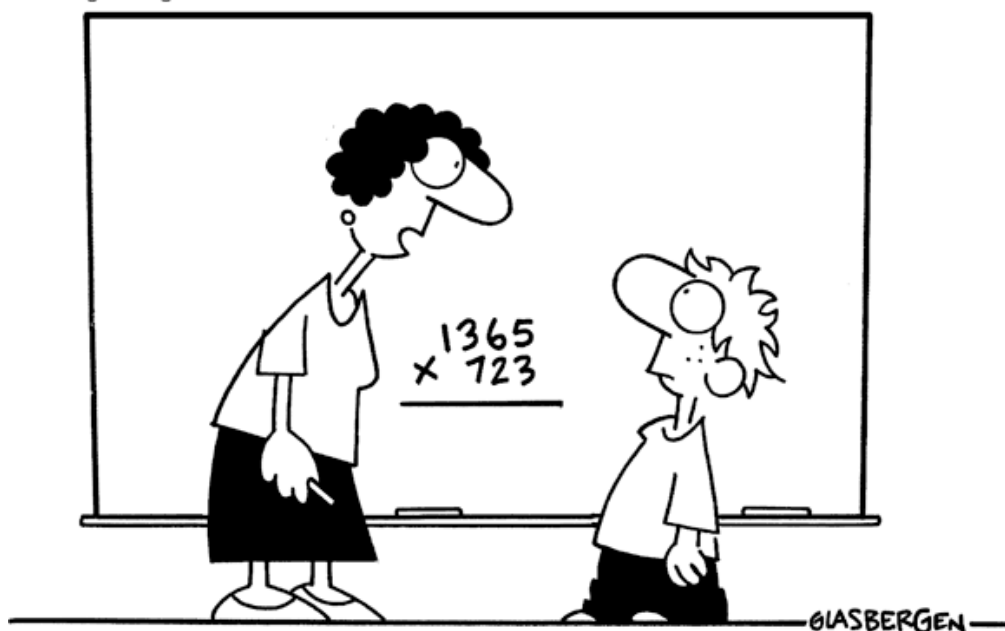


## Complete ACE questions starting on page 12 # 1-6 and 18-26

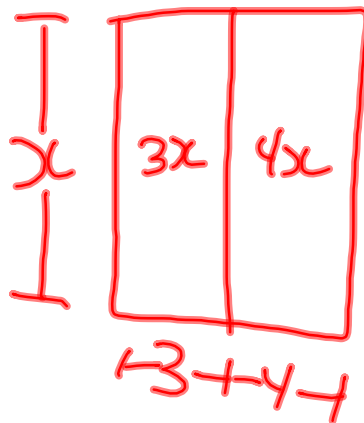
Complete ACE questions starting on

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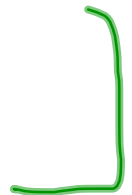


**"Pretend you're starring in a reality show about a kid who can make his dreams come true if he works hard and gets good grades."**

$$23) \quad 3x + 4x$$



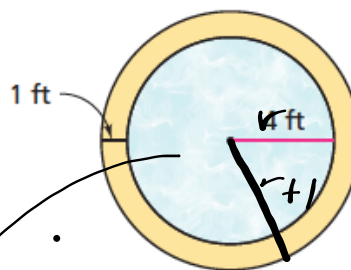
$$x(3+4)$$



,

24. A circular pool with a radius of 4 feet has a 1-foot border.

- What is the surface area of the circular pool?
- What is the surface area of the border?
- Write an expression for the surface area of a circular pool with a radius of  $r$  feet.
- Write an expression for the area of a 1-foot border around a circular pool with a radius of  $r$  feet.



$2\pi r + \pi$

Large  $\Rightarrow A = 25\pi$

Border =  $25\pi - 16\pi = 9\pi \text{ ft}^2$

$A = \pi r^2$

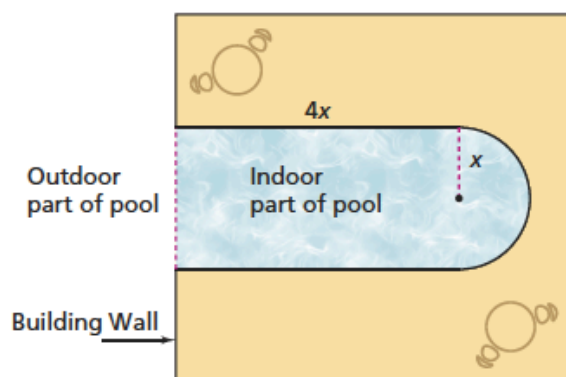
$A = \pi(r+1)^2 - \pi r^2$

$= \pi(r^2 + 2r + 1) - \pi r^2 = \cancel{\pi r^2} + 2\pi r + \pi$

### 1.3 The Community Pool Problem

In this problem, we will interpret symbolic statements and use them to make predictions.

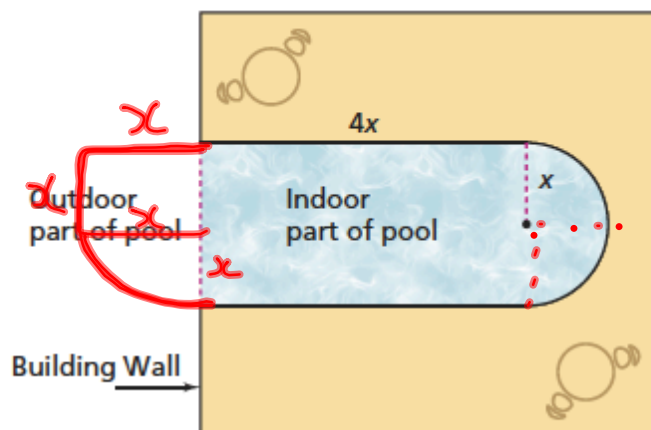
A community center is building a pool, part indoor and part outdoor. A diagram of the indoor part of the pool is shown. The indoor shape is made from a half-circle with radius  $x$  and a rectangle with length  $4x$ .



#### Problem 1.3 Interpreting Expressions

The exact dimensions of the community center pool are not available, but the area  $A$  of the whole pool is given by the equation:

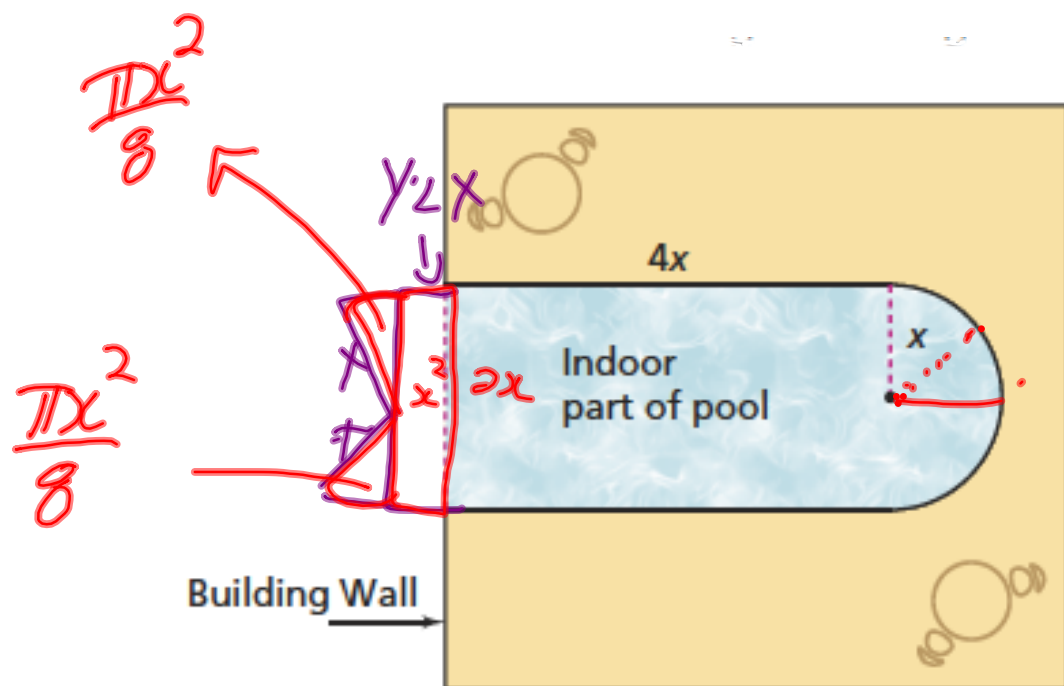
$$A = x^2 + \frac{\pi x^2}{2} + 8x^2 + \frac{\pi x^2}{4}$$

**Problem 1.3 Interpreting Expressions**

The exact dimensions of the community center pool are not available, but the area  $A$  of the whole pool is given by the equation:

$$A = x^2 + \frac{\pi x^2}{2} + 8x^2 + \frac{\pi x^2}{4}$$

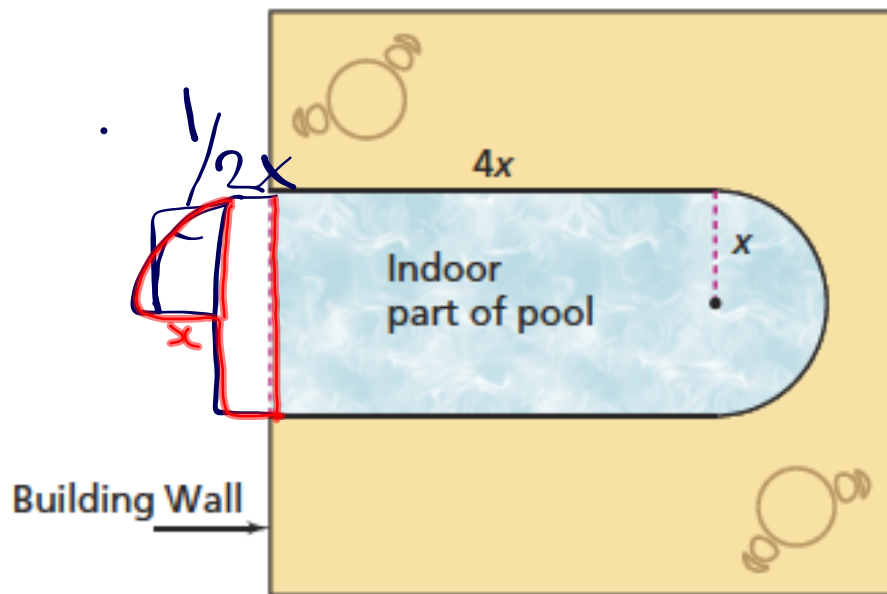
- A.** Which part of the expression for area represents
1. the area of the indoor part of the pool? Explain.
  2. the area of the outdoor part of the pool? Explain.
- B.**
1. Make a sketch of the outdoor part. Label the dimensions.
  2. If possible, draw another shape for the outdoor part of the pool. If not, explain why not.



$$A = x^2 + \frac{\pi x^2}{2} + 8x^2 + \frac{\pi x^2}{4}$$

Labels with arrows pointing to terms in the equation:

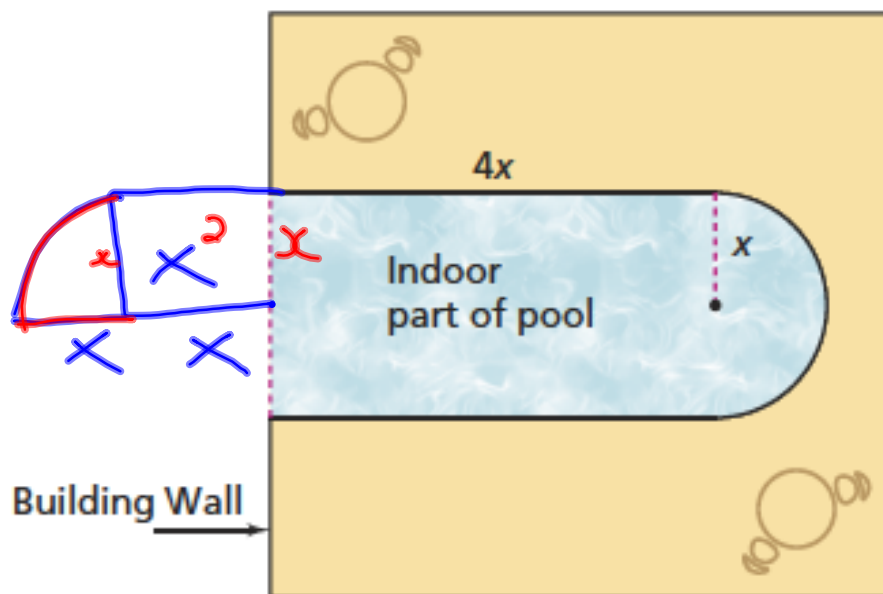
- "outdoor part" points to  $x^2$
- "indoor part" points to  $\frac{\pi x^2}{2}$
- The term  $\frac{\pi x^2}{4}$  is circled in red.



$$A = x^2 + \frac{\pi x^2}{2} + 8x^2 + \frac{\pi x^2}{4}$$

Diagram illustrating the components of the area  $A$ :

- The terms  $x^2$  and  $\frac{\pi x^2}{2}$  are grouped under the label "outdoor part".
- The terms  $8x^2$  and  $\frac{\pi x^2}{4}$  are grouped under the label "indoor part".



$$A = x^2 + \frac{\pi x^2}{2} + 8x^2 + \frac{\pi x^2}{4}$$

Diagram showing the breakdown of the area formula  $A$  into outdoor and indoor parts:

- The terms  $x^2$  and  $\frac{\pi x^2}{2}$  are grouped under the label "outdoor part".
- The terms  $8x^2$  and  $\frac{\pi x^2}{4}$  are grouped under the label "indoor part".



$A = x^2 + \frac{\pi x^2}{2} + 8x^2 + \frac{\pi x^2}{4}$

outdoor part (points to  $x^2$  and  $\frac{\pi x^2}{4}$ )  
 indoor part (points to  $\frac{\pi x^2}{2}$  and  $8x^2$ )

equation for the area of the outdoor catch.

Stella:  $x^2 + \frac{\pi x^2}{8} + \frac{\pi x^2}{8}$

Jerri:  $\left(\frac{1}{2}x\right)(2x) + \frac{\pi x^2}{4}$

1. Explain the reasoning each person may have used to write their expression.
  2. Decide if these expressions are equivalent to the original expression in Question A, part (2). Explain your reasoning.
- D.** Does the equation for the area of the pool represent a linear, exponential, or quadratic relationship, or none of these? Explain.

**ACE** Homework starts on page 12.

## 1.4 Diving In

In the pool tile problems, you found patterns that could be represented by several different but equivalent symbolic expressions, such as:

$$\begin{aligned} 4s + 4 \\ 4(s + 1) \\ s + s + s + s + 4 \\ 2s + 2(s + 2) \end{aligned}$$

The equivalence of these expressions can be shown with arrangements of tiles. Equivalence also follows from properties of numbers and operations.

An important property is the **Distributive Property:**

For any real numbers  $a$ ,  $b$ , and  $c$ :

$$a(b + c) = ab + ac \text{ and } a(b - c) = ab - ac$$

For example, this property guarantees that  $4(s + 1) = 4s + 4$  for any  $s$ .

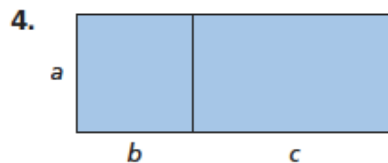
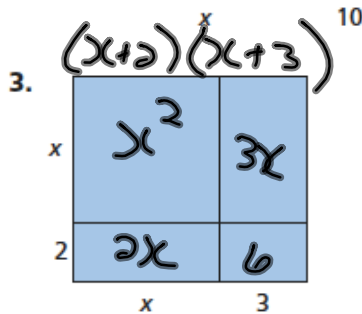
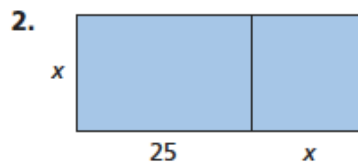
We say that  $a(b + c)$  and  $4(s + 1)$  are in *factored form* and  $ab + ac$  and  $4s + 4$  are in *expanded form*.

The next problem reviews the Distributive Property.

### Getting Ready for Problem 1.4

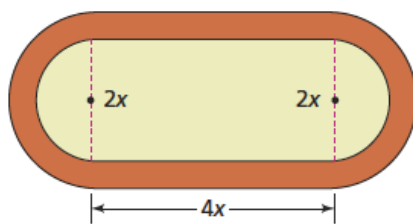
Swimming pools are sometimes divided into sections that are used for different purposes. A pool may have a section for swimming laps and a section for diving, or a section for experienced swimmers and a section for small children.

Below are diagrams of pools with swimming and diving sections. The dimensions are in meters.



- For each pool, write two different but equivalent expressions for the total area.
- Explain how these diagrams and expressions illustrate the Distributive Property.

50. Below is a diagram of Otter Middle School's outdoor track. The shape of the interior region (shaded green) is a rectangle with two half circles at each end.



$$4x + 4x + 2\frac{\pi x}{2} + \frac{2\pi x}{2}$$

- Find an expression that represents the area of the interior region.
- Find the perimeter of the interior region as if you wanted to put a fence around it. Explain how you found your answer.
- Write an expression equivalent to the one in part (b).

50. a. Possible answers:  $(2x)(4x) + \pi(x)^2$  or  $\frac{1}{2}\pi(x)^2 + \frac{1}{2}\pi(x)^2 + 8x^2$
- b. The fencing needed for the rectangular region is  $4x + 4x = 8x$  since you don't count the two shorter sides. The two half circles each have a perimeter of  $\frac{1}{2}\pi(2x)$ , which is half of the circumference  $\pi(2x)$ . So the perimeter is  $8x + 2[\frac{1}{2}\pi(2x)]$  or  $2\pi x + 8x$ .
- c. Possible answers:  $\pi x + \pi x + 4x + 4x$  or  $(2\pi + 8)x$ .

$$A = 8x^2 + \pi x^2$$

The Distributive Property, as well as the Commutative Property and other properties for numbers, are useful for writing equivalent expressions. The Commutative Property states that  $a + b = b + a$  and  $ab = ba$ , where  $a$  and  $b$  are real numbers. These properties were discussed in previous units.

### Problem 1.4 Revisiting the Distributive Property

A. Write each expression in expanded form.

1.  $3(x + 5) = 3x + 15$

2.  $2(3x - 10) = 6x - 20$

3.  $2x(x + 5) = 2x^2 + 10x$

4.  $(x + 2)(x + 5) = x^2 + 7x + 10$

B. Write each expression in factored form.

1.  $12 + 24x = 12(1 + 2x)$

2.  $x + x + x + 6 = 3x + 6 = 3(x + 2)$

3.  $x^2 + 3x = x(x + 3)$

4.  $x^2 + 4x + 3 = (x + 3)(x + 1)$

C. The following expressions all represent the number of border tiles  $N$  for a square pool with side length  $s$ .

$$\left. \begin{array}{l} 4(s + 1) \\ s + s + s + s + 4 \\ 2s + 2(s + 2) \\ 4(s + 2) - 4 \\ (s + 2)^2 - s^2 \end{array} \right\} 4s + 4$$

Use the Distributive and Commutative properties to show that these expressions are equivalent.

Expressions are equivalent.

- D. Three of the following expressions are equivalent. Explain which expression is not equivalent to the other three.

1.  $2x - 12x + 10 = -10x + 10$       2.  $12x - 2x + 10 = 10x + 10$   
 3.  $10 - 10x = -10x + 10$       4.  $10(1 - x) = -10x + 10$

- E. Copy each equation. Insert one set of parentheses in the expression to the left of the equal sign so that it is equivalent to the expression to the right of the equal sign.

1.  $6(p + 2) - 2p = 4p + 12$   
 2.  $6p + (2 - 2)p = 6p$

**ACE** Homework starts on page 12.

## Investigation

## 2

### Combining Expressions

**I**n the last investigation, you found several ways to write equivalent expressions to describe a quantity. You also learned several ways to show that two expressions are equivalent. We will continue to answer the questions:

- Are the expressions equivalent? Why?
- What information does each equivalent expression represent?

We will also look at ways to create new expressions and to answer the question:

- What are the advantages and disadvantages of using one equation rather than two or more equations to represent a situation?

#### 2.1

#### Walking Together

## 2.1 Walking Together

In *Moving Straight Ahead*, Leanne Gilberto, and Alana enter a walkathon as a team. This means that each person will walk the same number of kilometers. The walkathon organizers offer a prize to the three-person team that raises the most money.

- Leanne has walkathon pledges from 16 sponsors. All of her sponsors pledge \$10 regardless of how far she walks.
- Gilberto has pledges from 7 sponsors. Each sponsor pledges \$2 for each kilometer he walks.
- Alana has pledges from 11 sponsors. Each sponsor pledges \$5 plus \$0.50 for each kilometer she walks.





### Problem 2.1 Adding Expressions

- A. 1. Write equations to represent the money  $M$  that each student will raise for walking  $x$  kilometers.

a.  $M_{\text{Leanne}} = 16(10) = 160$

b.  $M_{\text{Gilberto}} = 7(2x) = 14x$

c.  $M_{\text{Alana}} = 11(5 + 0.50x) = 55 + 5.5x$

2. Write an equation for the total money  $M_{\text{total}}$  raised by the three-person team for walking  $x$  kilometers.

$$M_{\text{Total}} = 160 + 14x + 55 + 5.5x$$

- B. 1. Write an expression that is equivalent to the expression for the total amount in Question A, part (2). Explain why it is equivalent.

2. What information does this new expression represent about the situation?

*215 is the up front \$  
19.5 is the \$/km walked*

$$215 + 19.5x$$

3. Suppose each person walks 10 kilometers. Explain which expression(s) you would use to calculate the total amount of money raised.

*215 + 19.5x b/c it is simple*

- C. Are the relationships between kilometers walked and money raised linear, exponential, quadratic, or none of these? Explain.

## 2.2 Predicting Profit

**T**he manager of the Water City amusement park uses data collected over the past several years to write equations that will help her make predictions about the daily operations of the park.

The daily concession-stand profit in dollars  $P$  depends on the number of visitors  $V$ . The manager writes the equation below to model this relationship.

$$P = 2.50V - 500$$

She uses the equation below to predict the number of visitors  $V$  based on the probability of rain  $R$ .

$$V = 600 - 500R$$

- What information might each of the numbers in the equations represent?

$$P = 2.50V - 500$$

o predict the numl

$$V = 600 - 500R$$

(A1) 1st  $V = 600 - 500(.25) = 475$

$$P = 2.50(475) - 500 = \$687.50$$

(A2)  $625 = 2.5V - 500$

$$\Rightarrow 1125 = 2.5V$$

$$\Rightarrow 450 = V$$

$$450 = 600 - 500R$$

$$\Rightarrow -150 = -500R$$

$$\Rightarrow (300 = R)$$

**Problem 2.2 Substituting Equivalent Expressions**

- A.** 1. Suppose the probability of rain is 25%. What profit can the concession stand expect? Explain.
2. What was the probability of rain if the profit expected is \$625? Explain your reasoning.
- B.** 1. Write an equation that can be used to predict the concession-stand profit  $P$  from the probability of rain  $R$ .
2. Use this equation to predict the profit when the probability of rain is 25%. Compare your answer with your result in Question A, part (1).
- C.** 1. Write an equivalent expression for the profit in Question B. Explain why the two expressions are equivalent.
2. Predict the probability of rain on a day when the concession-stand profit is \$625. Compare your answer with the result you found in Question A, part (2).
3. Predict the profit when the probability of rain is 0%. Does your answer make sense? Explain.
4. Predict the profit when the probability of rain is 100%. Does your answer make sense?
- D.** Do the equations in Questions B and C represent a linear, exponential, or quadratic relationship, or none of these? Explain.

- B. 1. Write an equation that can be used to predict the concession-stand profit  $P$  from the probability of rain  $R$ .

$$P = 2.50(V) - 500$$

to predict the numl

$$V = (600 - 500R)$$

$$\begin{aligned} P &= 2.50(600 - 500R) - 500 \\ &= 1500 - 1250R - 500 \\ &= 1000 - 1250R \end{aligned}$$

$$625 = 1000 - 1250R$$

$$\Rightarrow -375 = -1250R$$

$$\Rightarrow .3 = 30\% = R$$

$$P = 100$$

$$\boxed{A = l(s_0 - l)} \quad l$$

$s_0 - l$

## 2.3 Area and Profit—What's the Connection?

In the next problem, you will explore two familiar situations that have an interesting connection.

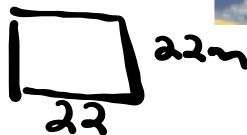
Tony and Paco will operate the water tube concession stand at Water City. Tony is responsible for designing the building that will store the rafts. Paco is responsible for deciding the rental fee for the tubes.

### Problem 2.3 Using Equations

- A. Every concession stand must have a rectangular floor space and a perimeter of 88 meters. Tony wants the greatest area possible.

- Write an equation for the area in terms of the length.  $A = l(44 - l)$
- What is the maximum area for the rectangular floor space?

$$484 \text{ m}^2$$



**B.** Paco knows that on a typical day, the number of tube rentals  $n$  is related to the price to rent each tube  $p$ . Records from other water park locations suggest:

If the tubes are free (no price), there will be 54 rentals.

- Each increase of \$1 in the price will result in one less tube rented.

Paco uses this information to write the following equations:

- Equation 1:  $n = 54 - (1)p$
- Equation 2:  $I = np$ , where  $I$  is the daily income

1. Do these equations make sense? Explain.

2. Write an equation for income in terms of the number of rentals  $n$ .



STOP



- Equation 1:  $n = 54 - (1)p$
- Equation 2:  $I = np$ , where  $I$  is the daily income

2. Write an equation for income in terms of the number of rentals  $n$ .

1st solve for  $p$  so you can substitute into other equations

$$\begin{array}{r}
 n = 54 - p \\
 + p \quad + p \\
 \hline
 n + p = 54 \\
 - n \quad - n \\
 \hline
 p = 54 - n
 \end{array}$$

This one is income in terms of rentals

$$\begin{aligned}
 I &= np \\
 &= n(54 - n) \\
 &= 54n - n^2
 \end{aligned}$$

This one is income in terms of price

$$\begin{aligned}
 I &= np \\
 I &= (54 - p)p
 \end{aligned}$$