

## Practice Problems

1. Write an equation for the line passing through the points  $(-2, 3)$  and  $(1, -3)$ .

$$\begin{aligned} \text{Slope} &= \frac{\Delta y}{\Delta x} \\ +3 \swarrow (-2, 3) \searrow -6 & \quad \frac{\Delta y}{\Delta x} = \frac{-6}{3} \\ (1, -3) & \quad = -2 \\ \text{OR} \\ m &= \frac{3 - (-3)}{-2 - 1} = \frac{6}{-3} = -2 \end{aligned}$$

$$\begin{aligned} y &= mx + b \\ y &= -2x + b \\ 3 &= -2(-2) + b \\ 3 &= 4 + b \\ -4 \quad -4 \\ \hline -1 &= b \end{aligned}$$

$$\boxed{y = -2x - 1}$$

2. Write an equation for the line passing through the points  $(17, 8)$  and  $(17, -2)$ .

$$+10 \swarrow (17, 8) \searrow -10 \\ (17, -2)$$

$$\frac{\Delta y}{\Delta x} = \frac{-10}{0}$$

CAN'T DIVIDE BY ZERO!

No slope, which means a vertical line.

$$m = \frac{8 - (-2)}{17 - 17} = \frac{10}{0}$$

$$\boxed{x = 17}$$

3. Write an equation for the line passing through the points  $(-3, 5)$  and  $(-7, 8)$ .

$$-4 \swarrow (-3, 5) \searrow +3 \\ (-7, 8)$$

$$\frac{\Delta y}{\Delta x} = \frac{3}{-4} = -\frac{3}{4}$$

$$m = \frac{5 - 8}{-3 - (-7)} = \frac{-3}{4}$$

$$y = mx + b$$

$$y = -\frac{3}{4}x + b$$

$$5 = -\frac{3}{4}(-3) + b$$

$$5 = \frac{+9}{4} + b$$

$$-\frac{9}{4} \quad -\frac{9}{4}$$

$$\frac{11}{4} = b$$

$$\boxed{y = -\frac{3}{4}x + \frac{11}{4}}$$

4. Find the equation of the line that has a slope of  $m=4$  and passes through the point  $(-1, -6)$ .

$$y = mx + b$$

$$y = 4x + b$$

$$-6 = 4(-1) + b$$

$$-6 = -4 + b$$

$$+4 \quad +4$$

$$-2 = b$$

OR

$$y - y_1 = m(x - x_1)$$

$$y - (-6) = 4(x - (-1))$$

$$y + 6 = 4(x + 1)$$

$$y + 6 = 4x + 4$$

$$-6 \quad -6$$

$$y = 4x - 2$$

$$\boxed{y = 4x - 2}$$

5. Find the equation of the line that passes through the points  $(-2, 4)$  and  $(1, 2)$ .

$$+3 \begin{pmatrix} (-2, 4) \\ (1, 2) \end{pmatrix} \rightarrow -2$$

$$\frac{\Delta y}{\Delta x} = \frac{-2}{3}$$

$$m = \frac{4-2}{-2-1} = \frac{2}{-3} = -\frac{2}{3}$$

$$y = mx + b$$

$$y = -\frac{2}{3}x + b$$

$$2 = -\frac{2}{3}(1) + b$$

$$2 = -\frac{2}{3} + b$$

$$+\frac{2}{3} \quad +\frac{2}{3}$$

$$\frac{8}{3} = b$$

$$y = -\frac{2}{3}x + \frac{8}{3}$$

6. Find an equation of the line that passes through the points  $(4, 5)$  and  $(7, -1)$ .

$$+3 \begin{pmatrix} (4, 5) \\ (7, -1) \end{pmatrix} \rightarrow -6$$

$$\frac{\Delta y}{\Delta x} = \frac{-6}{3} = -2$$

$$m = \frac{5-(-1)}{4-7} = \frac{6}{-3} = -2$$

$$y = mx + b$$

$$y = -2x + b$$

$$5 = -2(4) + b$$

$$5 = -8 + b$$

$$+8 \quad +8$$

$$13 = b$$

$$y = -2x + 13$$

7. Write an equation for the line that passes through the points  $(2, 7)$  and  $(6, 15)$ .

$$+4 \begin{pmatrix} (2, 7) \\ (6, 15) \end{pmatrix} \rightarrow +8$$

$$\frac{\Delta y}{\Delta x} = \frac{8}{4} = 2$$

$$m = \frac{15-7}{6-2} = \frac{8}{4} = 2$$

$$y = mx + b$$

$$y = 2x + b$$

$$7 = 2(2) + b$$

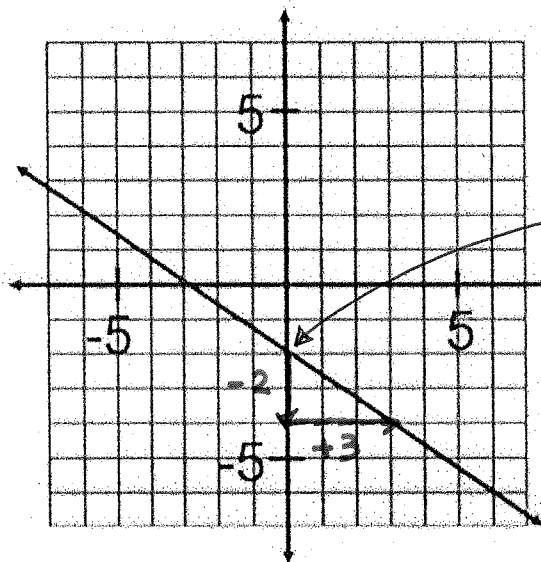
$$7 = 4 + b$$

$$-4 \quad -4$$

$$3 = b$$

$$y = 2x + 3$$

8. Write the equation of the line below.



y-intercept  
 $(0, -2)$

$$\frac{\Delta y}{\Delta x} = \frac{-2}{3}$$

or find slope from  
two points algebraically  
 $(0, -2)$  and  $(3, -4)$

$$\text{slope} = \frac{-4 - (-2)}{3 - 0}$$

$$= \frac{-4 + 2}{3} = -\frac{2}{3}$$

$$y = -\frac{2}{3}x - 2$$

Given the following equations, solve for x:

$$9. \quad 5\left(\frac{7(2x+3)}{5}\right) = (21)5$$

$$\frac{7(2x+3)}{7} = \frac{105}{7}$$

$$\frac{2x+3}{-3} = \frac{15}{-3}$$

$$\frac{2x}{2} = \frac{12}{2}$$

$$x = 6$$

$$\boxed{x = 6}$$

$$10. \quad \frac{2}{3}x - 5 = 3x + 7$$

$$\frac{2}{3}x = 3x + 12$$

$$\left(-\frac{3}{7}\right) \cdot \frac{2}{3}x = 12 \left(-\frac{3}{7}\right)$$

$$x = -\frac{36}{7}$$

$$\boxed{x = -\frac{36}{7}}$$

$$11. \quad 6x - (3x + 8) = 16$$

$$6x - 3x - 8 = 16$$

$$\frac{3x - 8}{+8 +8} = \frac{16}{+8 +8}$$

$$\frac{3x}{3} = \frac{24}{3}$$

$$x = 8$$

$$\boxed{x = 8}$$

$$12. \quad 5x + 2(x + 4) = 64$$

$$5x + 2x + 8 = 64$$

$$\frac{7x + 8}{-8 -8} = \frac{64}{-8 -8}$$

$$\frac{7x}{7} = \frac{56}{7}$$

$$x = 8$$

$$\boxed{x = 8}$$

$$13. \quad 13 - (2x + 2) = 2(x + 2) + 3x$$

$$13 - 2x - 2 = 2x + 4 + 3x$$

$$\frac{11 - 2x}{+2x +2x} = \frac{5x + 4}{+2x +2x}$$

$$\frac{11}{-4} = \frac{7x + 4}{-4}$$

$$\frac{7}{7} = \frac{7x}{7}$$

$$\boxed{x = 1}$$

$$14. \quad 5x - 6 = \frac{2-x}{3} + 4$$

$$3(5x - 10) = \left(\frac{2-x}{3}\right)3$$

$$\frac{15x - 30}{+x} = \frac{2-x}{+x}$$

$$\frac{16x - 30}{+30 +30} = \frac{2}{+30 +30}$$

$$\frac{16x}{16} = \frac{32}{16}$$

$$\boxed{x = 2}$$

$$15. \quad 3(2x - 5) + 4 = 5 - 2x$$

$$6x - 15 + 4 = 5 - 2x$$

$$\frac{6x - 11}{+2x +2x} = \frac{5 - 2x}{+2x +2x}$$

$$\frac{8x - 11}{+11 +11} = \frac{5}{+11 +11}$$

$$\frac{8x}{8} = \frac{16}{8}$$

$$x = 2$$

$$\boxed{x = 2}$$

$$16. \quad 4x - 5 - 2x = 3 - 10 + 3x$$

$$2x - 5 = -7 + 3x$$

$$\frac{-2x}{-5 -5} = \frac{-7 + 3x}{-2x -2x}$$

$$\frac{-5}{+7 +7} = \frac{-7 + x}{+7 +7}$$

$$2 = x$$

$$\boxed{x = 2}$$

$$17. \quad 10 - \frac{1}{2}x = 4 + \frac{1}{3}x + 1$$

$$\frac{10 - \frac{1}{2}x}{-5} = \frac{5 + \frac{1}{3}x}{-5}$$

$$\frac{5 - \frac{1}{2}x}{+ \frac{1}{2}x + \frac{1}{2}x} = \frac{\frac{5}{6}x}{+ \frac{1}{2}x + \frac{1}{2}x}$$

$$\frac{6(5)}{6} = \left(\frac{5}{6}x\right)6$$

$$\frac{30}{5} = \frac{5x}{5}$$

$$6 = x$$

$$\boxed{x = 6}$$

$$18. \quad \frac{x}{-2} + 5 = 3$$

$$\frac{-x}{-5 -5} = \frac{-2(-2)}{-5 -5}$$

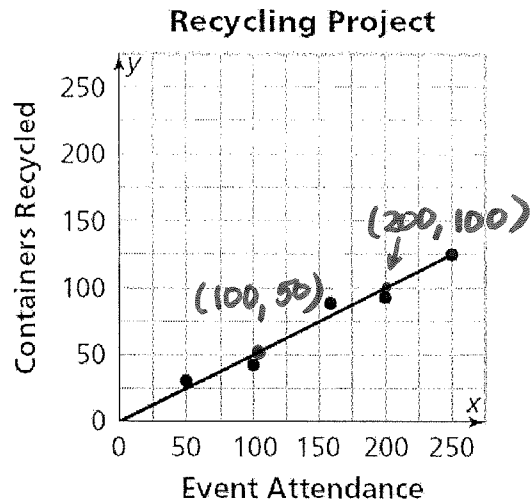
$$\left(\frac{-2}{1}\right) \frac{x}{-2} = -2(-2)$$

$$x = 4$$

$$\boxed{x = 4}$$

19. The Metropolis Middle School makes money by recycling cans and bottles after school events. Below is the data they collected for the number of cans collected for different event attendances; the data is also graphed with the line of best fit drawn.

(X) Event Attendance	(Y) # of Containers Recycled
50	28
100	46
160	88
200	90
250	125



- a. Write the equation for the line of best fit that model:

Use points on the line, not from the table.

$$\begin{aligned}\frac{\Delta y}{\Delta x} &= \frac{100 - 50}{200 - 100} \\ &= \frac{50}{100} \\ &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}y &= mx + b \\ y &= \frac{1}{2}x + b \\ 50 &= \frac{1}{2}(100) + b \\ 50 &= 50 + b\end{aligned}$$

$$\begin{aligned}50 &= 50 + b \\ -50 & \quad -50 \\ 0 &= b\end{aligned}$$

$$y = \frac{1}{2}x$$

- b. Using your model, estimate how many containers will be recycled if 160 people attend a chorus concert?

X: # of people attending  
y: # of containers recycled

$$\begin{aligned}\text{If } x = 160: \quad y &= \frac{1}{2}x \\ &= \frac{1}{2}(160) \\ &= 80\end{aligned}$$

~80 containers will be recycled.

- c. If 160 containers were recycled after the basketball game, about how many people attended?

$$\text{If } y = 160$$

$$\begin{aligned}y &= \frac{1}{2}x \\ 2(160) &= (\frac{1}{2}x)(\frac{2}{1}) \\ 320 &= x\end{aligned}$$

~320 people attended the game

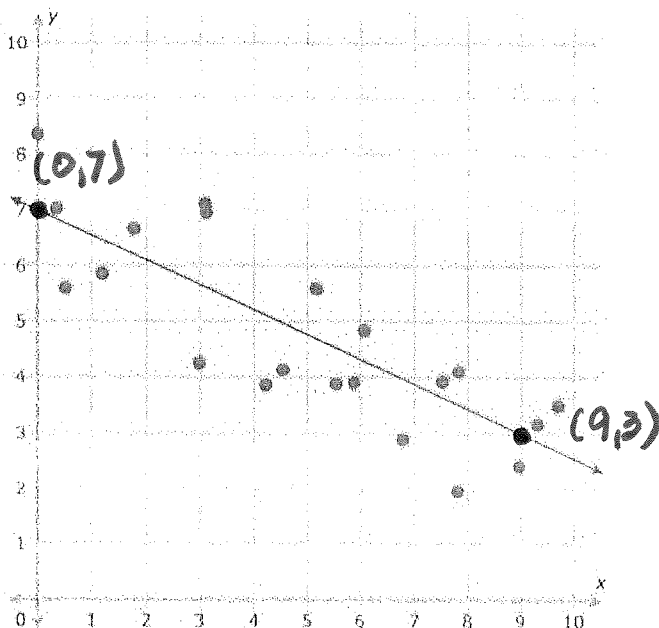
- d. If 500 people attended a game, about how many containers would you expect to be recycled?

$$\text{If } x = 500$$

$$\begin{aligned}y &= \frac{1}{2}x \\ y &= \frac{1}{2}(500) \\ y &= 250\end{aligned}$$

~250 containers would be recycled.

20. What is the equation of the trend line in the scatter plot?



$$\frac{\Delta y}{\Delta x} = \frac{7-3}{0-9} = \frac{4}{-9} = -\frac{4}{9}$$

$$y = mx + b$$

$$y = -\frac{4}{9}x + b$$

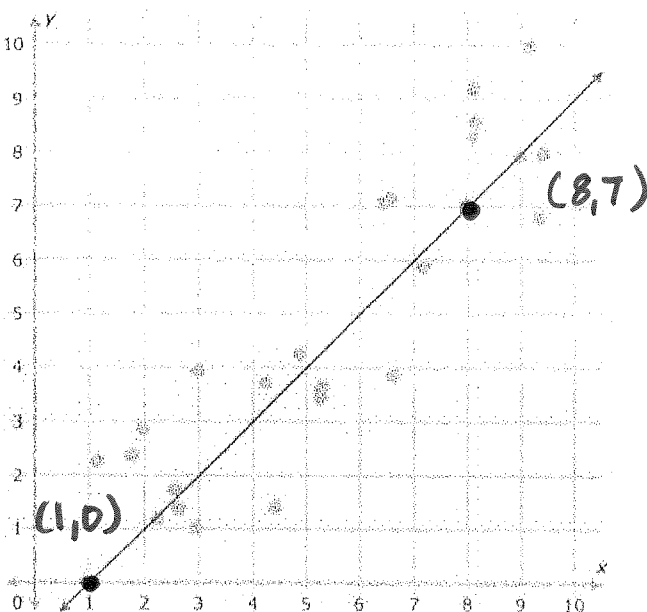
$$7 = -\frac{4}{9}(0) + b$$

$$7 = b$$

$$\boxed{y = -\frac{4}{9}x + 7}$$

Use the two red points to write the equation in slope-intercept form. Write any coefficients as integers, proper fractions, or improper fractions in simplest form.

21. What is the equation of the trend line in the scatter plot?



$$\frac{\Delta y}{\Delta x} = \frac{7-0}{8-1} = 1$$

$$y = mx + b$$

$$y = (1)x + b$$

$$7 = (1)(8) + b$$

$$7 = 8 + b$$

$$\begin{array}{r} -8 \quad -8 \\ 7 = 8 + b \\ \hline -1 = b \end{array}$$

$$-1 = b$$

$$\boxed{y = x - 1}$$

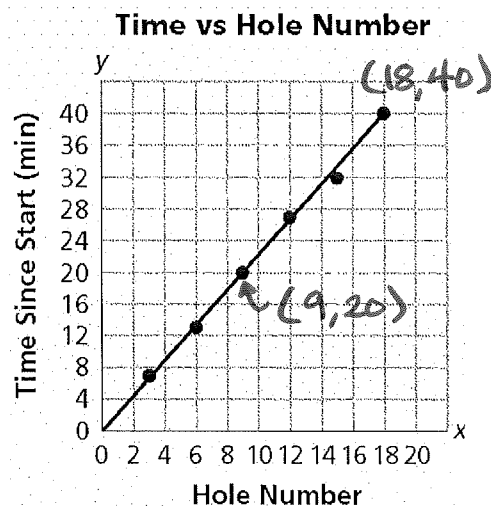
Use the two red points to write the equation in slope-intercept form. Write any coefficients as integers, proper fractions, or improper fractions in simplest form.

22. Jamal and Alisha played a round of miniature golf. They made some notes of the time it took to play. Their data are shown in the next table:

(h)

Hole Number	3	6	9	12	15	18
Time Since Start (minutes)	7	13	20	27	32	40

(t)



- a. Write an equation for the line of best fit modeling hole number,  $h$ , and time since start,  $t$ .

We have 2 points that fall on the line (model).  
 $(9, 20)$  and  $(18, 40)$

$$\frac{\Delta y}{\Delta x} = \frac{40-20}{18-9} = \frac{20}{9}$$

$$y = mx + b$$

$$y = \frac{20}{9}x + b$$

$$20 = \frac{20}{9}(9) + b$$

$$20 = 20 + b$$

$$0 = b$$

$$y = \frac{20}{9}x$$

$$t = \frac{20}{9}h$$

- b. Use your equation or graph model to estimate the time it took Jamal and Alisha to play the first 7 holes.

$$t = \frac{20}{9}h$$

When  $h = 7$ :  $t = \frac{20}{9}(7)$

$$t = \frac{140}{9}$$

$$t = 15.5\bar{5}$$

It will take approximately  $15\frac{1}{2}$  minutes to play the first 7 holes.

- c. Use your equation or graph model to estimate the time it would take Jamal and Alisha to play 27 holes.

When  $h = 27$ :

$$t = \frac{20}{9}h$$

$$t = \frac{20}{9}(27)$$

$$t = 60$$

I will take approximately one hour to play 27 holes of mini golf.

23. The Frederick Douglass Middle School chorus always has a party after their first concert. The cost per person for this party depends on the number of chorus members who attend. The following table shows some sample (*number attending*, *cost per person*) values.

(x)	Number Attending	5	10	15	20	25	30
(y)	Cost per Person	\$24	\$12	\$8	\$6	\$4.80	\$4

$$\begin{aligned} 5 \cdot 24 &= 120 \\ 10 \cdot 12 &= 120 \\ 15 \cdot 8 &= 120 \\ 20 \cdot 6 &= 120 \\ 25 \cdot 4.8 &= 120 \\ 30 \cdot 4 &= 120 \end{aligned}$$

INVERSE!

Write an equation to model this situation.

$$x \cdot y = 120 \quad \text{OR} \quad y = \frac{120}{x}$$

Using your model (and algebraic work), calculate the cost per person if 40 people decide to attend the party.

$$\begin{aligned} \text{if } x &= 40: & y &= \frac{120}{x} \\ & y &= \frac{120}{40} \\ & y &= 3 \end{aligned}$$

If 40 people decide to attend it will cost \$3 /person

24. The Phams like to go camping. Jen drives the camper. Grace follows behind on her motorcycle. Leim, Jen's brother, meets them at the campground later in his hybrid car. The table shows data they collected to keep track of the gas each vehicle used.

	camper	hybrid car	motorcycle
Fuel Efficiency (miles/gallon)	8	32	50
Amount of Gas Used (gallons)	25	6.25	4

- a. How many miles is the campground from the Pham family's house?

$$\left( \frac{\# \text{miles}}{\text{gallon}} \right) (\text{gallons}) = \# \text{ of miles}$$

$$\begin{aligned} 8 \cdot 25 &= 200 \\ 32 \cdot 6.25 &= 200 \\ 50 \cdot 4 &= 200 \end{aligned}$$

The campground is 200 miles from the house.

- b. Write an equation relating fuel efficiency to amount of gas used. Define your variables and describe what the numbers in the equation tell you about the situation.

$$y = \frac{200}{x}$$

# of gallons used to drive to the campground →  $y$

← the campground is 200 miles from the house

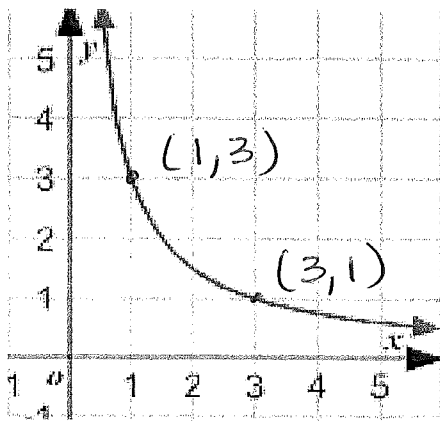
← fuel efficiency for the vehicle in miles/gallon →  $x$

- c. Grace's friend Wynona wants to show off her new sports car, so she drives the same distance to meet them at the campground. Her sports car gets 18 miles to the gallon. How many gallons of gas does she use on the trip?

$$\begin{aligned} \text{if } x &= 18 & y &= \frac{200}{x} \\ & y &= \frac{200}{18} \\ & y &= 11 \frac{1}{9} \end{aligned}$$

Wynona's car uses  $11 \frac{1}{9}$  gallons of gas.

25. What is the equation for the graph below?



• Based on the shape of the graph we know this is an inverse relationship.

• In an inverse relationship

$$x \cdot y = k$$

• Using points (1, 3) and (3, 1) we find  $x \cdot y = 3$

Equation:  $x \cdot y = 3$  or  $y = \frac{3}{x}$

26. As the speed increases, the time for the trip decreases. What type of relationship is shown in the table? Find the equation. (Note some of the times have been rounded)

Speedometer reading (km/hr)	Time for 100-km trip (hr)
20	5
30	3.33
40	2.5
50	2
60	1.67
70	1.43

$$x \cdot y$$

$$= 100$$

Inverse Relationship because  $x \cdot y = k$

$$x \cdot y = 100$$

or

$$y = \frac{100}{x}$$

27. Find the value of  $c$  for which both ordered pairs satisfy the same inverse variation. Then, write an equation for the relationship.

(3, 12), (9,  $c$ ) • If this is inverse  $x \cdot y = k$

• Using first pair (3, 12)  $x \cdot y = 3 \cdot 12 = 36$

• Same must be true for (9,  $c$ )

$$9 \cdot c = 36$$

$$c = 4$$

Equation:  $x \cdot y = 36$  or  $y = \frac{36}{x}$



28. Write an equation to relate base  $b$  and height  $h$  of rectangles with area  $60 \text{ cm}^2$ .

$$A = b \cdot h$$

$$60 = b \cdot h$$

↑ in this case.

$$\text{Equation: } b \cdot h = 60 \text{ or } b = \frac{60}{h}$$

29. What is the value of  $k$  for the following inverse relationship?

$x$	1	2	3	4
$y$	24	12	8	6

$$x \cdot y = k$$

$$1 \cdot 24 = 24$$

$$2 \cdot 12 = 24$$

$$3 \cdot 8 = 24$$

$$4 \cdot 6 = 24$$

$$k = 24$$

(8R)

30. Suppose that  $y$  varies inversely with  $x$  such that  $y = 5$  when  $x = 3$ . Write a general form equation to represent the relationship between any  $x$  and  $y$ .

• If  $y$  varies inversely with  $x$ :  $x \cdot y = k$

• If  $y = 5$  when  $x = 3$ ,  $(3, 5)$  is a point on the curve, and

$$x \cdot y = k$$

$$3 \cdot 5 = k$$

$$15 = k$$

$$\text{Equation: } xy = 15$$

or

$$y = \frac{15}{x}$$

31. Garden City introduced a recycling program. The goal of the program is to reduce the number of pounds of trash sent to landfills by 25% each year. In 2000, Garden City produced 100,000 tons of trash. If the recycling program were to reach its goal, how many tons of trash can Garden City expect to produce in the year 2020?

$T$  = tons of trash

$$T = 100,000 (0.75)^{20}$$

$y$  = # of years

$$T \approx 317 \text{ tons of trash in 2020}$$

25% = decay rate

100,000 = starting trash

32. Jasmine wins \$5000 on a scratch ticket and invests it at a rate of 3.5% compounded annually. How much money will she have after 15 years?

$M$  = money

$$M = 5000 (1.035)^{15}$$

$y$  = year

3.5% = growth rate

$$M \approx \$8376.74$$

\$5000 = start \$

8R

33. At a national park, the decay factor for the bear population is 0.87 each year. The decay rate for the fox population is 17% per year. Which population has the greatest percent of their population remaining each year?

$1 - \text{decay factor} = \text{decay rate}$

$$1 - 0.87 = 0.13 = 13\% \text{ decay rate}$$

$17\% > 13\%$  so the fox population has greater decay rate

thus bear population has greatest % left

34. Given the equation  $y = 250(.65)^x$ , what is the decay rate?

$$1 - 0.65 = 0.35 \text{ or } 35\% \text{ decay rate}$$

35. Fill in the missing values in the table for this exponential relationship

(H)	(B)
# of Hours	# of Bacteria
2	176
3	704
4	2816
5	11264
6	45056

What is the equation?

$$B = 11(4)^H$$

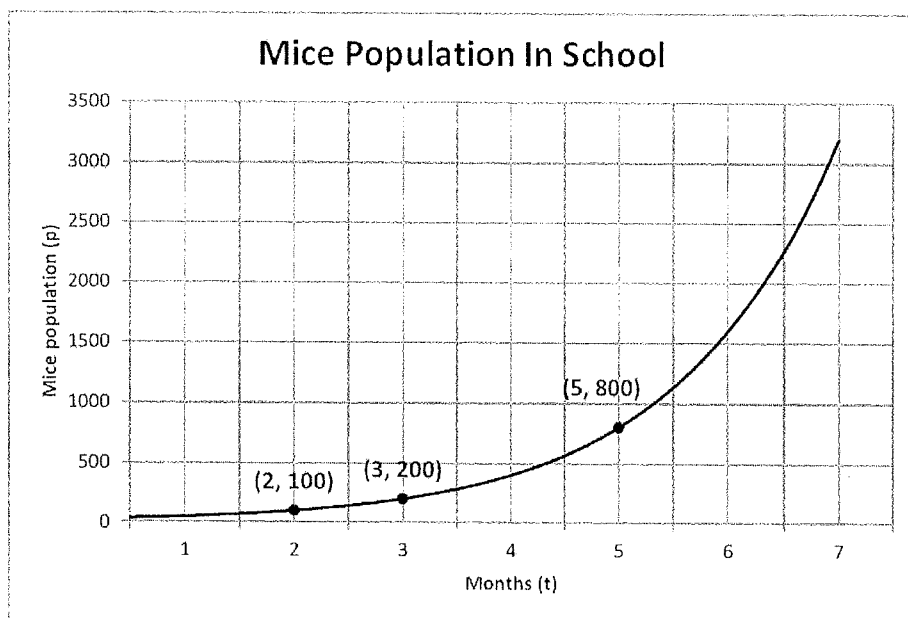
$$y = a(b)^x$$

$\swarrow$  start       $\searrow$  multiplier

$$176 \div 4 = 44 \text{ (\# of bacteria after 1 hour)}$$

$$44 \div 4 = 11 \text{ (\# of bacteria at start)}$$

36.



Write an exponential equation that models the number of mice ( $p$ ) for a given number of months ( $t$ ).

$$200 \div 100 = 2 \text{ so growth factor (multiplier) is 2}$$

$$100 \div 2 = 50 \text{ population after 1 month}$$

$$50 \div 2 = 25 \text{ population at start}$$

$$P = 25(2)^t$$

37. A population of bugs has a growth factor of 4. After year 2, there are 480 bugs. After year 3, there are 1,920 bugs. Write the equation that models the population growth.

(y) Year	(B) Bugs
0	30
1	120
2	480

$$B = 30(4)^y$$

38. Which of the following is growing at the fastest rate: a growth factor of 2.3, a growth rate of 230%, the equation  $y = 30(1.99)^x$ , or a growth rate of 199%? Explain.

Factor	2.3		1.99	
Rate	130%	230%	99%	199%

fastest rate 230%

change all to rate

39. What is the decay factor for the following table? What is the decay rate?

x	3	4	5	6
y	190	142	107	80

$$\frac{142}{190} \approx \frac{107}{142} \approx \frac{80}{107} \approx \text{decay factor } 0.75$$

$$1 - 0.75 = 0.25$$

so decay rate is 25%

40. A boat costs \$15,500 and decreases in value by 10 percent per year. How much will the boat be worth after 5 years?

W = worth  
y = years

$$W = 15,500(0.9)^y$$

start      decay factor

$$W = 15,500(0.9)^5 \approx \$9,152.60$$

41. The equation  $y = 2(3^x)$  might represent the growth pattern for a population of mice. Complete the following sentences. Your sentence should describe the pattern in words.

i. The population started with 2 mice.

ii. The population grew at a rate of 200 percent.

iii. In 8 years, the equation predicts the population of mice to be  $y = 2(3)^8$ .

42. In 1995, there were 85 rabbits living in the Sprague lower field woods. The population increased by 12% each year. How many rabbits were in the Sprague woods in 2005?

$$R = \text{rabbits} \quad R = 85(1.12)^y$$

$$y = \text{year} \quad R = 85(1.12)^{10} \approx 264$$

1995 = year 0      so about 264 rabbits in woods in 2005.

43. Mr. Clarke has discovered a strain of bacteria! The bacteria culture initially contained 1000 bacteria and the bacteria are doubling every half hour. Write an equation to match this situation and then determine how many bacteria are present after 3 hours?

Hour (H)	Bacteria (B)
0	1000
.5	
1	4000
1.5	
2	16,000
3	64,000

$$B = 1000(4)^H$$

so 64,000 bacteria  
after 3 hours

Study the patterns in the following tables. For each table:

- Tell whether the relationship between  $x$  and  $y$  is linear, inverse, exponential, or neither.
- Explain how you know the relationship is linear, inverse, exponential, or neither.
- If the relationship is linear, inverse, exponential, write an equation for it.

44.

x	5	-5	-13	-17
y	-2	3	7	9

- LINEAR  $\Delta y$   $\Delta x$
- $\frac{\Delta y}{\Delta x}$  all give  $-\frac{1}{2}$  or  $-0.5$

using  $(5, -2)$  and  $m = -\frac{1}{2}$

$$y = mx + b$$

$$(-2) = (-\frac{1}{2})(5) + b$$

$$-2 = -1.5 + b$$

$$-0.5 = b$$

$$y = -\frac{1}{2}x + \frac{1}{2}$$

OR

$$y = -\frac{1}{2}x + \frac{1}{2}$$

45.

x	2	3	5	9	15
y	225	150	90	50	30

- INVERSE
- decrease is decreasing &  $xy = k$

so  $xy = 450$  or  $y = \frac{450}{x}$

\*8R  
46.

x	0	1	2	3	4
y	0	2	4	8	16

- look exponential at first glance, But need to start with something to multiply  
SO NEITHER

47.

x	1	2	3	4	5
y	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{9}{4}$	$\frac{27}{4}$

- Exponential Growth
- increase is increasing at explosive rate & has a growth factor

- has multiplier of 3

- start of  $(0, \frac{1}{36})$

$$y = \frac{1}{36}(3)^x$$

48.

x	0	1	2	3	4	5
y	2.3	3.8	5.3	6.8	8.3	9.8

- LINEAR

$$y = 1.5x + 2.3$$

- $\frac{\Delta y}{\Delta x}$  all give 1.5

49.

x	0	1	2	3	4	5
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16	64

- EXPONENTIAL GROWTH

$$y = \frac{1}{16}(4)^x$$

- increase is increasing & has a growth factor  $\frac{y_1}{y_0} = \frac{y_2}{y_1} = \frac{y_3}{y_2} = \frac{y_4}{y_3} = \frac{y_5}{y_4} = 4$

50.

x	y
-2	-4
-1	-1
0	2
1	5
2	8
3	11

- LINEAR

-  $\frac{\Delta y}{\Delta x}$  all constant + 3

$$y = 3x + 2$$

51.

Tennis Tournament:

Rounds	1	2	3	4
Players left	64	32	16	8

- EXPONENTIAL DECAY

$$\frac{y_4}{y_3} = \frac{y_3}{y_2} = \frac{y_2}{y_1} = 0.5$$

- decrease is decreasing & has a decay factor

$$64 \div 0.5 = 128 \text{ START}$$

$$y = 128(0.5)^x$$

52.

x	3	4	5	6
y	11	8.25	6.6	5.5

- INVERSE

$$xy = 33 \text{ or } y = \frac{33}{x}$$

-  $xy = K$  for all

53.

x	-2	2	4	6
y	-7	*1	2	6

- NEITHER

- not LINEAR because not a constant rate of change  
 not INVERSE because  $xy \neq K$   
 not Exponential because no multiplier

54. Each of the four relationships below is represented by a situation, equation, table, and graph. Complete the table to match the different representations to the correct relationship.

Relationship	Situation	Equation	Table	Graph
Linear	B	F	G	Q
Inverse Variation	A	D	H	P
Exponential	C	E	I	O

### Situations

- A. The area of a rectangular enclosure is 240 square meters. The dimensions can change, but the area is fixed.
- B. Every week, you add \$10 to your piggy bank. There was \$240 in your piggy bank to start.
- C. There were 240 wolves in Northern Michigan last year. Every year, the population grows by 10%.

### Equations

D.  $y = \frac{240}{x}$

E.  $y = 240(1.10)^x$

F.  $y = 10x + 240$

### Tables

G.

x	y
1	250
2	260
3	270
4	280
5	290

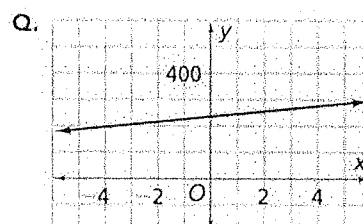
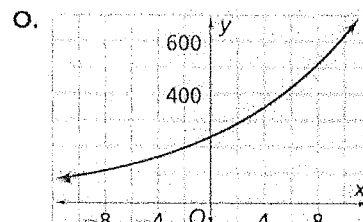
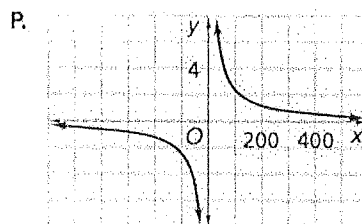
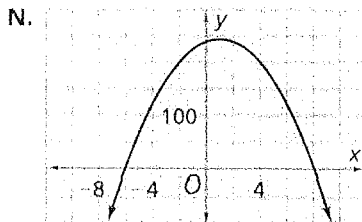
H.

x	y
1	240
2	120
3	80
4	60
5	48

I.

x	y
1	264
2	290
3	319
4	351
5	386

### Graphs





Simplify the following. All final answers must contain positive exponents.

55.

$$x^5 \left( \frac{3}{x^4} \right)^{-3}$$

$$x^5 \left( \frac{x^4}{3} \right)^3 = x^5 \left( \frac{x^{12}}{27} \right)$$

$$= \frac{x^{17}}{27}$$

56.

$$\frac{(-2x^{-1}y^2)^{-3}}{4x^{-6}y^4}$$

$$= \frac{x^6}{4y^4(-2x^{-1}y^2)^3}$$

$$= \frac{x^6}{4y^4 \cdot -8x^{-3}y^6}$$

$$= \frac{x^6 x^3}{4 \cdot -8 \cdot y^4 \cdot y^6} = \frac{x^9}{-32y^{10}}$$

57.

$$\frac{5x^{-3}y^2}{x^5y^{-1}} \cdot \frac{(2xy^3)^{-2}}{xy}$$

$$= \frac{5y^2y^1}{x^5x^3} \cdot \frac{1}{xy(2xy^3)^2}$$

$$= \frac{5y^3}{x^8 \cdot xy \cdot 4x^2y^6} = \frac{5y^3}{4x^{11}y^7}$$

$$= \frac{5}{4x^{11}y^4}$$

58.

$$\left( \frac{-6x^2y}{2xy^3} \right)^3$$

$$= \left( \frac{-3x}{y^2} \right)^3 = \frac{-27x^3}{y^6}$$

59.

$$\frac{7x^2y^5}{4xy^9} \cdot \frac{8x^{10}y}{-2x^4y^4}$$

$$= \frac{56x^{12}y^6}{-8x^5y^{13}} = \frac{-7x^7}{y^7}$$

60.

$$6a^2(-2ab^4)^3$$

$$6aa(-2abbbb)(-2abbbb)(-2abbbb)$$

$$= 6 \cdot -2 \cdot -2 \cdot -2 \cdot aaaaa \cdot bbbbbb$$

$$= -48a^5b^{12}$$

61.

$$\left[ \left( 3x^{4y^{7z^{12}}} \right)^5 \left( -5x^{9y^{3z^4}} \right)^2 \right]^0 = 1$$

62.

$$(3x)^{-2}$$

$$= \frac{1}{(3x)^2} = \frac{1}{(3x)(3x)} = \frac{1}{9x^2}$$

Write the following numbers in proper Scientific Notation form:

63. 4,500, 200

$$4.5002 \times 10^6$$

64. 0.00013

$$1.3 \times 10^{-4}$$

65.  $27 \times 10^3$

$$2.7 \times 10^4$$

66.  $43 \times 10^{-7}$

$$4.3 \times 10^{-6}$$

67. 1,308,000,000

$$1.308 \times 10^9$$

68.  $5,250 \times 10^{-2}$

$$5.25 \times 10$$

Write the following numbers in Standard form:

69.  $3.201 \times 10^2$

$$320.1$$

70.  $1.17 \times 10^{-5}$

$$0.0000117$$

71.  $4.785 \times 10^{-6}$

$$0.000004785$$

72.  $6.03458 \times 10^4$

$$60345.8$$

73. The diameters of some atoms are  $5 \times 10^{-10}$  m. What is the diameter in standard form?

$$0.0000000005 \text{ m}$$

74. Simplify the following expression and express your answer in scientific notation form.

$$\begin{array}{c} (4.0 \times 10^4)(1.6 \times 10^5) \\ \hline (4.0 \times 1.6)(10^4 \times 10^5) \\ \downarrow \quad \quad \quad \swarrow \\ 6.4 \times 10^9 \end{array}$$



84. You are planning a skating party at a rink that charges a \$38 rental fee plus an additional \$6.50 per person. You don't want to spend more than \$175. Write and solve an inequality to determine the maximum number of friends you can invite.

Let  $p$ : # of people attending the party.  
Then,

$$\begin{array}{r} \$38 + \$6.50p \leq \$175 \\ -\$38 \quad \quad -\$38 \\ \hline \end{array}$$

$$\frac{\$6.50p}{\$6.50} \leq \frac{\$137}{\$6.50} \rightarrow p \leq 21.08$$

The maximum number of friends you can invite is 21.

Solve the inequality and graph your solution on a number line.

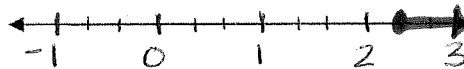
85.  $3x - 14 < 5x + 2$

$$\begin{array}{r} \phantom{3x} +14 \quad \phantom{5x} +14 \\ \hline 3x < 5x + 16 \\ -5x \quad -5x \\ \hline -2x < 16 \\ \underline{-2} \quad \underline{-2} \\ \hline x > -8 \end{array}$$



86.  $-6x + 15 \leq 1$

$$\begin{array}{r} -6x + 15 \leq 1 \\ \underline{-15} \quad \underline{-15} \\ -6x \leq -14 \\ \underline{-6} \quad \underline{-6} \\ \hline x \geq \frac{7}{3} \text{ or } x \geq 2\frac{1}{3} \end{array}$$



87.  $\frac{2}{3}(5 - 3x) \leq 12x$

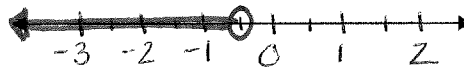
$$\begin{array}{r} \frac{10}{3} - \frac{6}{3}x \leq 12x \\ 3\left[\frac{10}{3} - \frac{6}{3}x \leq 12x\right] \\ 10 - 6x \leq 36 \\ \underline{-10} \quad \underline{-10} \\ \hline \end{array}$$



$$\begin{array}{r} -6x \leq 26 \\ \underline{-6} \quad \underline{-6} \\ \hline x \geq -4\frac{1}{3} \end{array}$$

88.  $-2(x + 4) > 6x - 4$

$$\begin{array}{r} -2x - 8 > 6x - 4 \\ +2x \quad +2x \\ \hline -8 > 8x - 4 \\ +4 \quad +4 \\ \hline -4 > 8x \\ \underline{8} \quad \underline{8} \\ \hline -\frac{1}{2} > x \end{array}$$



To find x intercept, set  $y=0$ .

Write each of the following equation in slope-intercept form. Identify the slope, x-intercept, and y-intercept.

89.  $\frac{2}{3}x - \frac{1}{5}y = 2$

$-\frac{2}{3}x \quad -\frac{2}{3}x$

$(-5) - \frac{1}{5}y = \left[-\frac{2}{3}x + 2\right](-5)$

$y = \frac{10}{3}x - 10$

$\begin{cases} x \text{ int.} = 3 \\ y \text{ int.} = -10 \\ \text{slope} = 10/3 \end{cases}$

91.  $9x - 2y = 40$

$-9x \quad -9x$

$-2y = -9x + 40$   
 $-2 \quad -2$

$y = \frac{9}{2}x - 20$

$\begin{cases} y \text{ int.} = -20 \\ x \text{ int.} = 4\frac{4}{9} \\ \text{slope} = \frac{9}{2} \end{cases}$

90.  $3x + 4y = -12$

$-3x \quad -3x$

$4y = -3x - 12$   
 $4 \quad 4$

$y = -\frac{3}{4}x - 3$

$\begin{cases} y \text{ int.} = -3 \\ m = -3/4 \end{cases}$

$x \text{ int.} = -4$

To find x int.,  
Set  $y=0$ .

92.  $-2x + 6y - 4 = 0$

$+4 \quad +4$

$-2x + 6y = 4$   
 $+2x \quad +2x$

$6y = 2x + 4$   
 $6 \quad 6$

$y = \frac{1}{3}x + \frac{2}{3}$

$\begin{cases} \text{slope} = \frac{1}{3} \\ y \text{ int.} = \frac{2}{3} \end{cases} \quad x \text{ int.} = -2$

Write each of the following equation in Standard Form ( $Ax + By = C$ ). Identify the slope, x-intercept, and y-intercept.

93.  $y = \frac{2}{3}x - 12$

$-\frac{2}{3}x \quad -\frac{2}{3}x$

$\text{slope: } \frac{2}{3}$

$x \text{ int.} : 18$

$y \text{ int.} : -12$

$-3 \left[ -\frac{2}{3}x + y = -12 \right]$

$2x - 3y = 36$

94.  $y = \frac{3}{4}x + \frac{1}{3}$

$-\frac{3}{4}x \quad -\frac{3}{4}x$

$-12 \left[ -\frac{3}{4}x + y = \frac{1}{3} \right]$

$9x - 12y = -4$

95. Identify the slope, x-intercept, and y-intercept for the linear equation  $2x + 4y = 12$ .

Finding x intercept,

$2x + 4(0) = 12$

$2x = 12$   
 $\frac{2}{2} \quad \frac{2}{2}$

$x = 6$

Finding slope & y intercept,

$2x + 4y = 12$   
 $-2x \quad -2x$

$4y = -2x + 12$   
 $4 \quad 4$

$y = -\frac{1}{2}x + 3$

$\begin{cases} \text{slope} = -\frac{1}{2} \\ y \text{ int.} = 3 \end{cases}$

**\*\*8R only this point forward. (Algebra 8 students, you will see this on your final!)**

Solve the following systems of equations using the most efficient method.

Combination.

$$96. \begin{cases} (x + 3y = 34) \cdot -2 \\ -5x + 6y = 40 \end{cases}$$

$$+ \quad -2x - 6y = -68$$

$$\underline{-7x = -28}$$

$$\underline{-7 \quad -7}$$

$$x = 4$$

$$\begin{array}{r} 4 + 3y = 34 \\ -4 \quad -4 \end{array} \quad (4, 10)$$

$$\begin{array}{r} 3y = 30 \\ 3 \quad 3 \\ \hline y = 10 \end{array}$$

$$98. \begin{cases} y = \frac{1}{2}x + 3 \\ y = -2x - 7 \end{cases} \text{ Equivalent Forms.}$$

$$\frac{1}{2}x + 3 = -2x - 7$$

$$+2x \quad +2x$$

$$\begin{array}{r} 2\frac{1}{2}x + 3 = -7 \\ -3 \quad -3 \end{array}$$

$$\begin{array}{r} 2\frac{1}{2}x = -10 \\ 2\frac{1}{2} \quad 2\frac{1}{2} \end{array}$$

$$x = -4$$

$$y = \frac{1}{2}(-4) + 3$$

$$y = -2 + 3$$

$$y = 1$$

Substitution.

$$97. \begin{cases} y - x = -11 \\ 3x + y = 3 \end{cases} \rightarrow y = [x - 11]$$

$$3x + x - 11 = 3$$

$$4x - 11 = 3$$

$$+11 \quad +11$$

$$\begin{array}{r} 4x = 14 \\ 4 \quad 4 \end{array}$$

$$x = \frac{14}{4} = \frac{7}{2} \quad \left(\frac{7}{2}, -\frac{15}{2}\right)$$

$$y = \frac{7}{2} - 11$$

$$y = -\frac{15}{2}$$

$$99. \begin{cases} (3x = 1 + y) \cdot -2 \\ -6x + 2y = 5 \end{cases} \text{ Substitution.}$$

$$-6x = [-2 - 2y]$$

$$-2 - 2y + 2y = 5$$

$$-2 = 5$$

No solution.

Equivalent forms.

$$100. \begin{cases} y = 2x + 10 \\ y = 3x + 12 \end{cases}$$

$$\begin{array}{r} 2x + 10 = 3x + 12 \\ -2x \quad -2x \end{array}$$

$$\begin{array}{r} 10 = x + 12 \\ -12 \quad -12 \end{array}$$

$$-2 = x$$

$(-2, 6)$

$$y = 2(-2) + 10$$

$$y = -4 + 10$$

$$y = 6$$

Combination.

$$102. \begin{cases} (2x + 2y = 3) \cdot 2 \\ (x - 4y = -1) \end{cases}$$

$$+ 4x + 4y = 6$$

$$\begin{array}{r} 3x = 5 \\ \hline 5 \quad 5 \end{array}$$

$$x = 1$$

$(1, \frac{1}{2})$

$$1 - 4y = -1$$

$$\begin{array}{r} -1 \quad -1 \end{array}$$

$$\begin{array}{r} -4y = -2 \\ \hline -4 \quad -4 \end{array}$$

$$y = \frac{1}{2}$$

Substitution.

$$101. \begin{cases} 3x - 4y = 10 \\ \frac{-8y = 20 - 6x}{2} \end{cases}$$

$$-4y = 10 - 3x$$

$$3x + 10 - 3x = 10$$

$$10 = 10$$

Infinite solutions.

Substitution.

$$103. \begin{cases} y = [\frac{5}{4}x - 5] \\ 5x - 4y = -10 \end{cases}$$

$$5x - 4(\frac{5}{4}x - 5) = -10$$

$$5x - 5x + 20 = -10$$

$$20 = -10$$

No solution.

Substitution.

$$104. \begin{cases} 3x + 2y = 12 \\ x - 3y = 26 \end{cases}$$

$$\begin{array}{r} +3x + 3y \\ \hline x = [3y + 26] \end{array}$$

$$3(3y + 26) + 2y = 12$$

$$\begin{array}{r} 9y + 78 + 2y = 12 \\ -78 \quad -78 \end{array}$$

$$\begin{array}{r} 11y = -66 \\ \hline 11 \quad 11 \end{array}$$

$$y = -6$$

$$3x + 2(-6) = 12$$

$$\begin{array}{r} 3x - 12 = 12 \\ +12 \quad +12 \end{array}$$

$$\begin{array}{r} 3x = 24 \\ \hline 3 \quad 3 \end{array}$$

$$x = 8$$

$$(8, -6)$$

Combination.

$$105. \begin{cases} (5x + 8y = -4) \cdot 2 \\ (2x - 5y = -18) \cdot -5 \end{cases}$$

$$\begin{array}{r} 10x + 16y = -8 \end{array}$$

$$+ \begin{array}{r} -10x + 25y = 90 \end{array}$$

$$\begin{array}{r} 41y = 82 \\ \hline 41 \quad 41 \end{array}$$

$$y = 2$$

$$5x + 8(2) = -4$$

$$\begin{array}{r} 5x + 16 = -4 \\ -16 \quad -16 \end{array}$$

$$\begin{array}{r} 5x = -20 \\ \hline 5 \quad 5 \end{array} \quad x = -4$$

$$(-4, 2)$$

Write and solve a linear system of equations for each of the following problems.

106. For dinner, Randy had 10 chicken McNuggets and one medium fries for 840 calories. Jack had 6 chicken McNuggets and two medium fries for 1036 calories. How many calories are there in each item?

$c$  = chicken McNuggets

$f$  = fries

$$\text{Randy: } (10c + f = 840) \cdot -2$$

$$\text{Jack: } 6c + 2f = 1036$$

$$+ \begin{array}{r} -20c - 2f = -1680 \end{array}$$

$$\begin{array}{r} -14c = -644 \\ \hline -14 \quad -14 \end{array}$$

$$c = 46$$

$$10(46) + f = 840$$

$$460 + f = 840$$

$$\begin{array}{r} -460 \quad -460 \end{array}$$

$$f = 380$$

McNuggets have 46 calories each and fries have 380 calories for each order.



107. At Billy's preschool, they have bicycles and tricycles, with a total of 57 wheels. The number of bicycles is three less than three times the number of tricycles. How many of each kind of bike are there?

$b = \text{bicycle}$

$t = \text{tricycle}$

$$2b + 3t = 57$$

$$b = [3t - 3]$$

$$2(3t - 3) + 3t = 57$$

$$\begin{array}{r} 6t - 6 + 3t = 57 \\ +6 \quad \quad +6 \end{array}$$

$$\frac{9t}{9} = \frac{63}{9} \quad t = 7$$

$$b = 3(7) - 3$$

$$b = 21 - 3$$

$$b = 18$$

There are 18 bicycles and 7 tricycles.

108. A test has twenty questions worth 100 points. The test consists of True/False questions worth 3 points each and multiple choice questions worth 11 points each. How many multiple choice questions are there on the test?

$t = \text{true / false questions}$

$m = \text{multiple choice questions}$

$$t + m = 20 \rightarrow t = [20 - m]$$

$$3t + 11m = 100$$

$$3(20 - m) + 11m = 100$$

$$\begin{array}{r} 60 - 3m + 11m = 100 \\ -60 \quad \quad -60 \end{array}$$

$$8m = 40$$

$$\begin{array}{r} 8m = 40 \\ \underline{8 \quad 8} \\ m = 5 \end{array}$$

There are 5 multiple choice questions on the test.

109. Two small pitchers and one large pitcher can hold 8 cups of water. One large pitcher minus one small pitcher constitutes 2 cups of water. How many cups of water can each pitcher hold?

$s = \text{small pitcher}$

$l = \text{large pitcher}$

$$2s + l = 8$$

$$l - s = 2 \rightarrow l = [2 + s]$$

$$2s + 2 + s = 8$$

$$\begin{array}{r} -2 \quad \quad -2 \end{array}$$

$$\frac{3s}{3} = \frac{6}{3}$$

$$s = 2$$

$$l - 2 = 2$$

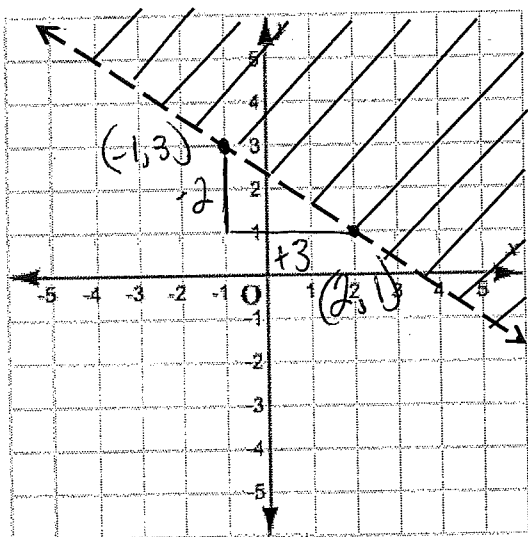
$$\begin{array}{r} +2 \quad +2 \end{array}$$

$$l = 4$$

The small pitcher holds 2 cups of water and the large pitcher holds 4 cups of water.

Write the inequality shown in each of the graphs.

110.



$$y = -\frac{2}{3}x + b$$

$$\frac{7}{3} = b$$

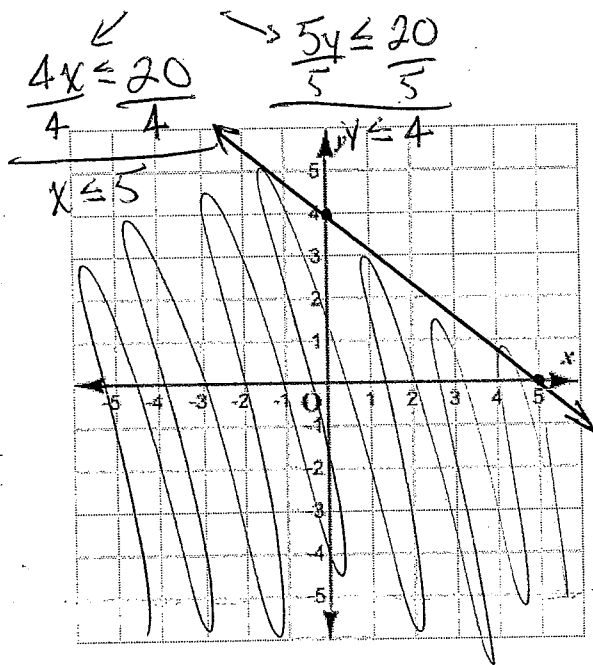
$$1 = -\frac{2}{3}(2) + b$$

$$1 = -\frac{4}{3} + b$$

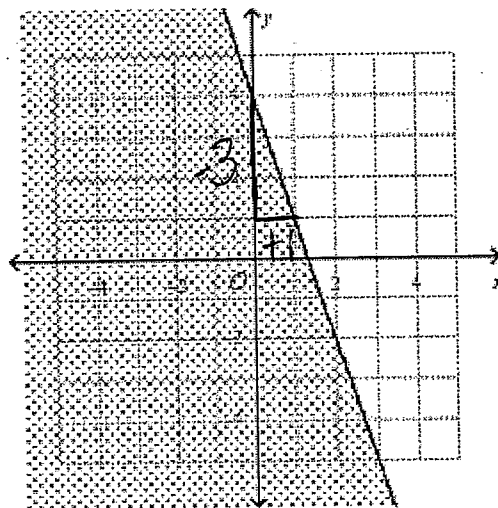
$$y > -\frac{2}{3}x + \frac{7}{3}$$

Graph the following inequalities.

112.  $4x + 5y \leq 20$

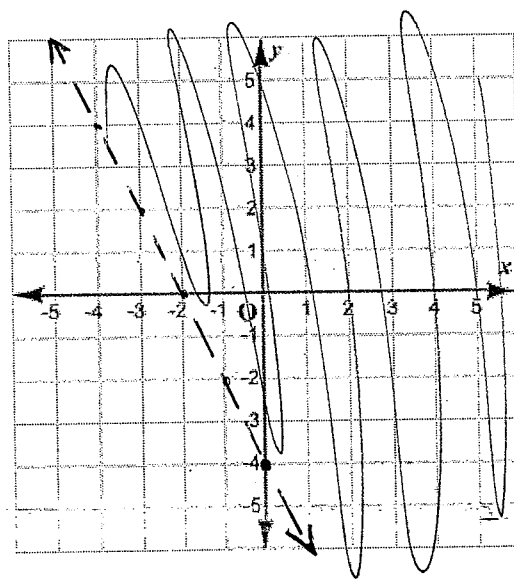


111.



$$y \leq -3x + 4$$

113.  $y > -2x - 4$



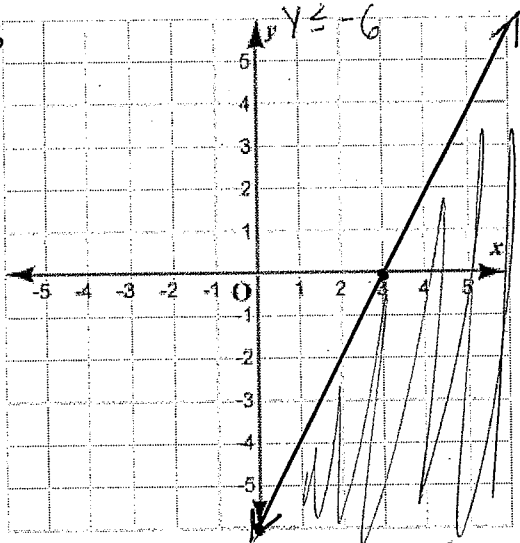
114.  $4x - 2y \geq 12$

$$\frac{4x}{4} \geq \frac{12}{4}$$

$$x \geq 3$$

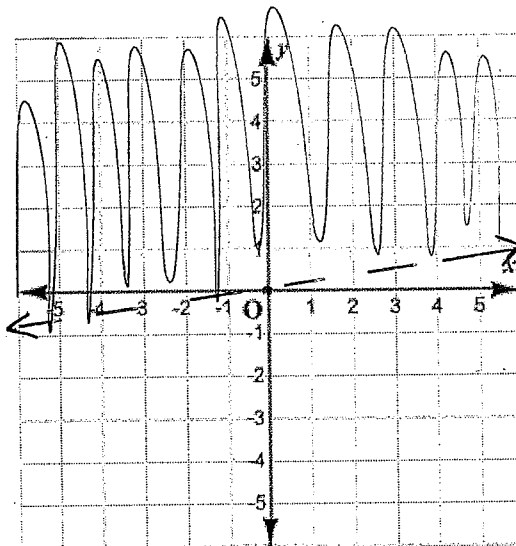
$$\frac{-2y}{-2} \geq \frac{12}{-2}$$

$$y \leq -6$$



115.  $\frac{\frac{1}{2}x}{3} < \frac{3y}{3}$

$$\frac{1}{6}x < y \rightarrow y > \frac{1}{6}x$$



Graph the following systems of inequalities.

116.

$$\begin{cases} 2x + y < 7 \\ x + 3y < 11 \end{cases}$$

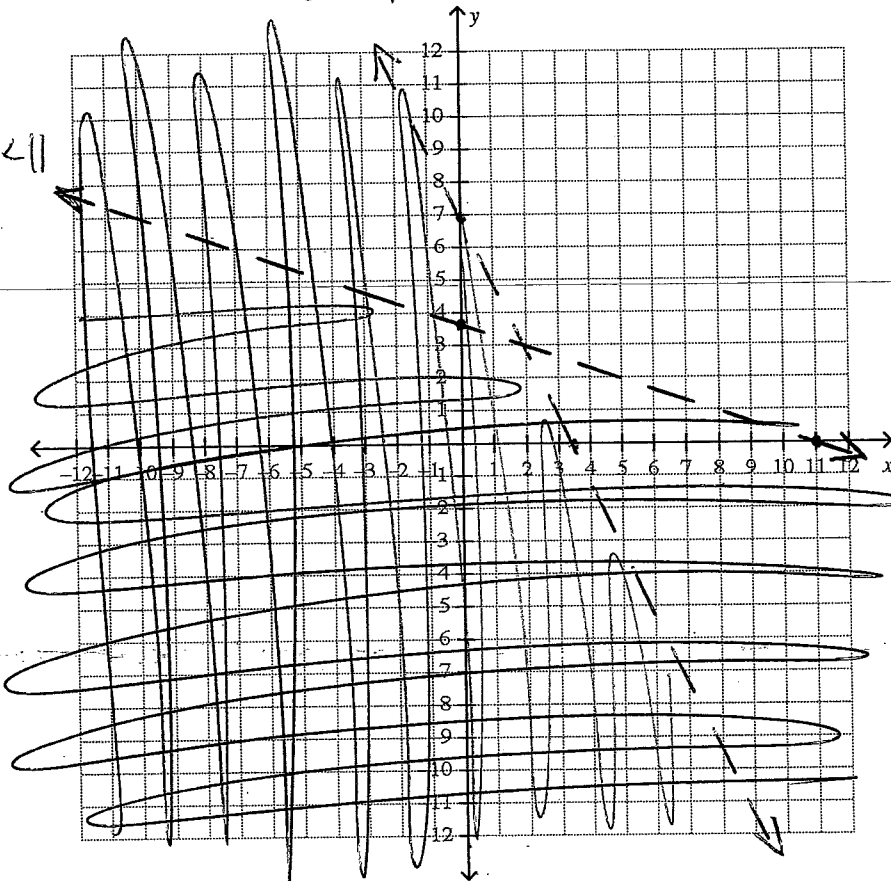
$$\begin{aligned} 2x &< 7 & y &< 7 \\ x &< \frac{7}{2} \end{aligned}$$

$$x < 11 \quad 3y < 11$$

$$y < \frac{11}{3}$$

$$x + 3y < 11$$

$$2x + y < 7$$

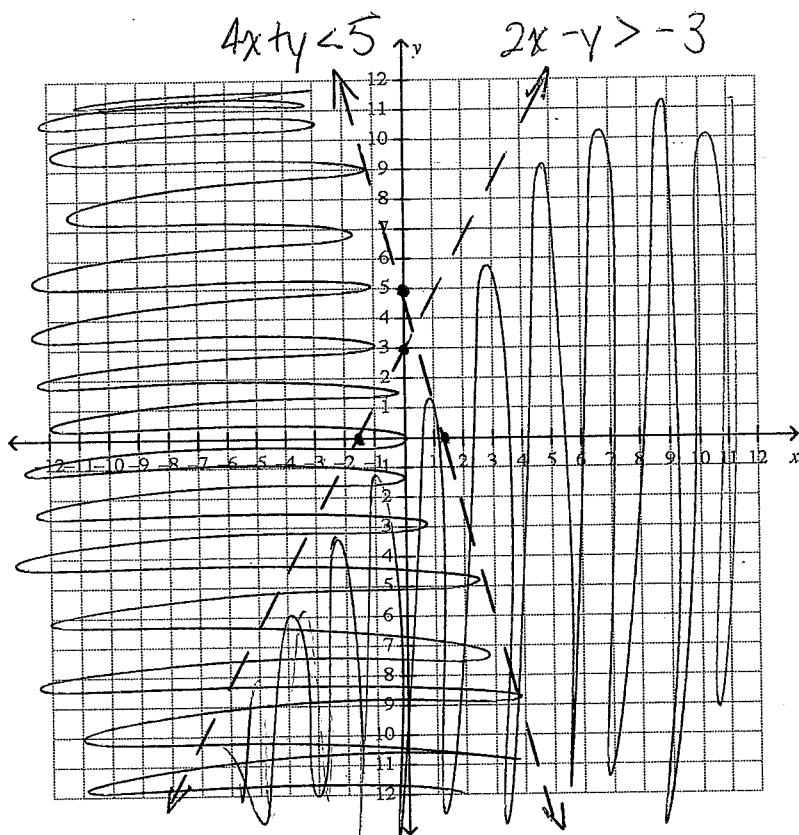


117.  $\begin{cases} 2x - y > -3 \\ 4x + y < 5 \end{cases}$

$x > -3$     $y > -3$

$x > -\frac{3}{2}$     $y < 3$

$x < 5$     $y < 5$   
 $x < \frac{5}{4}$



118.

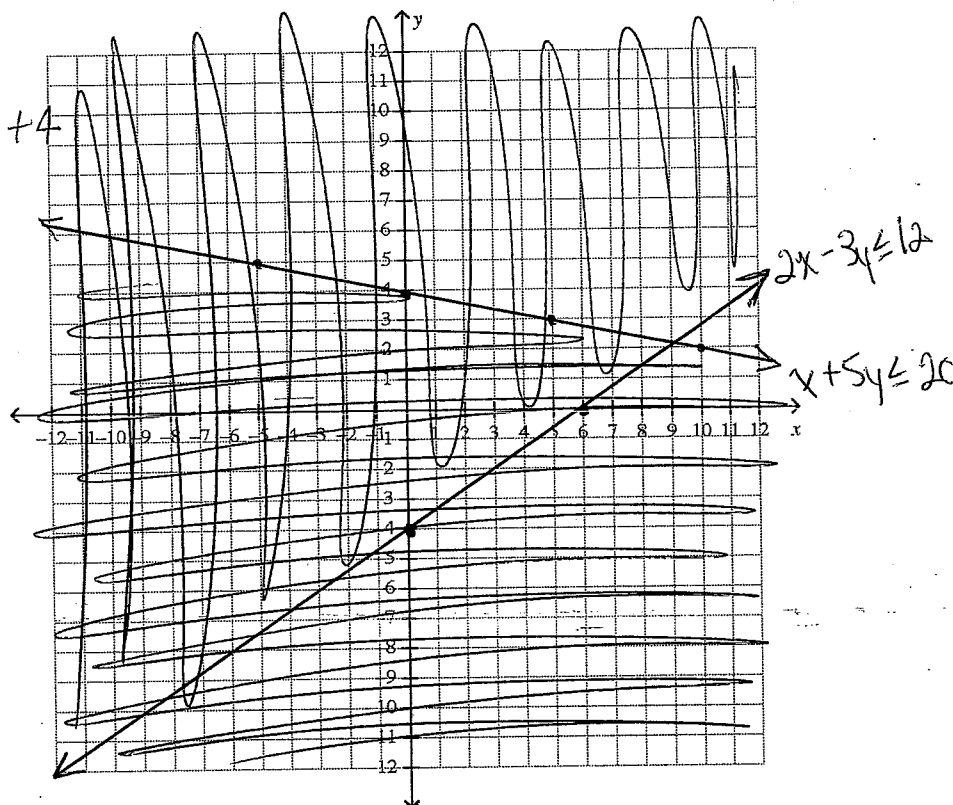
$2x - 3y \leq 12$

$x + 5y \leq 20 \Rightarrow y \leq \frac{-1}{5}x + 4$

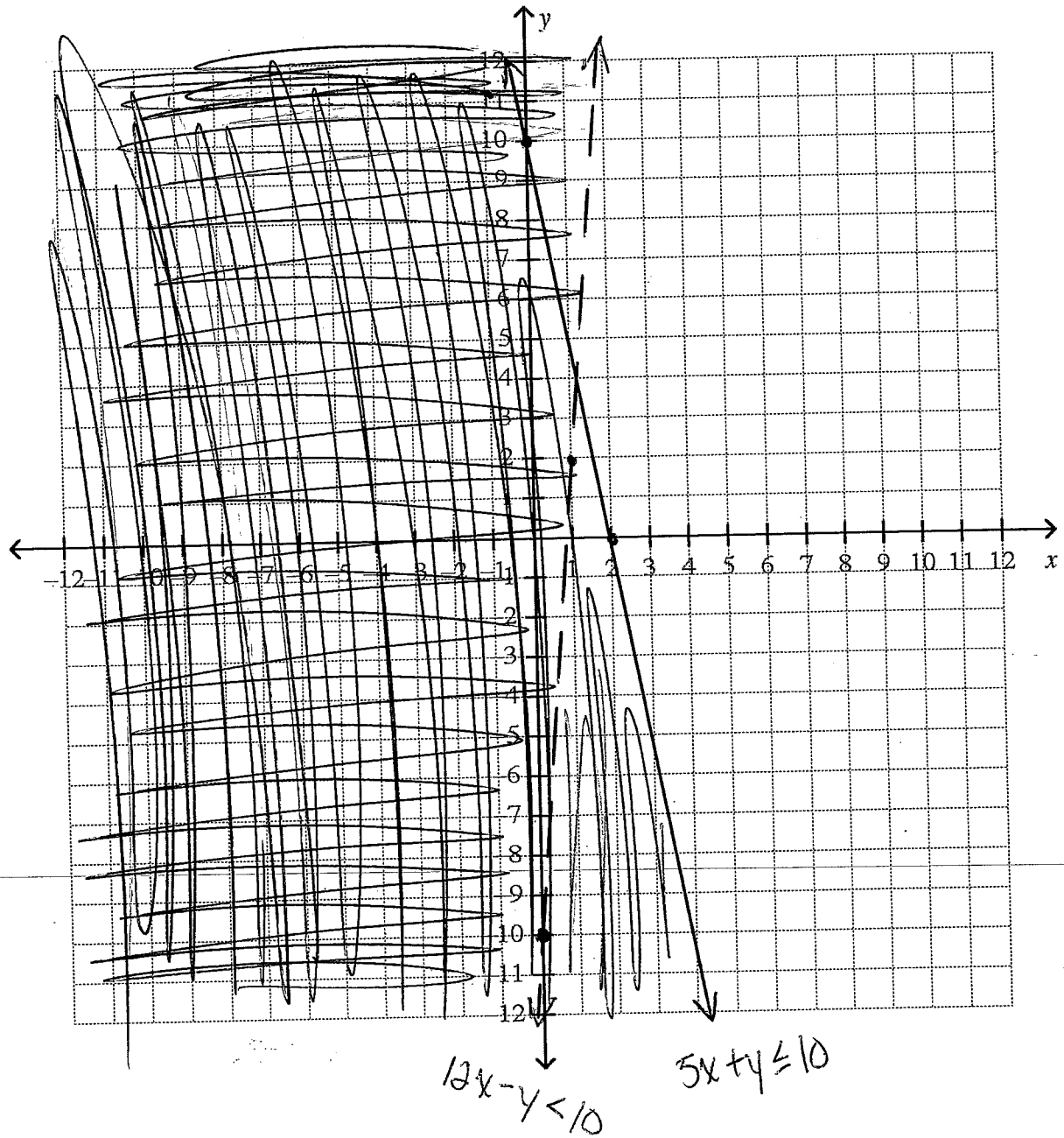
$x \leq 12$     $-3y \leq 12$

$x \leq 6$     $y \geq -4$

$x \leq 20$     $5y \leq 20$   
 $y \leq 4$



119.  $\begin{cases} 5x + y \leq 10 \\ 12x - y < 10 \end{cases} \rightarrow \begin{cases} 5x \leq 10 & y \leq 10 \\ x \leq 2 \end{cases}$   
 $y > 12x - 10$



120. The Maple Middle School Math Club wants to advertise their T-shirt and sweatshirt sale in the school paper. They know from experience (and common sense) that a front-page ad is much more effective than an ad inside the newspaper. They have a \$40 advertising budget. It costs \$5 to advertise on the front page and \$2 to advertise on an inside page. The Math Club would like to advertise at least 15 times.

- a. Write a system of linear inequalities to model this situation. Make sure to indicate what each variable represents.

$f$  = front page

$i$  = inside page

$$f + i \geq 15$$

$$5f + 2i \leq 40$$

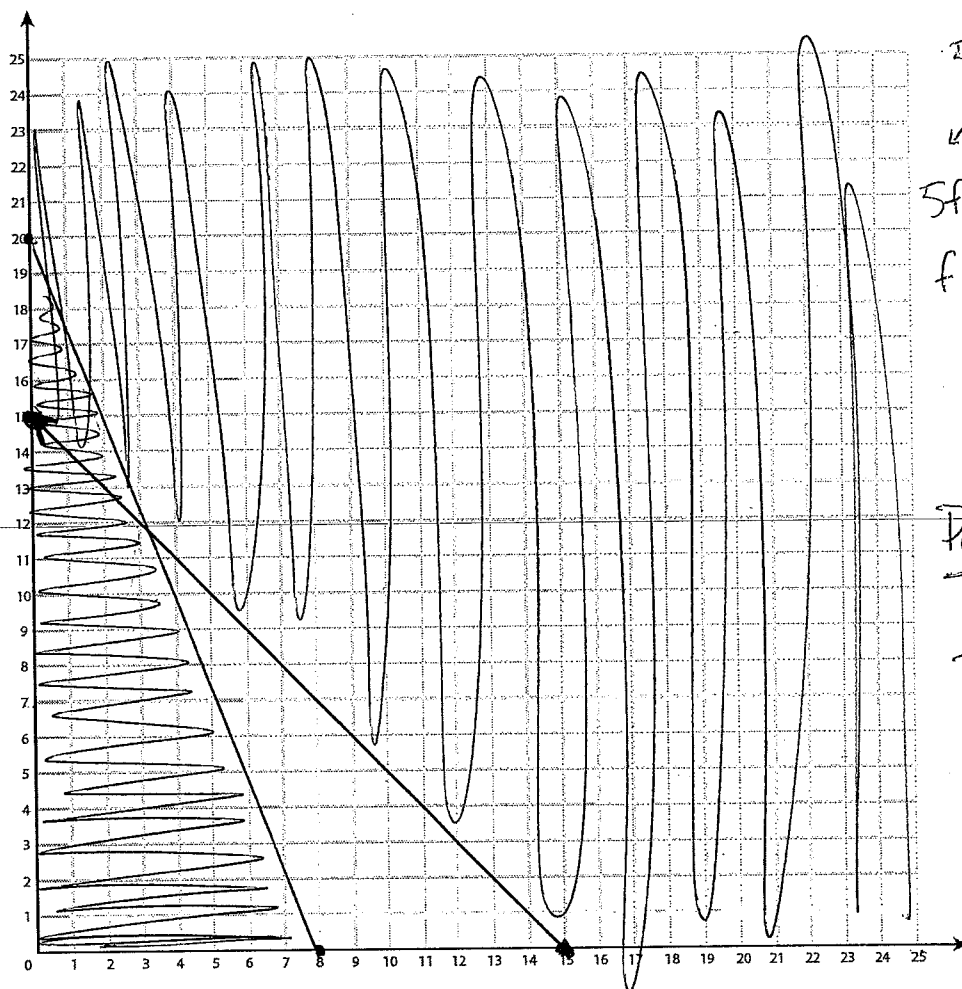
- b. Draw a graph illustrating the solutions to your system of linear inequalities on the coordinate plane below, and list three possible solutions.

$$f + i \geq 15$$

↙ ↘

$$f \geq 15 \quad i \geq 15$$

inside  
page



front page

$$5f + 2i \leq 40$$

↙ ↘

$$5f \leq 40 \quad 2i \leq 40$$

$$f \leq 8 \quad i \leq 20$$

Possible solutions:

front	inside
2	14
2	15
3	12
2	13
1	14
1	15
1	16
1	17

121. The Simons have a minivan and a sedan. The sedan emits 0.7 pounds of carbon dioxide ( $\text{CO}_2$ ) per mile, while the minivan emits 1.26 pounds of carbon dioxide ( $\text{CO}_2$ ) per mile. The Simons want to limit their emissions to at most 450 pounds per month.

Write an inequality modeling the possibilities for the number of miles the Simons can drive their minivan and the number of miles they can drive their sedan.

$S$  = Sedan miles

$m$  = Minivan miles

$$0.7S + 1.26m \leq 450$$