

# Algebra 8 Midterm Linear and Exponential Study Guide

## Linear Relationships

### FACTS:

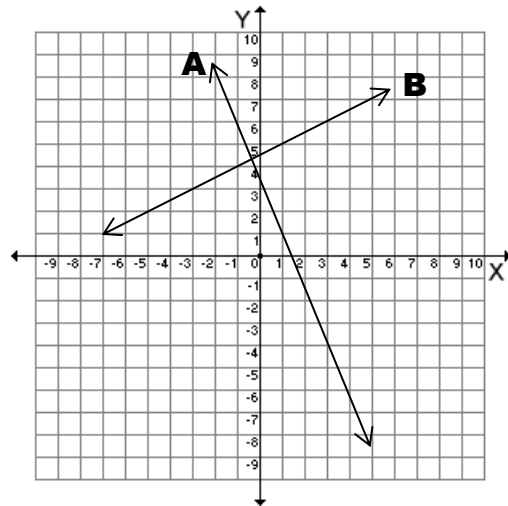
- Linear relationships are used to model real life situations.
- A relationship is linear NOT because the "line is straight" but because **as the x-value changes by a constant amount the y-value also changes by its own constant amount.**
- A relationship is linear if the **slope** between any two points is constant.
- Every point on a line is a SOLUTION for the relationship.
- Linear relationships can be written in Slope-Intercept Form:  
 $y = mx + b$  where  $m$  = slope and  $b$  = y-intercept.

### How to calculate slope:

#### Using a graph

1. Choose two points on the line. Make sure you pick places where the line crosses easy to read places on the grid.
2. Start with the left point.
3. Since the formula is  $m = \text{rise/run}$ , we do the "rise" first since it is in the numerator (the top part of the fraction). You either rise up or rise down. SO, count how many spaces it takes to up (or down) to get to the second point. **(Make sure to check your scale!)**
4. If you "rise up", the number is positive; if you "rise down", the number is negative.
5. Now, you ALWAYS "run over"! If you follow the procedure correctly, you will always run over to the right. Count how many spaces it takes to go "run over" to the second point.
6. Take the numbers you have and put in in the rise/run formula. YOU NOW HAVE THE SLOPE!
7. Check to see if you have the positive or negative sign correct. If the line is uphill, it will have a positive slope. If it is downhill, it will have a negative slope.

Find the slope of lines A and B.



#### Using two points on the line

1. Using the 2 data points we want to calculate the change in the y-value going from one point to the other, and the change in the x-value going from one point to the other.  
$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_2}{x_1 - x_2}$$
2. MAKE SURE YOU KEEP YOUR COORDINATE PAIRS IN THE RIGHT ORDER!
3. You can also put your points in a mini table if that makes finding the change in x and y easier.

Find the slope of the line connecting (5,9) and (7,4)

Slope = \_\_\_\_\_ = \_\_\_\_\_

OR

x	y

### Using a table of data

1. Determine the change in x and the change in y for all the values in the table.
2. Calculate slope using:  

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$
3. **NOTE:** Don't just assume that the change in x is always 1!

x	4	5	6	7
y	15	9	3	-3

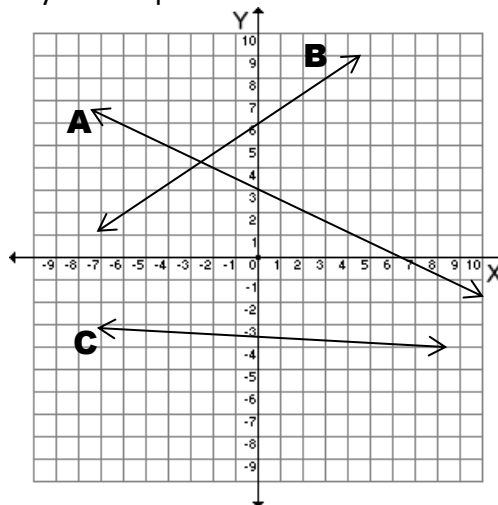
x	3	5	7	11
y	2	8	14	26

### How to find the y-intercept (the value of y when x=0):

#### Using a graph

- This is the point where the line crosses the y-axis
- The y-intercept is written as a coordinate pair ( 0, y )

Find the y-intercepts:



#### From an Equation

- Slope-Intercept Form:  

$$y = mx + b$$
- The "b" in the equation is the y-intercept

What is the y-intercept for the graphs of the following equations?

$$y = -3x + 9$$

$$y = 2x - 7$$

#### Using a table of data

- Look on the table for where **x=0** and find the y-value.
- If the point where x=0 is not on the table, you can follow the pattern of changes in x and y to find the point.

x	0	2	4	6
y	15	9	3	-3

x	6	9	12	15
y	10	12	14	16

## How to find the y-intercept (cont.):

### Using a table of data (continued)

- If the x values in the table are far from 0, and it would take a long time to “count back” to where  $x=0$ , you can find the y-intercept algebraically.
- You need to find the slope of the line from the data table and use a data point

Example:

x	48	51	54	57
y	72	66	60	54

$$\text{Slope} = \frac{-6}{3} = -2 \quad \text{Data point: } (48, 72)$$

Use the basic slope intercept form of a linear equation, and substitute in the slope from the table.

$$y = -2x + b$$

Substitute in a data point from the table (make sure you put the x and y values in the right place!) and now solve for “b” the y-intercept.

$$y = -2x + b$$

$$72 = -2(48) + b$$

$$72 = -96 + b$$

$$\begin{array}{r} +96 \quad +96 \\ \hline 168 = b \end{array}$$

y-intercept: ( 0, 168)

Find the y-intercept for each of the linear relationships represented in the tables below.

x	125	127	129	131
y	30	34	36	38

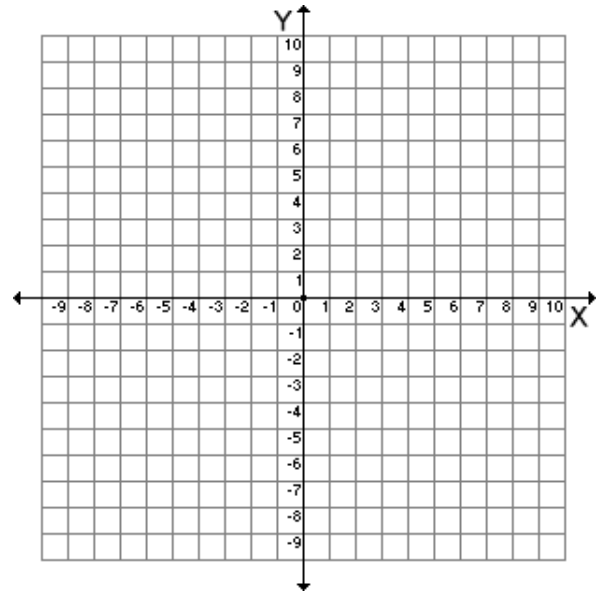
x	308	309	310	311
y	1000	995	990	985

## Ways to graph:

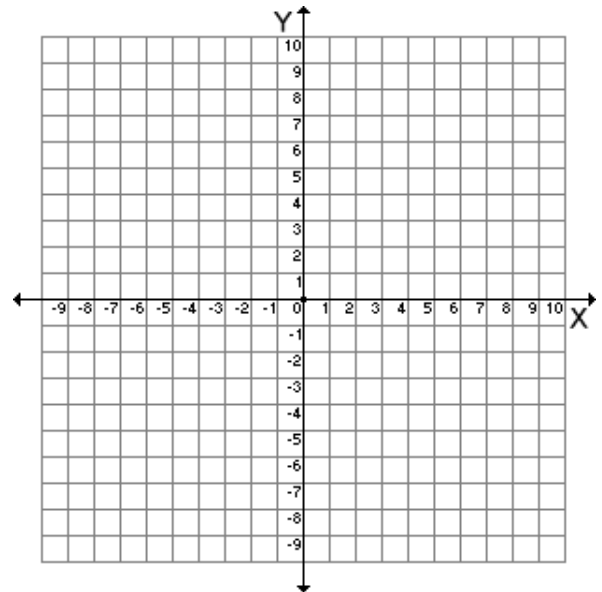
### Using Slope-Intercept form

- Plot a point at the y-intercept
- To find the next point on the line use the slope (rise/run)
- Plot the next point and draw the line (it should extend beyond the points you have plotted).

$$y = -3x + 1$$



$$y = \frac{1}{2}x - 2$$

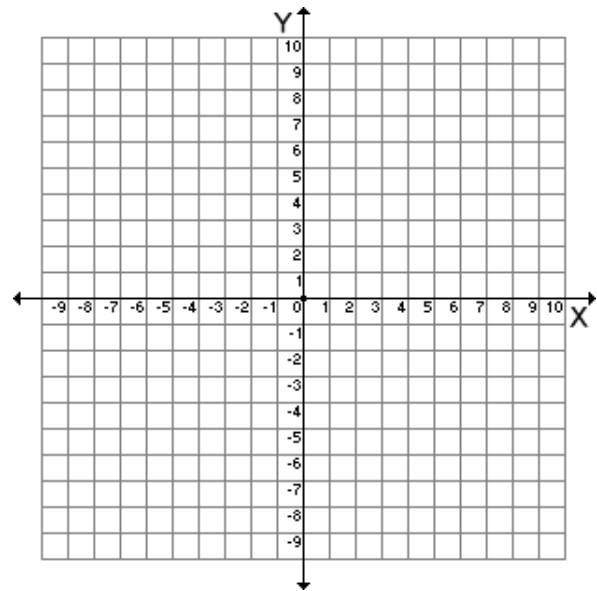


### Plotting Select Points

- Substitute select values for  $x$  into the equation and solve for  $y$ . (Remember to choose values of  $x$  that are easy to manipulate.)
- Plot these points.
- Draw the line (it should extend beyond the points you have plotted).

$x$	$\frac{3}{4}x + 2$	$y$	$(x, y)$
0			
4			
8			

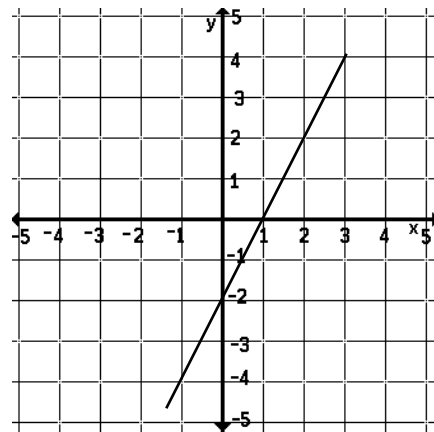
$$y = \frac{3}{4}x + 2$$



### How to write an equation (all you need to know is the slope and the y-intercept!):

#### From a graph

- Using a graph, you know how to find the slope and the y-intercept.
- Find the slope ( $m$ ) and y-intercept ( $b$ ) and plug the values into the Slope-Intercept equation  
 $y = mx + b$



#### From a data table

- Find the slope using the method mentioned above.
- Find the y-intercept using the method mentioned above.
- Substitute the values for slope ( $m$ ) and y-intercept ( $b$ ) into the Slope-Intercept equation  
 $y = mx + b$

$x$	22	26	32	36
$y$	23	29	38	44

**If given the slope and one point on the line**

- Say you were given the point (6,4) and the slope  $m=3$
- You know the equation for a line is  
 $y = mx + b$
- You already know the slope, so you can fill that in immediately.  
 $y = 3x + b$
- In order to solve for  $b$  we will use the point. The point (6,4) tells us that using the equation, when the x-value is 6, the y-value must = 4.
- Substitute in 6 for  $x$  and 4 for  $y$ , and solve for  $b$ .  
 $y = 3x + b$   
 $(4) = 3(6) + b$   
 $4 = 18 + b$   
 $-14 = b$
- Substitute this value for  $b$  back into the equation.
- The equation of the line is:  $y = 3x - 14$

Find the equation of the line with slope = -2 that contains the point (6,2)

**If given 2 points on the line**

- This uses the exact same method you would use if finding the equation given the slope and one point on the line, except **you have to find the slope first.**
- Say you were given the points (3,7) and (5,13)
- Find the slope:  $slope = \frac{7-13}{3-5} = \frac{-6}{-2} = 3$
- You know the equation for a line is  
 $y = mx + b$
- You know the slope is 3, so you can fill that in immediately.  
 $y = 3x + b$
- In order to solve for  $b$  we can use either point since both of them fall on the line.
- Substitute in 3 for  $x$  and 7 for  $y$ , and solve for  $b$ .  
 $y = 3x + b$   
 $(7) = 3(3) + b$   
 $7 = 9 + b$   
 $-2 = b$
- Substitute this value for  $b$  back into the equation.
- The equation of the line is:  $y = 3x - 2$

Find the equation of the line that passes through the points (10,4) and (8,-12)

## Solving Linear Equations

### Linear Relationships

- 1) Decide which side of the equal sign you want the variable to be on.
- 2) Expand and combine like terms if needed.
- 3) Get all of the variables on one side of the equation (by adding or subtracting one of the variables). It doesn't really matter which side, but solving is easier if you end up with a positive coefficient in front of the variable.
- 4) Eliminate the constant. Get all of the constants (numbers) on the other side of the equation (by adding or subtracting).
- 5) Divide both sides by the coefficient in front of the variable, and make sure you give a reduced answer.
- 6) Check your solution.

Solve for x:  $3(2x-5) = 9$

Solve for x:  $\frac{1}{4}x - 2 = 7$

Solve for x:  $4(2x + 3) = -3(x - 1) + 31$

## Exponential Relationships

### FACTS:

- Exponential relationships are used to model real life situations, most often population growth and investments, or blood drug level and radioactive decay.
- Every point on a line is a **SOLUTION** for the relationship.
- Exponential relationships are written:  
 $y = bg^x$  where  $b$  = y-intercept and  $g$  is the growth factor.

### How to recognize an exponential relationship:

#### From the equation: $y = bg^x$

- There is an exponent in the equation, and **the exponent is a variable**.
- $b$  represents the y-intercept. It is the initial value, which is the value of  $y$  when  $x = 0$ .
- $g$  is the growth factor.

Identify the y-intercept and growth factor in the following equations.

$$y = 7(3)^x$$

$$y = (.25)^x$$

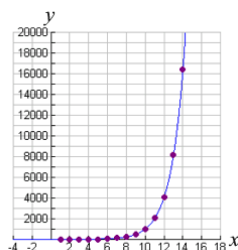
#### In words:

- The change in the y-value is a percent increase or decrease
- With an increase in  $x$ , the y-value is **doubled, tripled, halved, etc...**

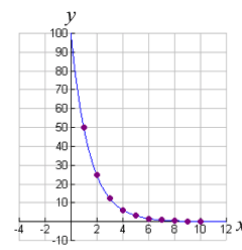
#### From a graph:

- Curved graph. If it is increasing it will start out slowly and then each increase will be greater than the one before.
- Curved graph. If it is decreasing it will start out quickly and then each decrease will be less than the one before.

#### Increasing – Growth



#### Decreasing - Decay



#### From a table:

- As the x-value changes by a constant amount, the y-value is repeatedly multiplied by the growth factor.

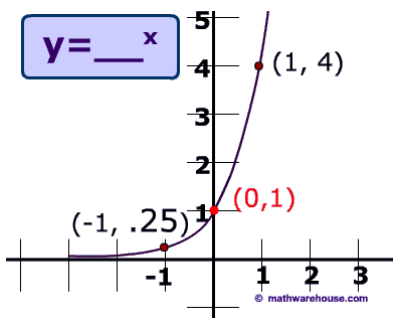
		+1	+1	+1
		^	^	^
x	0	1	2	3
y	6	12	24	48
	v	v	v	
	x2	x2	x2	

Are the following exponential relationships?

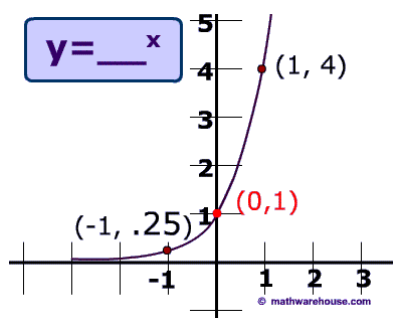
x	0	1	2	3
y	15	30	60	120

x	3	4	5	6
y	6	18	54	162

## How to find the growth factor

<b>Using the equation</b> <ul style="list-style-type: none"> <li><math>y = bg^x</math></li> <li>The “<math>g</math>” in the equation is the growth factor</li> </ul>																									
<b>From a table of data</b> <ul style="list-style-type: none"> <li>Always divide a <math>y</math>-value by the <math>y</math>-value that comes <b>before</b> it in the table. (Don’t forget to check the pattern of your <math>x</math>-values before you determine the factor!) We call this <b>Dividing Up The Table</b>.</li> </ul>	<table> <tr><th><math>x</math></th><th><math>y</math></th></tr> <tr><td>0</td><td>60</td></tr> <tr><td>1</td><td>72</td></tr> <tr><td>2</td><td>86.4</td></tr> <tr><td>3</td><td>103.68</td></tr> <tr><td>4</td><td>124.42</td></tr> </table> <table> <tr><th><math>x</math></th><th><math>y</math></th></tr> <tr><td>0</td><td>60</td></tr> <tr><td>1</td><td>48</td></tr> <tr><td>2</td><td>38.4</td></tr> <tr><td>3</td><td>30.72</td></tr> <tr><td>4</td><td>24.58</td></tr> </table>	$x$	$y$	0	60	1	72	2	86.4	3	103.68	4	124.42	$x$	$y$	0	60	1	48	2	38.4	3	30.72	4	24.58
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4	24.58																								
<b>From a graph</b> <ul style="list-style-type: none"> <li>Find the coordinates of points on the curve.</li> <li>Use <math>x</math>-values that increase by 1.</li> <li>Make a table of these points like above, and find the growth factor.</li> </ul>																									

## How to find the y-intercept (the value of $y$ when $x=0$ ):

<p><b>Using a graph</b></p> <ul style="list-style-type: none"><li>This is the point where the line crosses the y-axis</li></ul>																					
<p><b>Using a table of data</b></p> <ul style="list-style-type: none"><li>Look on the table for where <b>x=0</b> and find the y-value.</li><li>If the point where x=0 is not on the table, you can follow the pattern of changes in x and y to find the point. <b>Remember</b> though the x-values may be increasing by a constant number, the y-values are changing by a <b>factor</b> (being multiplied by a constant amount).</li></ul>	<p>Find the y-intercepts:</p> <table border="1"><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>y</td><td>6</td><td>12</td><td>24</td><td>48</td></tr></table> <table border="1"><tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>48</td><td>192</td><td>768</td><td>3072</td></tr></table>	x	0	1	2	3	y	6	12	24	48	x	2	3	4	5	y	48	192	768	3072
x	0	1	2	3																	
y	6	12	24	48																	
x	2	3	4	5																	
y	48	192	768	3072																	

## Growth/Decay Factor vs Rate

<b>Factor</b> <ul style="list-style-type: none"><li>• The factor is the number we multiply one y-value by to get the next y-value.</li><li>• For <b>growth</b> the factor is <math>&gt;1</math>.</li><li>• For <b>decay</b> the factor is between 0 and 1.</li><li>• <b>FACTOR</b> = <math>1 + \text{RATE}</math></li><li>• Example: If something increases by 20%, we write 20% as .2, and the <b>factor</b> is 1.2.</li><li>• Example: If something decreases by 15%, we write a decrease of 15% as -.15 the factor is .85.</li></ul>	Find the Growth <b>Rate</b> from the factors below.  Factor = 1.7  Factor = 3  Factor = 0.8  Factor = 0.21
<b>Rate</b> <ul style="list-style-type: none"><li>• The <b>rate</b> is the <b>percent change</b> we are observing as we go from one y-value to the next. Remember that the percent change can be either <b>positive or negative</b>.</li><li>• The rate is always written as a percent, and we indicate if it is an increase or decrease.</li></ul>	Write the growth factor given the following percent changes.  17% increase  200% increase  40% decrease  95% decrease

# Laws of Exponents

## Multiplying Powers with the Same Base

**Part 1: Write in expanded form:**

A.  $2^2 \cdot 2^4 = \underbrace{2 \cdot 2}_{2^2} \cdot \underbrace{2 \cdot 2 \cdot 2 \cdot 2}_{2^4} = 2^6$

B.  $x^3 \cdot x^5 \cdot x^2 = \underbrace{x \cdot x \cdot x}_{x^3} \cdot \underbrace{x \cdot x \cdot x \cdot x \cdot x}_{x^5} \cdot \underbrace{x \cdot x}_{x^2} = x^{10}$

**Property:** When multiplying powers with the same base, **add the exponents.**

**Part 2: Examples:**

A.  $y^3 \cdot y^4 = y^7$  Since the bases are the same (y), you can add the exponents:  $3+4 = 7$ .

B.  $d^2 \cdot d^3 \cdot d = d^6$  Since the bases are the same (d), you can add the exponents:  $2 + 3 + 1 = 6$ . \*\*Note:  $d = d^1$

C.  $p^2 \cdot p^4 \cdot q^3 = p^6 \cdot q^3$  You can only add the exponents of the terms with the same base ( $p^2 \cdot p^4$ ). You cannot add the exponent 3, because the base (q) is not the same!

Simplify the following:

$$x^9 \cdot x^4$$

$$2x^2 \cdot x^5$$

$$x^3y^2 \cdot x^9y^5$$

$$4x^8y^5 \cdot xy^3$$

$$5xy^7 \cdot 7x^3y^4$$

## Power of a Power Property

**Part 1: Write in expanded form:**

A.  $(2^3)^2 = \underbrace{2 \cdot 2 \cdot 2}_{2^3} \cdot \underbrace{2 \cdot 2 \cdot 2}_{2^3} = 2^6$

B.  $(a^2)^4 = \underbrace{a \cdot a}_{a^2} \cdot \underbrace{a \cdot a}_{a^2} \cdot \underbrace{a \cdot a}_{a^2} \cdot \underbrace{a \cdot a}_{a^2} = a^8$

**Property:** To find the power of a power, **multiply the exponents.**

**Part 2: Examples:**

A.  $(a^3)^5 = a^{15}$  Multiply the exponents.

B.  $(b^2)^6 = b^{12}$  Multiply the exponents.

Simplify the following:

$$(x^4)^3$$

$$(6^2)^3$$

## Power of a Product Property

$$(3xy)^2$$

Multiplication problem  $(3 \cdot x \cdot y)$  \*\*Remember when you don't see a mathematical sign between variables, it means **multiply**.

The 3, x, and y are called factors. **Factors** are the **numbers (or variables)** that you **multiply together** and the **answer** is called the **product**.

**Part 1: Write in expanded form.**

A.  $(-3a)^2 = -3 \cdot a \cdot -3 \cdot a = (-3)^2 \cdot a^2 = 9a^2$

B.  $(4b^2)^2 = 4 \cdot b^2 \cdot 4 \cdot b^2 = 16b^4$

$$\underbrace{4 \cdot 4}_{4^2 = 16} \cdot \underbrace{b^2 \cdot b^2}_{b^4} = 16b^4 \quad \text{Rewrite like terms together.}$$

**Property:** To find the power of a product, **find the power of each factor and multiply**.

**Think of it as distributing the exponent to each factor!**

**Part 2: Examples:**

A.  $(2xy)^3 = 2^3 x^3 y^3 = 8x^3y^3$   
 $\downarrow \quad \downarrow \quad \downarrow$   
 $8x^3y^3$   $2^3 = 8$ .  $x^3y^3$  cannot be combined because the bases are not the same.

B.  $(-3q^2rs)^2 = (-3)^2 \cdot (q^2)^2 \cdot r^2 \cdot s^2$   
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$   
 $9 \quad q^4 \quad r^2 \quad s^2$   
 $\therefore (-3q^2rs)^2 = 9q^4r^2s^2$  Raise each factor to the second power.

C.  $(-3x^3y^2z)^3 = (-3)^3 (x^3)^3 (y^2)^3 (z)^3$   
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$   
 $-27 \quad x^9 \quad y^6 \quad z^3$   
 $(-3x^3y^2z)^3 = -27x^9y^6z^3$  Raise each factor to the third power.

$$(2x^5)^3$$

$$(4x^6y^7)^2$$

## Dividing Monomials

Examples:

Original Problem	Expanded Form	Answer
$\frac{x^2}{x} =$	$\frac{\cancel{x} \cdot \cancel{x}}{\cancel{x}} = x$	$x$
$\frac{y^4}{y^2} =$	$\frac{\cancel{y} \cdot \cancel{y} \cdot y \cdot y}{\cancel{y} \cdot \cancel{y}} =$	$y^2$
$\frac{r^6}{r^2} =$	$\frac{\cancel{r} \cdot \cancel{r} \cdot r \cdot r \cdot r \cdot r}{\cancel{r} \cdot \cancel{r}} =$	$r^4$

- When you divide powers that have the same base, you subtract the exponents.
- When you have numbers as well as variables, you **MUST** divide the numbers, and then treat exponents as discussed above.
- Final answers need positive exponents.

Simplify:

$$\frac{6x^5y^3}{2x^3y}$$

$$\frac{4x^7y^2}{20x^4y^3}$$

# Scientific Notation

Scientific notation must always be written with the same components as the following model:

$$1.5876 \times 10^6$$

A number in the ones' place.

decimal

x10<sup>?</sup> (Any positive or negative exponent)

As many numbers as necessary after the decimal

Proper format:

Numeral	Is it written in scientific notation?	Why
19.5625 x 10 <sup>3</sup>	No	19.5625 x 10 <sup>3</sup> There are two places before the decimal – there should only be one.
837 x 10 <sup>2</sup>	No	837 x 10 <sup>2</sup> There is no decimal.
5.8938 x 5 <sup>2</sup>	No	5.8938 x 5 <sup>2</sup> Must be multiplied by 10
2.894 x 10 <sup>-3</sup>	YES	2.894 x 10 <sup>-3</sup> Numerical written correctly with decimal Multiplied by 10 to a power.

## Putting Numbers into SN Form

$$8,790,000,000 = 8.79 \times 10^9$$

Step 1: Take out the commas and put in a decimal after 8.

$$8.790000000$$

Step 2: Count how many spaces there are from the decimal to the end of the number.

$$\begin{array}{cccccccccc} 8 & . & 7 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & \end{array}$$

Step 3: Your exponent is going to be 9 since you counted 9 spaces from the decimal. Write your number in scientific notation.

$$8.79 \times 10^9$$

You must include all nonzero numbers, but do not include the 0's in scientific notation.

Your exponent is 9 since you counted 9 spaces to the right.

$$.000458 = 4.58 \times 10^{-4}$$

Step 1: You are going to be moving the decimal point to the right, but think about where you want to place it! Behind the first nonzero number which is 4.

$$.000458$$

You want the decimal to be here!

Step 2: Move the decimal point to the spot behind the first nonzero number! Count the number of spaces.

$$\begin{array}{cccccc} . & 0 & 0 & 0 & 4 & 58 \\ 1 & 2 & 3 & 4 & & \end{array}$$

Stop here!

Step 3: Your exponent is going to be negative 4 since you counted 4 spaces to the right! Write the number in scientific notation.

$$4.58 \times 10^{-4}$$

Write the following numbers in proper Scientific Notation Form:

4,507,321

181

0.0000829

107.23

0.02005

### Putting Numbers into Standard Form

- When you multiply a number by a positive power of 10, you can simply move the decimal point to the right the same number of spaces as the exponent

$$1.4958 \times 10^6 = 1.4958 \overbrace{000000}^{123456} = 1,495,800$$

- Step 1: Move the decimal point 6 spaces to the right since the power of 10 is 6.

$$1.4958 \overbrace{000000}^{123456}$$

- Step 2: Add 2 zeros since you have 2 extra spaces.

$$1.4958 \overbrace{000000}^{123456}$$

- $1.4958 \times 10^6$  is equal to 1,495,800.

- If you have a negative power of 10 you must move the decimal point to the left. This will be a small number.

$$8.2 \times 10^{-7} = 0.00000082 = .00000082$$

- Step 1: Move the decimal point 7 spaces to the left since the power of 10 is negative 7.

$$0.00000082$$

- Step 2: Add 6 zeros since you have 6 extra spaces.

$$0.00000082$$

- $8.2 \times 10^{-7} = .00000082$

Write the following numbers in standard form.

$$2 \times 10^5$$

$$6.025 \times 10^7$$

$$6 \times 10^{-4}$$

$$3.514 \times 10^{-2}$$