  $5(0.75)^n$   
 circumference =  $15.7(0.75)^n$   
 area =  $19.6(0.5625)^n$
- e.  $0.75 = \frac{3}{4}$ ;  $0.5625 = \frac{9}{16}$

Figure 2

Reduction Number	Diameter (in.)	Circumference (in.)	Area (in. <sup>2</sup> )
0	5.0	15.71	19.63
1	4.5	14.14	15.9
2	4.05	12.72	12.88
3	3.65	11.47	10.46
4	3.28	10.3	8.45
5	2.95	9.27	6.83

- f. Possible answer: Yes; a 10% reduction can be represented by the expression  $x - 0.10x$ ; 90% of original size can be represented by  $0.9x$ . These expressions are equivalent.

**Note to the Teacher** Common language is somewhat ambiguous about the meaning of “reduction in size.” If we mean reduction in dimensions, the reasoning above applies. If we mean reduction in area, it does not apply.

12. a. 0.8. This is less than 0.9, so its product with any number will be less than the product of the same number and 0.9.  
b.  $\frac{2}{10}, \frac{2}{9}, (0.8)^4, (0.9)^4, (0.9)^2, 0.84, 90\%$

## Extensions

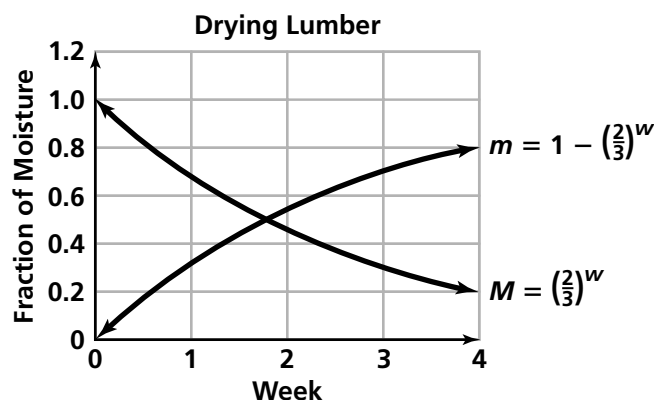
13. (Note: A table is helpful for answering these questions. See Figure 3. Also, this would be a good time for students to learn how to display an answer in fractional form on their calculators. The decimal form of  $(\frac{2}{3})^5$  is 0.1316872428, which is not very helpful when one is looking for patterns. See page 15 for a description of how to convert decimals to fractions with a calculator.)

- a.  $\frac{32}{243}$                       b.  $1 - \frac{32}{243} = \frac{211}{243}$

c.  $m = (\frac{2}{3})^w$

d.  $m = 1 - (\frac{2}{3})^w$

- e. The graphs are mirror images of each other around the line  $y = 0.5$ . One approaches the  $x$ -axis, showing that the moisture remaining approaches 0; the other approaches the line  $y = 1$ , showing that the moisture removed approaches 100%.



- f. moisture remaining =  $(\frac{3}{4})^w$   
moisture removed =  $1 - (\frac{3}{4})^w$

Figure 3

Week	Fraction of Moisture Removed	Total Fraction of Moisture Removed	Fraction of Moisture Remaining
1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
2	$\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$	$\frac{1}{3} + \frac{2}{9} = \frac{5}{9}$	$\frac{4}{9}$
3	$\frac{1}{3} \times \frac{4}{9} = \frac{4}{27}$	$\frac{5}{9} + \frac{4}{27} = \frac{19}{27}$	$\frac{8}{27}$
4	$\frac{1}{3} \times \frac{8}{27} = \frac{8}{81}$	$\frac{19}{27} + \frac{8}{81} = \frac{65}{81}$	$\frac{16}{81}$
5	$\frac{1}{3} \times \frac{16}{81} = \frac{16}{243}$	$\frac{65}{81} + \frac{16}{243} = \frac{211}{243}$	$\frac{32}{243}$

- g. These graphs are also mirror images about the line  $y = 0.5$ . They are stretched out farther to the right, which indicates that the moisture removal proceeds more slowly. (Figure 4)
- h. Possible answer: we need to go from a moisture content of 40% to one of 10%. For the first kiln, the equation is  $0.1 = 0.4\left(\frac{2}{3}\right)^w$ . Because  $0.4\left(\frac{2}{3}\right)^3 \approx 11.9\%$  and  $0.4\left(\frac{2}{3}\right)^4 = 7.9\%$ , the first kiln would produce this loss in 3 to 4 wk. For the second kiln, the equation is  $0.1 = 0.4\left(\frac{3}{4}\right)^w$ . Because  $0.4\left(\frac{3}{4}\right)^4 \approx 12.7\%$  and  $0.4\left(\frac{3}{4}\right)^5 \approx 9.5\%$ , the second kiln would produce this loss in 4 to 5 wk.

### Possible Answers to Mathematical Reflections

1. If the  $x$ -values are equally spaced, and if there is a constant ratio between each  $y$ -value and the previous  $y$ -value and that ratio is between 0 and 1, then the data show an exponential decay pattern.
2. The pattern is a curve that drops downward from left to right, eventually becoming almost horizontal.
3. Exponential decay patterns have equations of the form  $y = a \cdot b^x$ , with  $a > 0$  and  $b$  between 0 and 1.

4. Exponential growth and decay both have equations of the form  $y = a \cdot b^x$ , where  $a > 0$ . For exponential growth,  $b$  is greater than 1. For exponential decay,  $b$  is between 0 and 1. A graph of exponential decay is decreasing, and a graph of exponential growth is increasing. In a table, both exponential growth and decay are indicated by a constant ratio between each  $y$ -value and the previous  $y$ -value (assuming the  $x$ -values increase by a constant amount). However, in a growth situation, the ratio is greater than 1. In a decay situation, the ratio is between 0 and 1.
5. In both exponential decay situations and decreasing linear relationships,  $y$  decreases as  $x$  increases. In a table, we can see this difference easily. If the *difference* between consecutive  $y$ -values is constant and decreasing, the table exhibits a linear relationship. If the *ratio* of consecutive  $y$ -values is constant and between 0 and 1, the table exhibits an exponential decay pattern. A graph of a linear function looks like a straight line and has an  $x$ -intercept, and a graph of an exponential function is curved and does not have an  $x$ -intercept (if it is of the form  $y = a \cdot b^x$ , where  $a > 1$ ), although it may sometimes appear linear depending on the scale.

Figure 4

