

CRITICAL THINKING about the Lesson

1. True or False? The quadratic formula states that the solutions of the equation $ax^2 + bx + c = 0$ are "the opposite of b , plus or minus the square root of b minus $4ac$, all divided by $2a$." **False**
2. Describe the two models for vertical motion. **See top of page 474.**
3. State the values of a , b , and c from the standard form of the equation $5 = 6 + 9x - x^2$. **$a = -1$, $b = 9$, $c = 1$**
4. Solve $x^2 + x - 2 = 0$. **$1, -2$**
5. Sketch the graph of $y = x^2 + x - 2$ and label the x -intercepts. **See Additional Answers.**
6. Describe the relationship between the x -intercepts found in Exercise 5 and the solutions found in Exercise 4. **They are the same.**

Independent Practice

In Exercises 7–10, write the quadratic equation in standard form.

7. $-3x^2 + 5x = 9$ **$-3x^2 + 5x - 9 = 0$**
8. $5 - 2x + x^2 = 0$ **$x^2 - 2x + 5 = 0$**
9. $-4 + 3x + x^2 = 5$ **$x^2 + 3x - 9 = 0$**
10. $9x - 7x^2 = 16$ **$-7x^2 + 9x - 16 = 0$**

In Exercises 11–14, find the value of $b^2 - 4ac$ for the equation.

11. $2x^2 - 3x - 1 = 0$ **17**
12. $4x^2 + 4x + 1 = 0$ **0**
13. $3x^2 - 2x - 5 = 0$ **64**
14. $x^2 - 11x + 30 = 0$ **1**

Exercises 11–14, find the value of $b^2 - 4ac$ for the equation.

11. $2x^2 - 3x - 1 = 0$ 17

12. $4x^2 + 4x + 1 = 0$ 0

13. $3x^2 - 2x - 5 = 0$ 64

14. $x^2 - 11x + 30 = 0$ 1

18. $3 + \sqrt{2} \approx 4.41, 3 - \sqrt{2} \approx 1.59$

In Exercises 15–20, use the quadratic formula to solve the equation.

15. $4x^2 - 13x + 3 = 0$ $3, \frac{1}{4}$

16. $3y^2 + 11y + 10 = 0$ $-\frac{5}{3}, -2$

17. $2x^2 + 7x + 3 = 0$ $-\frac{1}{2}, -3$

18. $x^2 - 6x + 7 = 0$

19. $5y^2 + 2y - 2 = 0$

20. $2x^2 + 4x - 3 = 0$

19.–20. See below.

21. $\frac{-10 + \sqrt{70}}{6} \approx -0.27$

$\frac{-10 - \sqrt{70}}{6} \approx -3.06$

In Exercises 21–26, solve the quadratic equation by the most convenient method (finding square roots or the quadratic formula). Explain why you chose your method.

21. $6x^2 + 20x + 5 = 0$

22. $t^2 = 27$

23. $x^2 - 625 = 0$ 25, -25

24. $4u^2 - 49 = 0$ $\frac{7}{2}, -\frac{7}{2}$

25. $-2x^2 + 6x + 1 = 0$

26. $x^2 + 14x + 49 = 0$ -7

In Exercises 27–32, find the x-intercepts of the graph of the equation.

27. $y = x^2 + 2x + 15$ None

28. $y = x^2 - 6x - 7$ 7, -1

29. $y = x^2 + x - 20$ 4, -5

30. $y = x^2 + 8x + 12$ -2, -6

31. $y = x^2 + x - \frac{3}{4}$ $\frac{1}{2}, -\frac{3}{2}$

32. $y = x^2 + \frac{7}{3}x - 2$ $\frac{2}{3}, -3$

25. $\frac{3 + \sqrt{11}}{2} \approx 3.16, \frac{3 - \sqrt{11}}{2} \approx -0.16$

9.4 • The Quadratic Formula 475

19. $\frac{-1 + \sqrt{11}}{5} \approx 0.46; \frac{-1 - \sqrt{11}}{5} \approx -0.86$ 20. $\frac{-2 + \sqrt{10}}{2} \approx 0.58; \frac{-2 - \sqrt{10}}{2} \approx -2.58$

2. Describe the two models for vertical motion.
 3. State the values of a , h , and v for each model.

$$h = -16t^2 + s \quad \text{drop object}$$

$$h = -16t^2 + vt + s \quad \text{throwing object}$$

$$16. 3y^2 + 11y + 10 = 0$$

$$a = 3$$

$$b = 11$$

$$c = 10$$

$$x = \frac{-(11) \pm \sqrt{(11)^2 - 4(3)(10)}}{2(3)}$$

$$x = \frac{-11 \pm \sqrt{121 - 120}}{6}$$

$$x = \frac{-11 \pm \sqrt{1}}{6} \quad \frac{-11+1}{6} = \frac{-10}{6} = -\frac{5}{3}$$

$$x = \frac{-11 \pm 1}{6} \quad \frac{-11-1}{6} = \frac{-12}{6} = -2$$

$$18. x^2 - 6x + 7 = 0$$

$$a=1 \quad b=-6 \quad c=7 \quad x = \frac{(6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 28}}{2}$$

$$\therefore x = \frac{6 \pm \sqrt{8}}{2}$$

$$x = \frac{6 + \sqrt{8}}{2} \quad \text{or} \quad x = \frac{6 - \sqrt{8}}{2}$$

$$x \approx 4.41$$

$$x \approx 1.59$$

$$18. x^2 - 6x + 7 = 0$$

Complete square

$$\begin{aligned} \Rightarrow x^2 - 6x &= -7 \\ &\quad +9 \quad +9 \\ \hline x^2 - 6x + 9 &= 2 \\ \sqrt{(x-3)^2} &\quad \sqrt{2} \\ x-3 &= \pm\sqrt{2} \\ +3 \quad +3 & \\ \hline x &= 3 \pm \sqrt{2} \end{aligned}$$

20. $2x^2 + 4x - 3 = 0$

$a=2$

$b=4$

$c=-3$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{-4 \pm \sqrt{16+24}}{4}$$

$$x = \frac{-4 \pm \sqrt{40}}{4}$$

$$x = \frac{-4 + \sqrt{40}}{4} \text{ or}$$

$$x = \frac{-4 - \sqrt{40}}{4}$$

≈ 0.58

≈ -2.58

Solving a quadratic equation by using factoring and the zero-product property

$$0 = -8x^2 + 24x$$

$$0 = 8x(-x + 3)$$

$$\begin{array}{l} \text{Zero-product} \\ \text{Property /} \\ \text{or} \end{array}$$

$$\begin{array}{r} 8x = 0 \\ \hline 8 \quad 8 \\ \hline x = 0 \end{array}$$

$$\begin{array}{r} -x + 3 = 0 \\ +x \quad +x \\ \hline 3 = 3 \end{array}$$

roots are 0 or 3

$$\begin{aligned}
 x &= \frac{6 + \sqrt{8}}{2} = \frac{6}{2} + \frac{\sqrt{8}}{2} \\
 &= \frac{6 + \sqrt{4} \sqrt{2}}{2} = 3 + \frac{\sqrt{4} \sqrt{2}}{2} \\
 &= 3 + \frac{\cancel{2} \sqrt{2}}{\cancel{2}} \\
 &= 3 + \sqrt{2} \\
 &= \frac{6 + 2\sqrt{2}}{2} \\
 &= \frac{2(3 + \sqrt{2})}{2} \\
 &= 3 + \sqrt{2}
 \end{aligned}$$

$$\frac{3 + 12}{3} \neq 12$$

$$\frac{\cancel{3}(1+4)}{\cancel{3}} = 5$$

Square root method
 $ax^2 - c = 0$

$$4x^2 - 36 = 0$$

$$\frac{+36 \quad +36}{\quad}$$

$$\frac{4x^2}{4} = \frac{36}{4}$$

$$\sqrt{x^2} = \sqrt{9}$$

$$x = \pm 3$$

Solve each equation below and find your answer in the code. Each time the solution appears, write the letter of that exercise above it.

ⓐ $8u = 3u + 35$ $7u = 7$

Ⓝ $7y = 33 - 4y$ $3y = 3$

ⓔ $2x + 48 = 10x$ $6x = 6$

Ⓣ $5t - 26 = 18t$ $t = -2$

Ⓢ $k = 8k + 28$ $k = -4$

ⓖ $-30n = -27n - 63$ $n = 21$

ⓗ $4x + 4 = 2x + 36$ $x = 16$

ⓓ $9y - 1 = y - 25$ $y = -3$

Ⓟ $14p - 8 = 22 + 20p$ $p = -5$

Ⓛ $z + 81 = 9z - 7$ $z = 11$

Ⓨ $39 - 12w = 7 - 16w$ $w = -8$

Ⓒ $-15v - 40 = 23 - 8v$ $v = -9$

Ⓜ $63 - x = 2x + 3$ $x = 20$

Ⓤ $3n + 46 = 1 + 8n$ $n = 9$

Ⓑ $12r - 18 = 13r + 18$ $r = -36$

Ⓢ $-x - 1 = x - 21$ $x = 10$

$$\textcircled{1} \quad k = 8k + 28$$

$$\begin{array}{r} -8k - 8k \quad (SPE) \\ \hline -7k = 28 \\ \hline -7 \quad \quad -7 \quad (DPE) \\ \hline k = -4 \end{array}$$

Ⓓ $9y - 1 = y - 25$

$$\frac{-y}{-y} \quad (SPE)$$

$$8y - 1 = -25$$

$$\frac{+1}{+1} \quad (APE)$$

$$\frac{8y}{8} = \frac{-24}{8}$$

$$\frac{8}{8} \quad (DPE)$$

$$y = -3$$

$$\textcircled{P} \quad 14p - 8 = 22 + 20p$$

$$\begin{array}{r} -14p \quad -14p \quad (\text{STEP}) \\ \hline \end{array}$$

$$\begin{array}{r} -8 = 22 + 6p \\ -22 \quad -22 \quad (\text{STEP}) \\ \hline \end{array}$$

$$\begin{array}{r} -30 = 6p \\ \hline \end{array}$$

$$\begin{array}{r} -30 = 6p \\ \hline -5 = p \quad (\text{STEP}) \end{array}$$