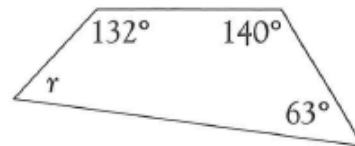




On January 7 of this year, the first presidential election was held in the United States. To learn the year, solve this puzzle.

- My hundreds, tens, and units digits are consecutive integers whose sum is 24.
- The sum of all of my digits is equal to  $r$ .



What year am I?

1789

Thousands

Hundreds

Tens

Units

Once, Chuck Norris only filled his name in on the SAT. He got a perfect score.



### Problem 3.4 Solving Quadratic Equations

- A.**
1. Write  $x^2 + 10x + 24$  in factored form.  $(x+4)(x+6)$
  2. How can you use the factored form to solve  $x^2 + 10x + 24 = 0$  for  $x$ ?  $(x+4)(x+6)=0$  use Z.P.P.
  3. Explain how the solutions to  $0 = x^2 + 10x + 24$  relate to the graph of  $y = x^2 + 10x + 24$ .  $x$ -int
- B.** Solve each equation for  $x$  without making a table or graph.
1.  $0 = (x + 1)(2x + 7)$   $-1$  or  $-\frac{7}{2}$
  2.  $0 = (5 - x)(x - 2)$   $5$  or  $2$
  3.  $0 = x^2 + 6x + 9$   $-3$
  4.  $0 = x^2 - 16$   $\pm 4$
  5.  $0 = x^2 + 10x + 16$   $-8$  or  $-2$
  6.  $0 = 2x^2 + 7x + 6$   $-\frac{3}{2}$  or  $-2$   
 $0 = (2x+3)(x+2)$
  7. How can you check your solutions without using a table or graph?

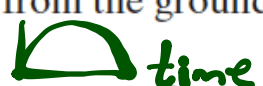
C. Solve each equation for  $x$  without making a table or graph. Check your answers.

1.  $0 = x(9 - x)$   $0$  or  $9$

2.  $0 = -3x(2x + 5)$   $0$  or  $-\frac{5}{2}$

3.  $0 = 2x^2 + 32x$   $0$  or  $-16$

4.  $0 = 18x - 9x^2$   $0$  or  $2$

D. You can approximate the height  $h$  of a pole-vaulter from the ground after  $t$  seconds with the equation  $h = 32t - 16t^2$ . 

1. Suppose the pole-vaulter writes the equation  $0 = 32t - 16t^2$ . What information is the pole-vaulter looking for?

\* 2. The pole-vaulter wants to clear a height of 17.5 feet. Will the pole-vaulter clear the desired height? Explain.

**ACE** Homework starts on page 45.

$$\text{L.O.S. } x = \frac{-b}{2a} = \frac{-32}{2(-16)} = \frac{-32}{-32} = 1$$

$$h = 32(1) - 16(1)^2 = 16 \quad (1, 16)$$

$$17.5 = 32t - 16t^2$$

$$\Rightarrow \odot = 32t - 16t^2 - 17.5$$

$$a = -16$$

$$b = 32$$

$$c = -17.5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Do Now:

Complete the following ACE 3 questions:

# 4 - 26 (evens), 37, 43, 51, and 52

OR

INR 4.1 Pumping  $H_2O$

16.  $2.75 - 7.75(5 - 2x) = 26$

$$2.75 - 38.75 + 15.5x = 26$$

$$-36 + 15.5x = 26$$

$$\begin{array}{r} +36 \\ \hline \end{array}$$

$$+36$$

$$\begin{array}{r} 15.5x \\ \hline 15.5 \end{array}$$

$$= \frac{62}{15.5}$$

$$\boxed{x = 4}$$

10.  $8x + 16 = 6x$

$$\begin{array}{r} -6x \quad -6x \\ \hline \end{array} \quad (SP\epsilon)$$

$$2x + 16 = 0$$

$$\begin{array}{r} -16 \quad -16 \\ \hline \end{array} \quad (SP\epsilon)$$

$$\begin{array}{r} 2x \quad = -16 \\ \hline 2 \quad 2 \end{array}$$

$$\begin{array}{r} \hline x = -8 \end{array} \quad (DP\epsilon)$$

check:

$$8(-8) + 16 = 6(-8)$$

$$-64 + 16 = 6(-8)$$

$$-48 = -48$$

14.  $2x - 3(x + 6) = -4(x - 1)$

$$2x - 3x - 18 = -4x + 4$$

$$-1x - 18 = -4x + 4$$

$$\begin{array}{r} +4x \\ \hline \end{array} \quad \begin{array}{r} +4x \\ \hline \end{array}$$

$$3x - 18 = 4$$

$$\begin{array}{r} +18 \\ \hline \end{array} \quad \begin{array}{r} +18 \\ \hline \end{array}$$

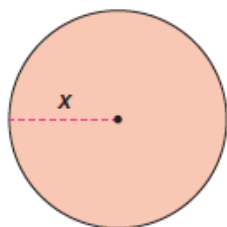
$$\frac{3x}{3} = \frac{22}{3}$$

$$x = \frac{22}{3}$$



36-40 has an area of 24 square meters.

37.



38.



$$A = \pi r^2$$

$$\frac{(24)}{\pi} = \frac{\pi r^2}{\pi} \quad (\text{DPE})$$


---


$$\frac{24}{\pi} = r^2$$

$$\sqrt{r^2} = \sqrt{\frac{24}{\pi}}$$

$$r = \sqrt{\frac{24}{\pi}} \approx 2.76 \text{ m}$$

43. John wants to know if he can bounce a superball over his house. You can approximate the height  $h$  of the superball on one bounce with the equation  $h = 48t - 16t^2$ , where  $t$  is the number of seconds after the ball hits the ground.

a. How long is the ball in the air? **3 sec.**

b. Suppose his house is 30 feet tall. Will the ball make it over his house? Explain.

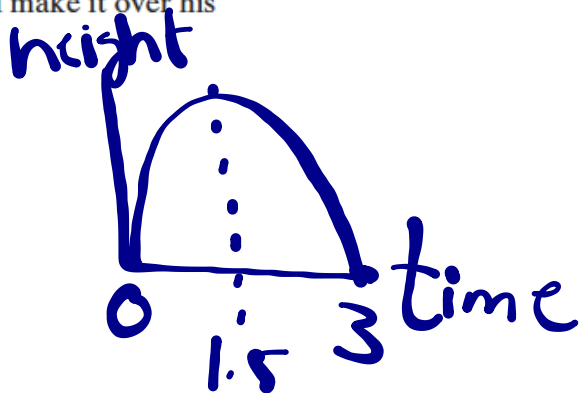
$$h = 48t - 16t^2$$

$$0 = 48t - 16t^2$$

$$0 = 16t(3 - t)$$

$\swarrow$   
 $16t = 0$   
 $t = 0$

$\searrow$   
 $3 - t = 0$   
 $t = 3$



$$h = 48(1.5) - 16(1.5)^2$$

$$= 36 \text{ feet}$$

Algebra

**On January 11** of this year, Chicago schools were closed in the wake of record-breaking  $-26^{\circ}\text{F}$  temperatures. Learn the year by solving this puzzle.

- 3 less than the sum of my units digit and 5 is 4.
- My tens digit is 1 less than my hundreds digit; their sum is 17.
- The sum of all of my digits is equal to the sum of the first five even numbers.

What year am I?

Page 151: The schools closed in 1982.

Thousands

Hundreds

Tens

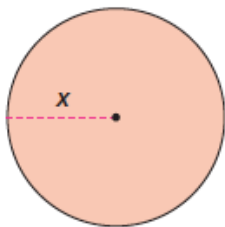
Units

Chuck Norris knows what is at the bottom of a bottomless pit!



36-40 has an area of 24 sq

37.



$$A = \pi r^2$$

$$\frac{24}{\pi} = \frac{\pi r^2}{\pi} \quad (\text{DP}\epsilon)$$

$$\sqrt{r^2} = \sqrt{\frac{24}{\pi}}$$

$$r \approx \sqrt{\frac{24}{\pi}} \approx 2.76$$

Meters

- 43.** John wants to know if he can bounce a superball over his house. You can approximate the height  $h$  of the superball on one bounce with the equation  $h = 48t - 16t^2$ , where  $t$  is the number of seconds after the ball hits the ground.

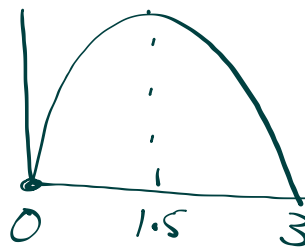
- a.** How long is the ball in the air? *3 seconds*
- b.** Suppose his house is 30 feet tall. Will the ball make it over his house? Explain.

$$0 = 48t - 16t^2$$

$$0 = 16t(3 - t)$$

$$\begin{array}{l} \swarrow \\ 16t = 0 \\ t = 0 \end{array}$$

$$\begin{array}{l} \searrow \\ 3 - t = 0 \\ t = 3 \end{array}$$



$$\begin{aligned} h &= 48(1.5) - 16(1.5)^2 \\ &= 36 \end{aligned}$$

# Investigation 4

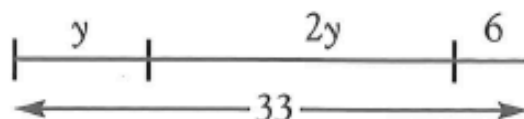
## Looking Back at Functions

**T**hroughout your work in algebra, you have identified patterns of change between variables as linear, exponential, and quadratic functions. You have used tables, graphs, and equations to represent and reason about these functions. In this unit, you have found that writing equivalent expressions for a quantity or variable can reveal new information about a situation. This investigation will help pull these ideas together.

### 4.1 Pumping Water

On January 10 of this year, the League of Nations was established. This was the predecessor of the United Nations and was dissolved 26 years after it was formed. The United States never joined. Learn the year it was established by solving this puzzle.

- My hundreds digit is the value of  $y$  in this diagram:



- The two-digit number formed by my tens and units digits is equal to  $\sqrt{100 \times 4}$ .  $= \sqrt{100} \times \sqrt{4}$   
 $10 \times 2$
- The sum of all of my digits is 1 less than the sixth prime number.

What year am I?

The League of Nations was established in 1920.

Thousands

Hundreds

Tens

Units

Chuck Norris once kicked a horse in the chin. Its descendants are known today as Giraffes.

150

Daily W

$$\begin{aligned}\sqrt{20} &= \sqrt{4} \sqrt{5} \\ &= 2\sqrt{5}\end{aligned}$$



d. For parts (a)–(c), which equations are linear? Explain.

22. A line has a slope of 1.5 and goes through the point  $(2, 5)$ .

a. Find the coordinates of three other points that lie on the line.

b. Find the coordinates of the y-intercept.  $(0, 2)$

c. Find the y-coordinate of the point whose x-coordinate is  $-4$ .

d. Write an equation for the line.

$$y = mx + b$$

$$y = (1.5)x + b$$

$$(5) = (1.5)(2) + b$$

$$\begin{array}{r} -3 \quad -3 \\ \hline \end{array}$$

$$2 = b$$

$$y = 1.5x + 2$$

x	y
1	2
2	3
-4	-1

## 4.1 Pumping Water

Every winter, Magnolia Middle School empties their pool for cleaning. Ms. Theodora's math class decides to collect data on the amount of water in the pool and how long it takes to empty it. They write an equation to represent the amount of water  $w$  (in gallons) in the pool after  $t$  hours.

$$w = -250(t - 5) = -250t + 1250$$

Handwritten arrows point from the equation to the problem questions: from  $-250$  to question 1, from  $(t - 5)$  to question 2, and from  $1250$  to question 3.

### Problem 4.1 Looking at Patterns of Change

A. Answer the following questions. Explain your reasoning.

1. How many gallons of water are pumped out each hour? **250**
2. How long will it take to empty the pool? **5 hours**
3. How many gallons of water are in the pool at the start? **1250 gal**

B. 1. Write an expression for the amount of water in the tank after  $t$  hours that is equivalent to the original expression.

2. What information does this new expression tell you about the amount of water in the tank?

3. Which expression is more useful in this situation? Explain.

C. 1. Describe the pattern of change in the relationship between the two variables  $w$  and  $t$ .

2. Without graphing the equation, <sup>linear</sup> describe the shape of the graph. Include as much information as you can.

D. Suppose the equation for the amount of water  $w$  (in gallons) in another pool after  $t$  hours is  $w = -450(2t - 7) = -900t + 3150$

1. How many gallons of water are pumped out each hour? 900

2. How long will it take to empty the pool? 3.5 hour

3. How many gallons of water are in the pool at the start? 3150

4. Write an expression that is equivalent to  $-450(2t - 7)$ . Which expression is more useful? Explain.  $-900t + 3150$

**ACE** Homework starts on page 60.



## 4.2 Generating Patterns

In this problem, you are given two data points for a linear, exponential, and quadratic relationship. You will use these points to find more data points. Then you will write an equation for each relationship.

linear  
 $y = mx + b$   
 ↓  
 slope  
 ↓  
 y-int

exponential  
 $y = a(b)^x$   
 ↓      ↓  
 y-int   factor

quadratic  
 $y = ax^2 + bx + c$   
 ↓      ↓  
 $\frac{1}{2}$  2nd y-int  
 difference

Find the equation for the table below:

x	y
0	-2
1	6
2	20
3	40
4	66
5	98

$$y = ax^2 + bx + c$$

$$y = 3x^2 + bx - 2$$

$$6 = 3(1)^2 + b(1) - 2$$

$$6 = 3 + b - 2$$

$$\Rightarrow 5 = b$$

$$y = 3x^2 + 5x - 2$$

Chuck Norris once killed a bird by throwing it off a cliff.

$$b = b + 3 - 2$$

$$6 = b + 1$$

$$\begin{array}{r} -1 \\ -1 \\ \hline 5 = b \end{array}$$

Find the equation for the table below:

x	y
0	-2
1	6
2	20
3	40
4	66
5	98

$\Delta y = 8$   
 $\Delta^2 y = 14$   
 $\Delta^3 y = 20$   
 $\Delta^4 y = 26$   
 $\Delta^5 y = 32$

$$y = ax^2 + bx + c$$

$$y = 3x^2 + bx + c$$

$$y = 3x^2 + bx - 2$$

$$(20) = 3(2)^2 + b(2) - 2$$

$$20 = 12 + 2b - 2$$

$$20 = 10 + 2b$$

$$10 = 2b \Rightarrow 5 = b$$

$$y = 3x^2 + 5x - 2$$

Chuck Norris once kicked a horse in the chin. Its descendants are known today as Giraffes.

## Problem 4.2 Linear, Exponential, Quadratic

- A. The first two rows in a table of numbers are given below. Write four more numbers in each column to make a linear relationship, an exponential relationship, and a quadratic relationship.

$$y = mx + b$$

Data Points

$$y = a(b)^x$$

x	Linear y	Exponential y	Quadratic y
1	1	1	1
2	4	4	4
3	7	16	11
4	10	64	22
5	13	256	37
6	16	1024	56

$$y = ax^2 + bx + c$$

$$y = 3x - 2 \quad y = \frac{1}{4}(4)^x \quad y = 2x^2 - 3x + 2$$

- B. Explain why the pattern in each column is correct.
- C. 1. Write an equation for each relationship. Explain what information the variables and numbers represent.
2. Compare your equations with those of your classmates. Do you all have the same equations? Explain.



Homework starts on page 60.

## 4.3 Sorting Functions

**I**n the following problem, a set of equations relating  $x$  and  $y$  is given. Some of the expressions for  $y$  are in factored form, and some are in expanded form.

*Which form is easier to use to determine whether a function is linear, exponential, quadratic, or none of these?*

*Which form is easier to use to determine the  $x$ - and  $y$ -intercepts, rates of change, and maximum or minimum points of the graph of the function?*



(1)  $y = x^2 + 8x + 16$

(2)  $y = \frac{1}{3}(3^x)$

(3)  $y = 10 - 2x$

(4)  $y = 2x^3 + 5$

(5)  $y = (x^2 + 1)(x^2 + 3)$

(6)  $y = 0.5^x$

(7)  $y = 22 - 2x$

(8)  $y = \frac{3}{x}$

(9)  $y = (x + 4)(x + 4)$

(10)  $y = (4x - 3)(x + 1)$

(11)  $y = 20x - 4x^2$

(12)  $y = x^2$

(13)  $y = 3^x - 1$

(14)  $y = 16 - 2(x + 3)$

(15)  $y = 4x^2 - x - 3$

(16)  $y = x + \frac{1}{x}$

(17)  $y = 4x(5 - x)$

(18)  $y = 2(x - 3) + 6(1 - x)$

**A.** Which equations represent functions that are

- 1.** linear?                      **2.** exponential?                      **3.** quadratic?

**B. 1.** For each function in Question A, find those equations that represent the same function.

**2.** Without graphing the equation, describe the shape of the graph of those equations in part (1). Give as much detail as possible, including patterns of change, intercepts, and maximum and minimum points.

**C.** Pick one linear, one quadratic, and one exponential equation. Describe a problem that could be represented by each equation.

**On January 19** of this year, the world's record snowfall fell on London, England. Drifts of 4.5 meters were measured.

(How many feet is 4.5 meters?) Solve this puzzle to learn the year.

- My date is a palindrome with a sum of 18.

What year am I?

Thousands

Hundreds

Tens

Units

