

## 4.3

## Putting It All Together

### Goals

- Identify the first and second differences for a quadratic relationship represented in a table
- Summarize the understandings about quadratic functions

Students continue to use tables to find patterns characteristic of quadratic relations. By focusing on the first and second differences of the  $y$ -values in a table for a relation, they are able to determine whether or not it is a quadratic relation.

### Launch 4.3

Put up the following table for a linear equation:

Table for a Linear Equation

$x$	$y$
0	1
1	4
2	7
3	10
4	13
5	16

### Suggested Questions

- *Do the data in the table represent a quadratic relationship? Explain why.* (No, each successive  $y$ -value is increasing by 3. It is a linear relationship whose slope is 3 and  $y$ -intercept is 1. The equation for this relationship is  $y = 3x + 1$ .)

The constant difference, 3, is called the *first difference*. Let's look at the first differences for quadratic relationships.

Go over the example in the Getting Ready.

Put the equation  $y = x^2$  and its table on the overhead.

- *Does this equation represent a quadratic relationship?* (yes)
- *Let's look at the first differences between successive values of  $y$ .*

Generate the table for values of  $x$  from  $-5$  to  $5$ . Then take the first difference.

- *Is the first difference constant?* (no)
- *What happens if you take the differences of the first differences?* (They are all equal to 2.)

This difference is called the second difference. For this quadratic function the second differences are constant; they are all 2.

In this problem we will investigate to see if other quadratic functions have a similar pattern.

Let the class work in groups of two to four.

### Explore 4.3

You could ask different groups to put their work for one of the equations in Question A on poster paper for the summary. Ask a couple of groups to put the work for all of the equations for Question B on poster paper.

### Going Further

Use your calculator to explore the effects of the  $b$  and  $c$  in the equation,  $y = ax^2 + bx + c$ . Start with the basic equation of  $y = x^2$  and then add the parameters,  $a$ ,  $b$ , and  $c$ , one at a time to determine their effects. (The coefficient,  $a$ , affects the width of the parabola and whether it is an upside down or an upright parabola. The constant,  $c$ , shifts the graph vertically up or down, while  $b$  shifts the graph horizontally to the left or right.)

### Summarize 4.3

**Suggested Questions** As a class, discuss a few of the equations in Problem 4.3A.

- *In any of the tables, is there a constant rate of change for  $y$ ?* (no)
- *What does this tell you about these equations?* (They are not linear.)

Each equation takes a second step to get the constant differences, and so none of the equations is linear.

- If you were to graph these equations, what would the graphs look like? How does the equation or table help you predict the graph? (Each one is a parabola. We expect a parabola if there is an  $x^2$  term or if the second differences are constant.)
- Which parabolas would open upward? Which would open downward? ( $y = 3x - x^2$  is the only one that opens downward. It has a maximum point. The others all have a minimum point. We expect a maximum if the coefficient of  $x^2$  is negative, or a minimum if the coefficient of  $x^2$  is positive.)

Be sure to discuss 4.3 B. See the answers for guidance. Then discuss another quadratic equation to check for understanding.

### Check for Understanding

Recognizing patterns of change from tables: Put a few tables on the overhead or board and ask what relationship each represents.

- What relationship does each set of data represent? Explain why.  
[Linear,  $y = 3 - 2x$ ; quadratic,  $y = 1 - x^2$ ; exponential,  $y = 2(3^x)$ ; quadratic,  $y = x^2 - 1$ ]
- Describe important features of the relationship, such as intercepts, maximum or minimum points, and symmetry.

Tables of Various Relationships

Table 1	
$x$	$y$
-1	5
0	3
1	1
2	-1
3	-3

Table 3	
$x$	$y$
0	2
1	6
2	18
3	54
4	162

Table 2	
$x$	$y$
-1	0
0	1
1	0
2	-3
3	-8

Table 4	
$x$	$y$
-1	0
0	-1
1	0
2	3
3	8

Recognizing patterns of change from graphs: Each of the graphs represents a different relationship.

- Analyze each graph below and indicate how the graph reflects the patterns of change for that particular relationship.
- For linear and exponential relationships, explain how the ratio of vertical change to horizontal change between two points on the graph is related to the pattern of change for that relationship.
- Does a similar pattern hold for quadratic relationships? Explain.

These graphs and tables can be found on Transparencies 1.2C and 1.2D.

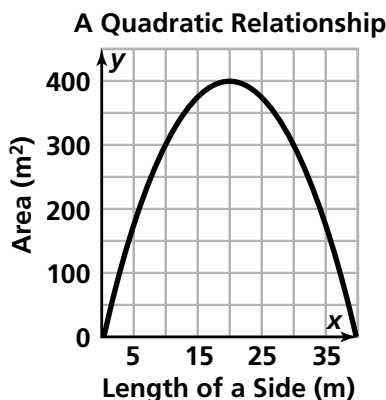
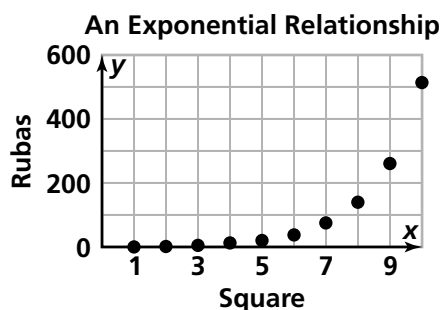
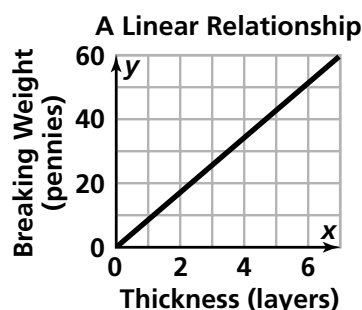


Table For a Linear Relationship

Thickness (layers)	Breaking Weight (pennies)
0	0
1	8.4
2	16.8
3	25.2
4	33.6
5	42.0
6	50.4
7	58.8

Table For an Exponential Relationship

Square	Rubas
1	1
2	2
3	4
4	8
5	16
6	32
7	64
8	128
9	256
10	512

Table For a Quadratic Relationship

Length of a side (m)	Area (m <sup>2</sup> )
0	0
5	175
10	300
15	375
20	400
25	375
30	300
35	175
40	0

### Going Further

You could have students examine the first, second, third (and so forth) differences for the power functions:  $y = x$ ,  $y = x^2$ ,  $y = x^3$ , and  $y = x^4$ , look for patterns and then predict what would happen for  $y = x^5$ ....  $y = x^n$ .



# 4.3

## Putting It All Together

**At a Glance**

PACING 1 day

### Mathematical Goals

- Identify the first and second differences for a quadratic relationship represented in a table
- Summarize the understandings about quadratic functions

### Launch

Put up a table for a linear equation with a rate of change of 3:

- *Does the data in the table represent a quadratic relationship? Explain.*

The constant difference, 3, is called the *first difference*. Let's look at the first differences for quadratic relationships.

Use Transparency 4.3 for the Getting Ready.

Let the class work in groups of two to four.

### Materials

- Transparency 4.3
- Poster paper (optional)
- Labsheets 4.3A and B (one per student)

### Explore

Ask different groups to put their work for one of the equations in Question A on poster paper for the summary. Ask a couple of groups to put the work for all three of the equations for Question B on poster paper.

### Summarize

As a class, discuss a few of the equations in Problem 4.3A.

- *In any of the tables, is there a constant rate of change?*
- *What does this tell you about these equations?*
- *If you were to graph these equations, what would the graphs look like? How does the equation or table help you predict the graph? Which parabolas would open upward? Downward?*

To check for understanding you might put a few tables on the overhead or board and ask what relationship each represents (for example table for  $y = 3 - 2x$ ,  $y = 1 - x^2$ ,  $y = 2(3^x)$  and  $y = x^2 - 1$ ).

- *What kind of relationship does each set of data represent? Explain why. Describe important features of the relationship, such as intercepts, maximum or minimum points, and symmetry.*

Each of the graphs represents a different relationship.

- *Analyze each graph and indicate how the graph reflects the patterns of change for that particular relationship.*
- *For linear and exponential relationships, explain how the ratio of vertical change to horizontal change between two points on the graph is related to the pattern of change for that relationship. Does a similar pattern hold for quadratic relationships? Explain.*

### Materials

- Student notebooks

## ACE Assignment Guide for Problem 4.3

**Differentiated  
Instruction**  
Solutions for All Learners

Core 18–22, 25, 26

Other Applications 23, 24; Connections 36–40;  
Extensions 55; unassigned choices from previous  
problems

**Adapted** For suggestions about adapting ACE  
exercises, see the *CMP Special Needs Handbook*.

### Answers to Problem 4.3

A. 1. a.

$$y = 2x(x + 3)$$

$x$	$2x(x + 3)$		First Differences	Second Differences
-5	20			
-4	8		-12	
-3	0		-8	4
-2	-4		-4	4
-1	-4		0	4
0	0		4	4
1	8		8	4
2	20		12	4
3	36		16	4
4	56		20	4
5	80		24	4

b.

$$y = 3x - x^2$$

$x$	$3x - x^2$		First Differences	Second Differences
-5	-40			
-4	-28		12	
-3	-18		10	-2
-2	-10		8	-2
-1	-4		6	-2
0	0		4	-2
1	2		2	-2
2	2		0	-2
3	0		-2	-2
4	-4		-4	-2
5	-10		-6	-2

c.

$$y = (x - 2)^2$$

$x$	$(x - 2)^2$		First Differences	Second Differences
-5	49			
-4	36		-13	
-3	25		-11	2
-2	16		-9	2
-1	9		-7	2
0	4		-5	2
1	1		-3	2
2	0		-1	2
3	1		1	2
4	4		3	2
5	9		5	2

d.

$$y = x^2 + 5x + 6$$

$x$	$x^2 + 5x + 6$		First Differences	Second Differences
-5	6			
-4	2		-4	
-3	0		-2	2
-2	0		0	2
-1	2		2	2
0	6		4	2
1	12		6	2
2	20		8	2
3	30		10	2
4	42		12	2
5	56		14	2

2. For  $y = 2x(x + 3)$ ,  $y = (x - 2)^2$ , and  $y = x^2 + 5x + 6$ , the  $y$ -value first decreases and then increases. For the equation  $y = 3x - x^2$  the  $y$ -value first increases and then decreases. In all four equations, the first differences are not constant: for  $y = 2x(x + 3)$ , they increase by 4; for  $y = (x - 2)^2$  and  $y = x^2 + 5x + 6$ , they increase by 2; and for  $y = 3x - x^2$  they decrease by 2.
3. In all four equations, the second differences are constant.

B. 1. a.  $y = x + 2$

$x$	$y$		
		First Differences	Second Differences
0	2		
1	3	1	0
2	4	1	0
3	5	1	0
4	6	1	0
5	7	1	0

b.  $y = 2x$

$x$	$y$		
		First Differences	Second Differences
0	0		
1	2	2	0
2	4	2	0
3	6	2	0
4	8	2	0
5	10	2	0

c.  $y = 2^x$

$x$	$y$		
		First Differences	Second Differences
0	1		
1	2	1	1
2	4	2	2
3	8	4	4
4	16	8	8
5	32	16	

d.  $y = x^2$

$x$	$y$		
		First Differences	Second Differences
0	1		
1	2	1	2
2	4	3	2
3	9	5	2
4	16	7	2
5	25	9	2

2. In all the tables, for  $x > 0$ , the  $y$ -value increases as the  $x$  value increases. For  $y = x + 2$  and  $y = 2x$ , the change in the  $y$ -value is constant, which means that the  $y$ -value increases at a constant rate. For  $y = 2^x$  and  $y = x^2$ , the  $y$ -value increases at an increasing rate. The second differences for  $y = x^2$  are constant, while the second differences for  $y = 2^x$  increase exponentially.
3. The equations  $y = x + 2$  and  $y = 2x$  fit the general form of linear equations,  $y = mx + b$ . In the table, the constant first differences tell that the equation is linear. The third equation,  $y = 2^x$ , fits the form of an exponential equation,  $y = b^x$ . Since the variable is in the exponent, the base 2 tells the factor by which the  $y$ -value grows. In the table, the growth factor of 2 shows up in the ratio of consecutive  $y$ -values: each difference is twice the previous difference. In  $y = x^2$ , the exponent is 2 and the base is the variable, so the  $y$ -values are the square numbers. In the table we note that first differences are not constant, but second differences are all 2.

